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Landmark Writings in Western Mathematics 1640-1940

Edited by

I. Grattan-Guinness

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Landmark Writings in Western Mathematics 1640–1940

CHAPTER 0

INTRODUCTION

I. Grattan-Guinness

1 WAVES IN THE SEA

For a very long time mathematical research has been circulated as a stream of books and papers, manuscripts and letters; in ancient times scrolls and tablets prevailed, and in recent ones emails and electronic files have joined in. Most writings duly took or take their modest or perhaps overlooked place in the flow; but some have made a major impact on the branches and aspects of mathematics to which they refer, and maybe also to other branches and even disciplines not originally within their purview. This book is devoted to a substantial number of the principal writings of this kind that were published during the period 1640–1940. The order of articles is that of the appearance of the writings involved: they are cited throughout the book by article number, in the manner '§21'. The table of contents indicates the range of topics to be covered; this introduction explains the scope and limitations of the choice of writings, and the manner of their treatment.

Usually the text discussed is a book; but sometimes it is one or more papers in a journal, such as N.H. Abel on the quintic equation in 1826 (§29). Thus the more general word 'writing' is used to describe the chosen text. This book is composed of 'articles', which are divided into 'sections' (whereas some of the original writings fall into 'Sections'). The articles contain cross-references to other articles; '§21.3' refers to section 3 of article 21.

Most articles deal with one writing each; but in a few cases more than one are taken together when they handle closely related topics and were published within a short time; for example, G.W. Leibniz launching his version of the calculus in three papers between 1684 and 1693 (§4), or two books by different pairs of authors in the mid 1930s (§76). A multi-volume work is considered in total even if the spread of time is as great as that needed by P.S. Laplace to assemble his mathematical astronomy (1796–1827: §18). Normally the first edition of a book catches the attention; but the second (1799–1802) edition of Etienne Montucla's history of mathematics is taken in §21, for it covered a wider range and enjoyed much more impact than did the first edition of 40 years earlier.

2 ORGANISATION OF THE ARTICLES

Each article begins bibliographically, with the publication history of the writing(s) as far as we have been able to track it down: first publication, later reprints and/or editions where

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applicable, and (photo)reprints and translations. (In the case of modern reprints that may have been reprinted several times themselves, such as with publishers such as Dover and Chelsea, we have given only the date of the *initial* reprinting.) We exclude short extracts or parts of a writing as reproduced in general anthologies. When known, the location of the manuscript of the original writing is recorded. Finally, cross-references to related articles are listed.

In the article proper the career of the author is briefly reviewed, with especial attention given to the place of the writing in his career. Its prehistory and content are surveyed, and for writings of some length a table of contents is supplied, divided up into suitable units such as parts or chapters. Page numbers are usually given, either of the first page number of each unit or the number of pages that it occupies; unless indicated otherwise, they pertain to the original printing. Then principal features of the writing are described and discussed, and on occasion omissions of topics that one might expect to have seen handled. In a few cases a portrait of an author is included, and the title page of a writing if it includes a nice design, say, or carries an interesting motto. Some original diagrams are included.

Then comes the impact of the writing—often the most difficult part of the history to assess. When impact was made fairly quickly, authors have concentrated upon the first 30–40 years or so, including where appropriate upon the later work of its author(s); striking cases of negative influence are noted. But for several writings the reception was quite tardy—the ripples from Hermann Grassmann's *Ausdehnungslehre* (1844) took nearly 40 years to propagate before being gaining a quantity of admirers, for instance (§32)—and such facts are noted, and where possible explanations are suggested. When different parts of the writing received rather different impacts, each one is discussed *in situ*. On literary style, some authors write in the past ('Newton showed') while others adopt the historic present ('Newton shows').

Some writings have appeared in various printings, editions and translations: thus several page numbers are involved. Where practical, passages in the writing are cited by article or chapter numbers. Otherwise the original printing is cited unless it has become very rare; for example, George Green's book of 1828 on potential theory (§30), where the reprint in the edition of his works is cited.

Each article ends with a bibliography of relevant items, mostly historical ones but some primary ones also. Several items, sometimes all of them, are cited in the text, in the style '[Smith, 1976]'. Reprints of these items are often indicated, especially in editions of works. Other items, especially of primary literature, are usually mentioned by name in the article, and with sufficient precision for the reader to be able to track them down.

The book ends with a list of affiliations of the authors and their articles, and an index.

3 SOME PRINCIPAL LIMITATIONS

3.1 Period

The chosen period begin around 1640, when mathematics (and science in general) was beginning to show the first signs of professional employment and diffusion of information as we know it; for example, somewhat more publication than before, the founding the

Royal Society of London and the *Académie des Sciences* in Paris in the 1660s, and the launch of scientific journals such as the *Acta Eruditorum* which was Leibniz's venue in 1684 and later. Some comments need to be made about the immediate pre-history.

In research up to and during the early 17th century, geometry was Euclidean, but the range of curves and surfaces had expanded far beyond the repertoire deployed in Euclid's *Elements*. Topics included methods of determining tangents to curves or surfaces and areas enclosed by them, in methods now often called 'pre-calculus'. Partly in these connections some functions and series were developed; also various numerical methods, especially logarithms. Algebra was much concerned with properties of polynomial equations. Mechanics was a major concern, usually with different theories obtaining in the terrestrial and celestial branches (the latter including technology of machines and artefacts). Trigonometry, planar and spherical, was part of the mathematical wardrobe, especially for cartography with navigation and astronomy, though its heyday for research was largely over. Much less developed topics include probability theory and mathematical statistics, and number theory. Professional support was modest; universities were best developed in Italy. Another important type of employer was the leader of a country or state. Few mathematicians made a living from their work; for several their research was a hobby.

Major figures from the immediately preceding generation include Johannes Kepler in Germany, Galileo Galilei (died 1642) in Italy, Simon Stevin in the Netherlands, and John Napier and Thomas Harriot in Britain. They and other contemporaries and immediate forerunners come at the end of a *different* European story, which begins with the transmission of ancient and Arabic sources into Europe in the 11th and 12th centuries and the translation into Latin of most of them, and then the reliance upon manuscripts being supplemented and later overtaken by the introduction of printing from the late 15th century onwards. That story is substantially different from ours, and needs a separate book; to encompass both would require more space than is available here. The same remark could be made about the history of writings elsewhere in the world, such as in the Far East.

The publication terminus of 1940 is chosen not only because of the Second World War but also the massive size of candidate later writings. Various survey books or encyclopaedias on branches of recent mathematics can be consulted: for example, [Pier, 1994, 2000], largely for pure mathematics.

3.2 Choice of the writings

It would have been easy but rather tedious and narrow-minded to dominate the list of writings with a procession of undoubtedly major treatises on mathematical analysis, algebras, mechanics and mathematical physics. A principal purpose of this book is to exhibit the *range* and *variety of theories* within mathematics as it has developed over the period covered. Thus writings have been selected from both pure and applied mathematics, including probability and statistics, and their selection was guided by their global place as well as by their intrinsic merits: for example, that a writing was not only important but also is the only representative of its rather unusual area (such as Stanley Jevons on mathematical economics in §41).

It was decided to have *articles* on the chosen writings, and not create a much more numerous list of dictionary-like summaries. This policy made selection even more severe;

77 articles cover 89 writings from across the mathematical board of the period. Cut down from an original list of more than twice the length, several of the final choices were difficult to make, and omissions do not entail criticism. The selected ensemble offers, we hope, a reasonable characterisation of the full ensemble. If Your Favourite Writing is missing, dear reader, then we mortals have offended.

Within the full sequence of articles occur some sub-sequences of articles on writings in and around the same branch of mathematics or topic. Table 1 indicates the main such sub-sequences.

The book is not offered as a *general history* of mathematics, even for its period. For various important developments have taken place without any one writing being significant enough to have gained an article here. Among many examples, Newton's 'fluxional' version of the calculus gradually became known in manuscript form from the late 1660s onwards and then in print from 1704, but no one (or even two) versions are sufficiently significant to enter our roll; however, Colin MacLaurin's *Treatise on fluxions* (1742) is the subject of §10. Again, Karl Weierstrass's lectures at the University of Berlin cast a huge influence upon his students and their own later endeavours for nearly 30 years from the late 1850s; but none was published at that time, and the line of influence from any one of them is too tenuous to be described, or to be highlighted over those of the other lecture courses. Among branches of mathematics, numerical methods are not well represented, as they have not generated major writings in our sense; however, several methods are mentioned in some articles.

Another criterion for selection was that the impact of the writing had to be reasonably (inter)national. This required that it be written in a widely read language or soon translated into one, or at least that much of its contents became known well beyond its geographical origin. Among writings that did not meet this criterion, some Russian works have been casualties, in particular several excellent Soviet achievements.

In some cases the impact of a writing was so late that the achievement involved was acknowledged as anticipation and maybe as a general inspiration for later work but not as an active source for its prosecution. For example, Leonhard Euler's solution in 1736 of the Königsberg bridge problem was a remarkable pioneering effort in graph theory and combinatorics; but it does not seem to have led to the development, much later, of both subjects [Biggs and others, 1976], and so is not given an article here.

Another excluded source is a short statement. For example, Pierre de Fermat's conjecture in number theory, which became known rather optimistically as his 'last theorem', was posthumously published in 1679; but his few lines involved have not been taken as a writing as such.

Also excluded are all manuscripts (including letters) that were published only much later or not (yet) at all; for while the achievements in them may have been remarkable, no broad impact was made. However, manuscripts pertinent to the history of a chosen writing (such as its own manuscript, as mentioned above) are noted in some articles.

3.3 Mathematical level

In the early 1820s A.-L. Cauchy launched his version of real-variable mathematical analysis, a landmark process indeed (§25). However, the writings involved are very unusual for

Table 1. Groupings of writing by principal branches of mathematics. Often a shortened title or indicative description is used.

Geometries	**Algebras**
1649 Descartes, *Geometria* (§1)	1649 Descartes, *Geometria* (§1)
1744 Euler on curves (§12)	1826 Abel on the quintic equation (§29)
1748 Euler, *Introductio* to analysis (§13)	1844 Grassmann, *Ausdehnungslehre* (§32)
1795 Monge, *Géométrie descriptive* (§17)	1853 Hamilton, *Lectures on quaternions* (§35)
1822 Poncelet on projective geometry (§27)	1854 Boole, *Laws of thought* (§36)
1844 Grassmann, *Ausdehnungslehre* (§32)	1863 Dirichlet, *Vorlesungen über Zahlentheorie* (§37)
1847 von Staudt, *Geometrie der Lage* (§33)	1872 Klein, Erlangen programme (§42)
1867 Riemann on geometries (§39)	1895–1896 Weber, *Lehrbuch der Algebra* (§53)
1872 Klein, Erlangen programme (§42)	1897 Hilbert on algebraic number fields (§54)
1899 Hilbert, *Grundlagen der Geometrie* (§55)	1930–1931 van der Waerden, *Moderne Algebra* (§70)
1905–34 Enriques and Chisini on algebraic geometry (§62)	
	Number theory
Calculus	1801 Gauss, *Disquisitiones arithmeticae* (§22)
1684–93 Leibniz, first papers on the calculus (§4)	1863 Dirichlet *Vorlesungen über Zahlentheorie* (§37)
1734 Berkeley, *The analyst* (§8)	1897 Hilbert on algebraic number fields (§54)
1742 MacLaurin, *Treatise on fluxions* (§10)	1919–1923 Dickson, *Number theory* (§65)
1744 Euler on curves (§12)	
1755 Euler, *Differentialis* (§14)	**Real and complex analysis**
1797 Lagrange, *Fonctions analytiques* (§19)	1823 Cauchy, *Résumé* of the calculus (§25)
1797–1800 Lacroix, *Traité du calcul* (§20)	1825, 1827 Cauchy, two main writings on complex analysis (§28)
	1851 Riemann on complex analysis (§34)
Functions, series, differential equations	1867 Riemann on trigonometric series (§38)
1656 Wallis, *Arithmetica infinitorum* (§2)	1904 Lebesgue, *Intégration* (§59)
1748 Euler, *Introductio* to analysis (§13)	1932 Bochner on Fourier integrals (§74)
1797 Lagrange, *Fonctions analytiques* (§19)	
1797–1800 Lacroix, *Traité du calcul* (§20)	**Set theory, foundations**
1799–1827 Laplace, *Mécanique céleste* (§18)	1872 Dedekind, *Stetigkeit und Irrationalzahlen* (§43)
1821 Cauchy, *Cours d'analyse* (§25)	
1822 Fourier on heat diffusion (§26)	
1829 Jacobi, *Functionum ellipticarum* (§31)	

Table 1. (*continued*)

1905–06 Baire on discontinuous functions and Lebesgue on trigonometric series (§59)

1932 Bochner on Fourier integrals (§74)

General mechanics

1687 Newton, *Principia* (§5)

1743 d'Alembert, *Dynamique* (§11)

1788 Lagrange, *Méchanique analitique* (§16)

1867 Thomson and Tait, *Treatise on natural philosophy* (§40)

1894 Hertz, *Prinzipien der Mechanik* (§52)

Astronomy

1687 Newton, *Principia* (§5)

1788 Lagrange, *Méchanique analitique* (§16)

1799–1827 Laplace, *Mécanique céleste* (§18)

1809 Gauss, *Theoria motus* (§23)

1890 Poincaré on the three-body problem (§48)

Probability and statistics

1713 James Bernoulli, *Ars conjectandi* (§6)

1718 De Moivre, *Doctrine of chances* (§7)

1764 Bayes on probability theory (§15)

1809 Gauss, *Theoria motus* (§23)

1812–1814 Laplace, *Probabilités* (§24)

1854 Boole, *Laws of thought* (§36)

1900 Pearson on the chi-squared test (§56)

1925 Fisher, *Statistical methods* (§67)

1931 Shewhart, *Economic quality control* (§72)

1933 Kolmogorov on the foundations of probability theory (§75)

1883 Cantor, *Grundlagen* of set theory (§46)

1888 Dedekind, *Was sind Zahlen?* (§47)

1889 Peano on axioms for arithmetic (§47)

1910–1913 Whitehead and Russell, *Principia mathematica* (§61)

1931 Gödel's incompletability theorem (§71)

1934, 1939 Hilbert and Bernays, *Grundlagen der Mathematik* (§77)

History, general

1799–1802 Montucla, *Histoire des mathématiques* (§21)

1892 Rouse Ball, *Mathematical recreations* (§50)

1901 Hilbert, paper on mathematical problems (§57)

Dynamics

1673 Huygens, *Horologium* (§3)

1738 Daniel Bernoulli, *Hydrodynamica* (§9)

1890 Poincaré on the three-body problem (§48)

1893 Lyapunov, *Stability theory* (§51)

1927 Birkhoff, *Dynamical systems* (§68)

Mathematical physics

1822 Fourier on heat diffusion (§26)

1828 Green, *Electricity and magnetism* (§30)

1844 Grassmann, *Ausdehnungslehre* (§32)

1873 Maxwell, *Electricity and magnetism* (§44)

1877–1878 Rayleigh, *Theory of sound* (§45)

1892 Heaviside, *Electrical theory* (§49)

1904 Thomson, *Baltimore lectures* (§58)

1909 Lorentz on electrons (§60)

Table 1. (*continued*)

Topology	1916 Einstein on general relativity theory (§63)
1889 Poincaré on the three-body problem (§48)	1930 Dirac, *Quantum mechanics* (§69)
1923–1926 Urysohn and Brouwer on dimensions (§66)	1932 von Neumann, *Quantenmechanik* (§69)
1934 Seifert and Threlfall, *Topologie* (§76)	
1935 Alexandroff and Hopf, *Topologie* (§76)	**Social and life sciences**
	1871 Jevons, *Theory of political economy* (§41)
	1917 Wentworth Thompson, *On growth and form* (§64)
	1931 Volterra on mathematical biology (§73)

this volume in being two textbooks; they were chosen because much of their content was new—and, as is revealed in the article, the students hated them! The writings for this book have been chosen for their *research content*: the novelties that they contained and/or 'the state of the art' which they comprehensively summarised. A comparable review of textbooks at various stages of education requires a companion volume, for a massive literature of its own is involved—in terms of print-runs, often much larger than those of the writings discussed here. While historians of mathematical education will find some material of interest here, the main audience is historically sympathetic mathematicians, members of kindred disciplines, and historians of mathematics.

3.4 Journals

Since the majority of research mathematics appears in journals, then their own inaugurations constitute landmarks. However, they have not been treated here, since they embody a different kind of history. It is not well covered: Erwin Neuenschwander provides a valuable short survey in [Grattan-Guinness, 1994, art. 11.12].

3.5 Landmarks as epitaphs

The writings discussed here either launched new phases of work, or consolidated the known state of theory on a topic, of both. But theories and traditions sometimes die, or at least die down: for example for mathematics, the last fluxional textbooks in the 1810s and 1820s, the fading away of quaternions (for a long time anyway) in the early 20th century, or the final calculations of massive invariants of high order. This volume bears upon declines only when a chosen writing treats theories that were soon to be noticeably eclipsed. An arresting example is Lord Kelvin's 'Baltimore lectures' of 1884 on aspects of classical mathematical physics (§58), which were fully published in 1904, just before the emergence of Albert Einstein.

ACKNOWLEDGEMENTS

A history of this 'Great Books' type has not been attempted for mathematics before, or maybe for any science; I am indebted to Arjen Sevenster of Elsevier for suggesting that the time was ripe for a try, and to him and to Mrs. Andy Deelen for bearing the brunt of the publisher's consequences. I thank the authors not only for preparing their articles but also for helping me and each other with all sorts of details; and my editorial board for advice on the choice of writings and of authors, and for reading articles. I am indebted to Dr. Ben Garling for agreeing to let me complete the editing of the article on Cauchy on complex-variable analysis by Frank Smithies (§28), who sent me the final version on the day before he died in November 2002. Those articles written in French or German were kindly translated into English by Dr. D.L. Johnson of Nottingham University, and checked with the authors.

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Internet

Many writings in all fields are available for downloading from some websites; for example, 'gallica', maintained by the *Bibliothèque Nationale* in Paris.
A valuable site with links to many other sites is run by the British Society for the History of Mathematics at www.dcs.warwick.ac.uk/bshm/.
A site devoted to general and biographical information in the history of mathematics is maintained by St. Andrew's University at the url: http://turnbull.mcs.st-and.ac.uk/history/.
Another site with more specialized focus on mathematics in culture is at Simon Fraser University: http://www.math.sfu.ca/histmath.

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In many cases a short title or a description of the writing is given. The date is that of first publication; the first edition is involved unless otherwise indicated.