

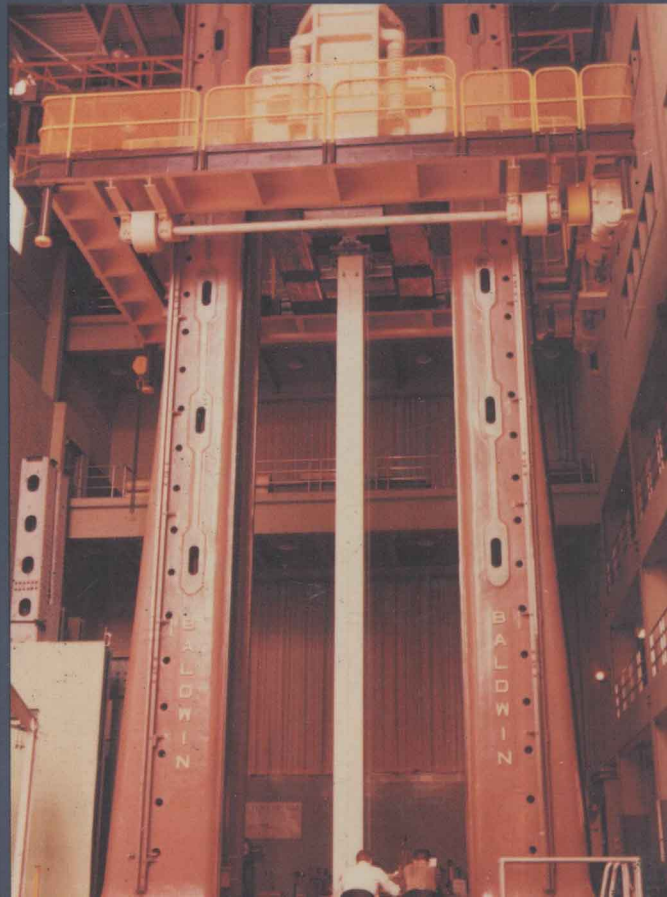
Third Edition

# MECHANICS OF MATERIALS

Ferdinand P. Beer

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Third Edition  
**MECHANICS OF MATERIALS**

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*The photograph on the cover shows a steel wide-flange column being tested in the five-million-pound universal testing machine at Lehigh University, Bethlehem, Pennsylvania. (Courtesy of Fritz Engineering Laboratory.)*

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This book is printed on acid-free paper.

International 3 4 5 6 7 8 9 0 VNH/VNH 0 9 8 7 6 5 4 3 2  
Domestic 3 4 5 6 7 8 9 0 VNH/VNH 0 9 8 7 6 5 4 3 2

ISBN 0-07-365935-5

Publisher: *Elizabeth A. Jones*  
Senior sponsoring editor: *Jonathan Plant*  
Associate editor: *Debra Matteson*  
Marketing manager: *Ann Caven*  
Project manager: *Kimberly D. Hooker*  
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Senior producer/developer, media technology: *Phillip Meek*  
Cover design: *Michael Warrell*  
Cover photo: *Courtesy of Fritz Engineering Laboratory*  
Interior design: *Maureen McCutcheon/TECHBOOKS*  
Compositor: **TECHBOOKS**/Production Manager: *E. Gail Downey*  
Typeface: *10/12 New Caledonia*  
Printer: *Von Hoffmann Press, Inc.*

## Library of Congress Cataloging-in-Publication Data

Beer, Ferdinand Pierre, 1915—  
Mechanics of materials / Ferdinand P. Beer, E. Russell Johnston, Jr., John T. DeWolf.—  
3rd ed.  
p. cm.  
ISBN 0-07-365935-5  
I. Strength of materials. I. Johnston, E. Russell (Elwood Russell), 1925— II. DeWolf, John T., 1943— III. Title.  
TA405 .B39 2002  
620.1'12—dc21

2001030518

INTERNATIONAL EDITION ISBN 0-07-112167-6

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# **MECHANICS OF MATERIALS**

## About the Authors

Ferd Beer and Russ Johnston are often asked how they happened to write their books together, with one of them at Lehigh and the other at the University of Connecticut.

The answer to this question is simple. Russ Johnston's first teaching appointment was in the Department of Civil Engineering and Mechanics at Lehigh University. There he met Ferd Beer, who had joined that department two years earlier and was in charge of the courses in mechanics. Born in France and educated in France and Switzerland (he holds an M.S. degree from the Sorbonne and an Sc.D. degree in the field of theoretical mechanics from the University of Geneva), Ferd had come to the United States after serving in the French army during the early part of World War II and had taught for four years at Williams College in the Williams-MIT joint arts and engineering program. Born in Philadelphia, Russ had obtained a B.S. degree in civil engineering from the University of Delaware and an Sc.D. degree in the field of structural engineering from MIT.

Ferd was delighted to discover that the young man who had been hired chiefly to teach graduate structural engineering courses was not only willing but eager to help him reorganize the mechanics courses. Both believed that these courses should be taught from a few basic principles and that the various concepts involved would be best understood and remembered by the students if they were presented to them in a graphic way. Together they wrote lecture notes in statics and dynamics, to which they later added problems they felt would appeal to future engineers, and soon they produced the manuscript of the first edition of *Mechanics for Engineers*. The second edition of *Mechanics for Engineers* and the first edition of *Vector Mechanics for Engineers* found Russ Johnston at Worcester Polytechnic Institute and the next editions at the University of Connecticut. In the meantime, both Ferd and Russ had assumed administrative responsibilities in their departments, and both were involved in research, consulting, and supervising graduate students—Ferd in the area of stochastic processes and random vibrations, and Russ in the area of elastic stability and structural analysis and design. However, their interest in improving the teaching of the basic mechanics courses had not subsided, and they both taught sections of these courses as they kept revising their texts and began

writing together the manuscript of the first edition of *Mechanics of Materials*.

Their collaboration has spanned the years of the revolution in computing. The first editions of *Mechanics for Engineers* and of *Vector Mechanics for Engineers* included notes on the proper use of the slide rule. With the advent of the pocket calculator, these notes were replaced by notes on the use of calculators. Now, problems requiring the use of a computer are included in each chapter of their texts.

Ferd and Russ's contributions to engineering education have earned them a number of honors and awards. They were presented with the Western Electric Fund Award for excellence in the instruction of engineering students by their respective regional sections of the American Society for Engineering Education, and they both received the Distinguished Educator Award from the Mechanics Division of the same society. In 1991 Russ received the Outstanding Civil Engineer Award from the Connecticut Section of the American Society of Civil Engineers, and in 1995 Ferd was awarded an honorary Doctor of Engineering degree by Lehigh University.

John T. DeWolf, Professor of Civil Engineering at the University of Connecticut, joined the Beer and Johnston team as a collaborator to the second edition of *Mechanics of Materials* and, starting with this third edition, is now a co-author of that text. John holds a B.S. degree in civil engineering from the University of Hawaii and M.E. and Ph.D. degrees in structural engineering from Cornell University. His research interests are in the area of elastic stability, bridge monitoring, and structural analysis and design. He is a member of the Connecticut Board of Examiners for Professional Engineers.

# PREFACE

The main objective of a basic mechanics course should be to develop in the engineering student the ability to analyze a given problem in a simple and logical manner and to apply to its solution a few fundamental and well-understood principles. This text is designed for the first course in mechanics of materials—or strength of materials—offered to engineering students in the sophomore or junior year. The authors hope that it will help instructors achieve this goal in that particular course in the same way that their other texts may have helped them in statics and dynamics.

In this text the study of the mechanics of materials is based on the understanding of a few basic concepts and on the use of simplified models. This approach makes it possible to develop all the necessary formulas in a rational and logical manner, and to clearly indicate the conditions under which they can be safely applied to the analysis and design of actual engineering structures and machine components.

Free-body diagrams are used extensively throughout the text to determine external or internal forces. The use of “picture equations” will also help the students understand the superposition of loadings and the resulting stresses and deformations.

It is expected that students using this text will have completed a course in statics. However, Chap. 1 is designed to provide them with an opportunity to review the concepts learned in that course, while shear and bending-moment diagrams are covered in detail in Secs. 5.2 and 5.3. The properties of moments and centroids of areas are described in Appendix A; this material can be used to reinforce the discussion of the determination of normal and shearing stresses in beams (Chaps. 4, 5, and 6).

The first four chapters of the text are devoted to the analysis of the stresses and of the corresponding deformations in various structural members, considering successively axial loading, torsion, and pure bending. Each analysis is based on a few basic concepts, namely, the conditions of equilibrium of the forces exerted on the member, the relations existing between stress and strain in the material, and the conditions imposed by the supports and loading of the member. The study

of each type of loading is complemented by a large number of examples, sample problems, and problems to be assigned, all designed to strengthen the students' understanding of the subject.

The concept of stress at a point is introduced in Chap. 1, where it is shown that an axial load can produce shearing stresses as well as normal stresses, depending upon the section considered. The fact that stresses depend upon the orientation of the surface on which they are computed is emphasized again in Chaps. 3 and 4 in the cases of torsion and pure bending. However, the discussion of computational techniques—such as Mohr's circle—used for the transformation of stress at a point is delayed until Chap. 7, after students have had the opportunity to solve problems involving a combination of the basic loadings and have discovered for themselves the need for such techniques.

In this new edition, the discussion in Chap. 2 of the relation between stress and strain in various materials has been expanded to include fiber-reinforced composite materials. Also, the study of beams under transverse loads is now covered in two separate chapters. Chapter 5 is devoted to the determination of the normal stresses in a beam and to the design of beams based on the allowable normal stress in the material used (Sec. 5.4). The chapter begins with a discussion of the shear and bending-moment diagrams (Secs. 5.2 and 5.3) and includes an optional section on the use of singularity functions for the determination of the shear and bending moment in a beam (Sec. 5.5). The chapter ends with an expanded optional section on nonprismatic beams (Sec. 5.6).

Chapter 6 is devoted to the determination of shearing stresses in beams and thin-walled members under transverse loadings. In this new edition, and at the suggestion of our reviewers, the formula for the shear flow,  $q = VQ/I$ , is derived in the traditional way. More advanced aspects of the design of beams, such as the determination of the principal stresses at the junction of the flange and web of a W-beam, have been moved to Chap. 8, an optional chapter that may be covered after the transformations of stresses have been discussed in Chap. 7. The design of transmission shafts has been moved to that chapter for the same reason, as well as the determination of stresses under combined loadings which, in this new position, can now include the determination of the principal stresses, principal planes, and maximum shearing stress at a given point.

Statically indeterminate problems are first discussed in Chap. 2 and considered throughout the text for the various loading conditions encountered. Thus, students are presented at an early stage with a method of solution which combines the analysis of deformations with the conventional analysis of forces used in statics. In this way, they will have become thoroughly familiar with this fundamental method by the end of the course. In addition, this approach helps the students realize that stresses themselves are statically indeterminate and can



be computed only by considering the corresponding distribution of strains.

Design concepts are discussed throughout the text whenever appropriate. A discussion of the application of the factor of safety to design can be found in Chap. 1, where the concepts of both allowable stress design and load and resistance factor design are presented.

The concept of plastic deformation is introduced in Chap. 2, where it is applied to the analysis of members under axial loading. Problems involving the plastic deformation of circular shafts and of prismatic beams are also considered in optional sections of Chaps. 3, 4, and 6. While some of this material can be omitted at the choice of the instructor, its inclusion in the body of the text will help students realize the limitations of the assumption of a linear stress-strain relation and serve to caution them against the inappropriate use of the elastic torsion and flexure formulas.

The determination of the deflection of beams is discussed in Chap. 9. The first part of the chapter is devoted to the integration method and to the method of superposition, with an optional section (Sec. 9.6) based on the use of singularity functions. (This section should be used only if Sec. 5.5 was covered earlier.) The second part of Chap. 9 is optional. It presents the moment-area method in two lessons instead of three as in our previous edition.

Chapter 10 is devoted to columns and contains new material on the design of wood columns. Chapter 11 covers energy methods, including Castigliano's theorem.

Additional topics such as residual stresses, torsion of noncircular and thin-walled members, bending of curved beams, shearing stresses in non-symmetrical members, and failure criteria, have been included in optional sections for use in courses of varying emphases. To preserve the integrity of the subject, these topics are presented in the proper sequence, wherever they logically belong. Thus, even when not covered in the course, they are highly visible and can be easily referred to by the students if needed in a later course or in engineering practice. For convenience all optional sections have been indicated by asterisks.

Each chapter begins with an introductory section setting the purpose and goals of the chapter and describing in simple terms the material to be covered and its application to the solution of engineering problems. The body of the text has been divided into units, each consisting of one or several theory sections followed by sample problems and a large number of problems to be assigned. Each unit corresponds to a well-defined topic and generally can be covered in one lesson. Each chapter ends with a review and summary of the material covered in the chapter. Notes in the margin have been included to help the students organize their review work, and cross references provided to help them find the portions of material requiring their special attention. The theory sections include many examples designed to illustrate the material being presented and facilitate its understanding. The sample problems are in-

tended to show some of the applications of the theory to the solution of engineering problems. Since they have been set up in much the same form that students will use in solving the assigned problems, the sample problems serve the double purpose of amplifying the text and demonstrating the type of neat and orderly work that students should cultivate in their own solutions. Most of the problems are of a practical nature and should appeal to engineering students. They are primarily designed, however, to illustrate the material presented in the text and help the students understand the basic principles used in mechanics of materials. The problems have been grouped according to the portions of material they illustrate and have been arranged in order of increasing difficulty. Problems requiring special attention have been indicated by asterisks. Answers to problems are given at the end of the book, except for those with a number set in italics.

Because it is essential that students be able to handle effectively both SI metric units and U.S. customary units, half the examples, sample problems, and problems to be assigned have been stated in SI units and half in U.S. customary units. Since a large number of problems are available, instructors can assign problems using each system of units in whatever proportion they find most desirable for their class. The availability of personal computers makes it possible for engineering students to solve a great number of challenging problems. In this new edition of *Mechanics of Materials*, a group of six or more problems designed to be solved with a computer can be found at the end of each chapter. Developing the algorithm required to solve a given problem will benefit the students in two different ways: (1) it will help them gain a better understanding of the mechanics principles involved; (2) it will provide them with an opportunity to apply the skills acquired in their computer programming course to the solution of a meaningful engineering problem.

The authors wish to acknowledge with gratitude the helpful comments and suggestions of the following colleagues from various universities: Candace A. Ammerman (Colorado School of Mines), Subhash C. Anand (Clemson University), Christopher M. Foley (Marquette University), Paul Heyliger (Colorado State University), D. A. Jenkins (University of Florida), Vistap M. Karbhari (University of California, San Diego), Ajit D. Kelkar (North Carolina A&T State University), Timothy C. Kennedy (Oregon State University), Robert M. Korol (McMaster University), Jack Lesko (Virginia Polytechnic Institute), Lee L. Lowery, Jr. (Texas A&M University), Ghyslaine McClure (McGill University), Amhad Pourmovahed (Kettering University), G. E. Ramey (Auburn University), Ting-Leung Sham (Rensselaer Polytechnic Institute), Stephen W. Swanson (The University of Utah), M. D. A. Thomas (University of Toronto), George Z. Voyiadjis (Louisiana State University), Douglas Winslow (Purdue University), and Manoochehr Zoghi (The University of Dayton).

Their special thanks go to two colleagues on the Lehigh University faculty: Dean Updike, of the Department of Mechanical Engineering

and Mechanics, who checked the solutions and answers of all the problems in this edition, and Le-Wu Lu, of the Department of Civil and Environmental Engineering, who provided the photograph used on the cover of this book.

*Ferdinand P. Beer*  
*E. Russell Johnston, Jr.*  
*John T. DeWolf*

# List of Symbols

$a$	Constant; distance	$P_L$	Live load (LRFD)
<b>A, B, C, . . .</b>	Forces; reactions	$P_U$	Ultimate load (LRFD)
$A, B, C, . . .$	Points	$q$	Shearing force per unit length; shear flow
$A, (f)$	Area	<b>Q</b>	Force
$b$	Distance; width	$Q$	First moment of area
$c$	Constant; distance; radius	$r$	Radius; radius of gyration
$C$	Centroid	<b>R</b>	Force; reaction
$C_1, C_2, . . .$	Constants of integration	$R$	Radius; modulus of rupture
$C_p$	Column stability factor	$s$	Length
$d$	Distance; diameter; depth	$S$	Elastic section modulus
$D$	Diameter	$t$	Thickness; distance; tangential deviation
$e$	Distance; eccentricity; dilatation	<b>T</b>	Torque
$E$	Modulus of elasticity	$T$	Temperature
$f$	Frequency; function	$u, v$	Rectangular coordinates
<b>F</b>	Force	$u$	Strain-energy density
$F.S.$	Factor of safety	$U$	Strain energy; work
$G$	Modulus of rigidity; shear modulus	<b>v</b>	Velocity
$h$	Distance; height	<b>V</b>	Shearing force
<b>H</b>	Force	$V$	Volume; shear
$H, J, K$	Points	$w$	Width; distance; load per unit length
$I, I_x, . . .$	Moment of inertia	<b>W, W</b>	Weight, load
$I_{xy}, . . .$	Product of inertia	$x, y, z$	Rectangular coordinates; distance; displacements; deflections
$J$	Polar moment of inertia	$\bar{x}, \bar{y}, \bar{z}$	Coordinates of centroid
$k$	Spring constant; shape factor; bulk modulus; constant	$Z$	Plastic section modulus
$K$	Stress concentration factor; torsional spring constant	$\alpha, \beta, \gamma$	Angles
$l$	Length; span	$\alpha$	Coefficient of thermal expansion; influence coefficient
$L$	Length; span	$\gamma$	Shearing strain; specific weight
$L_e$	Effective length	$\gamma_D$	Load factor, dead load (LRFD)
$m$	Mass	$\gamma_L$	Load factor, live load (LRFD)
<b>M</b>	Couple	$\delta$	Deformation; displacement
$M, M_x, . . .$	Bending moment	$\epsilon$	Normal strain
$M_D$	Bending moment, dead load (LRFD)	$\theta$	Angle; slope
$M_L$	Bending moment, live load (LRFD)	$\lambda$	Direction cosine
$M_U$	Bending moment, ultimate load (LRFD)	$\nu$	Poisson's ratio
$n$	Number; ratio of moduli of elasticity; normal direction	$\rho$	Radius of curvature; distance; density
$p$	Pressure	$\sigma$	Normal stress
<b>P</b>	Force; concentrated load	$\tau$	Shearing stress
$P_D$	Dead load (LRFD)	$\phi$	Angle; angle of twist; resistance factor
		$\omega$	Angular velocity

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