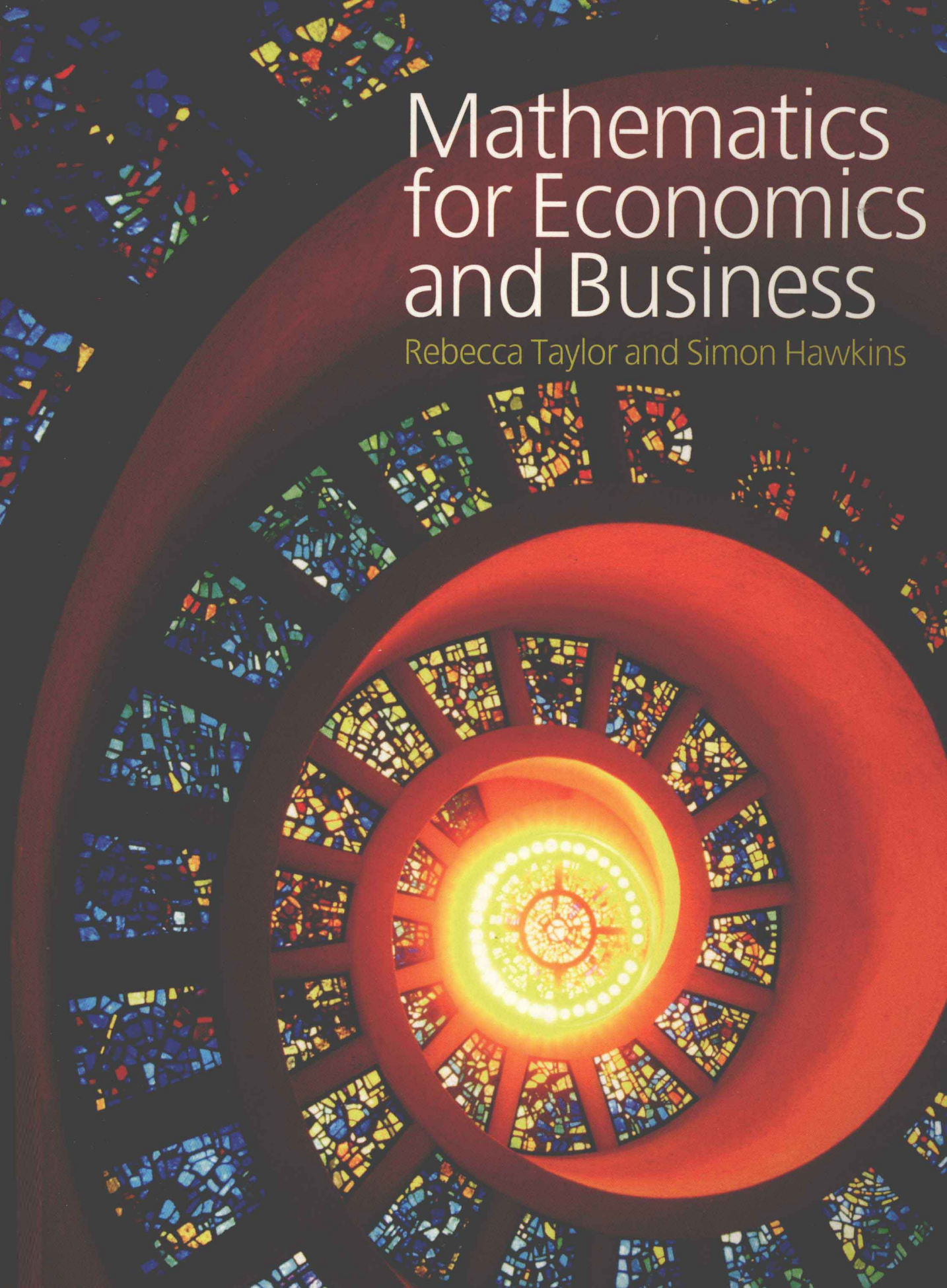
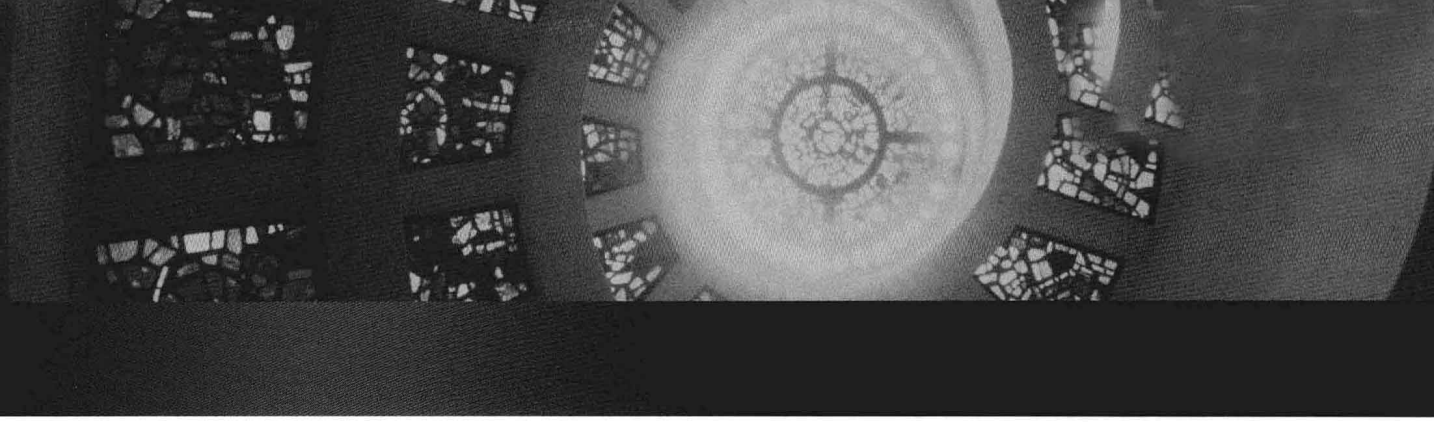


Mathematics for Economics and Business

Rebecca Taylor and Simon Hawkins



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Rebecca Taylor and
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Rebecca Taylor and Simon Hawkins

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Mathematics for Economics and Business

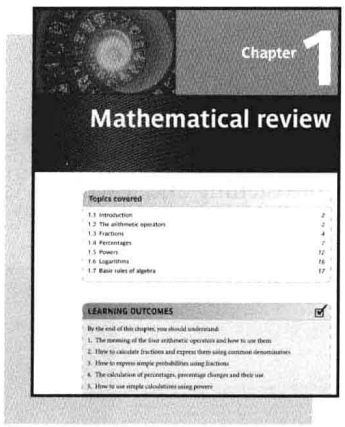
Preface

The American economist Heilbroner once said that, 'Mathematics has given economics rigour, but alas, also mortis' and that might be a surprising and bold opening to a textbook which has both 'mathematics' and 'economics' in its title. But, in a way, Heilbroner was right to imply that too often mathematics can overtake the economics leaving us with long pages of equations which are perfectly described but which we cannot relate to the world we all observe and experience. Both authors are economists and both have a passion for teaching and learning. As students, both authors have sat through hours of dreary lectures of mathematics which threw little if any light on the fascinating, dynamic and puzzling world of business and economics. It was against this backdrop that we set about creating a textbook which genuinely provided an interesting and engaging introduction to the world of business and economics *through* mathematics, and not the other way around.

The book is aimed at students who are embarking on a course of economics or business at first or second year undergraduate level and no mathematical proficiency is presumed beyond perhaps having undertaken a GCSE. The techniques we cover can be applied to a wide variety of related undergraduate modules and topics including accounting and finance, operational research, pure mathematics, statistics, marketing and geography.

Readers should feel that the mathematics here is a tool or perhaps a 'way-in' to better understand the contemporary problems in the world of business and economics, rather than an end in itself. In this way, we hope to secure the rigour but avoid any of the mortis.

Guided tour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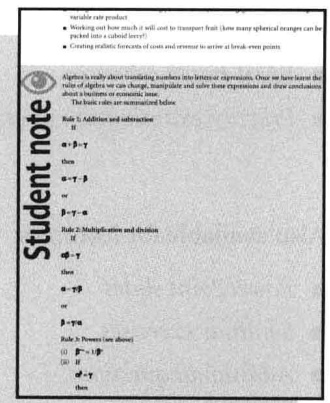
LEARNING OUTCOMES
By the end of this chapter, you should understand:
1. The meaning of the four arithmetic operators and how to use them
2. How to calculate fractions and express them using common denominators
3. How to express simple probabilities using fractions
4. The calculation of percentages, percentage changes and their use
5. How to use simple calculations using powers

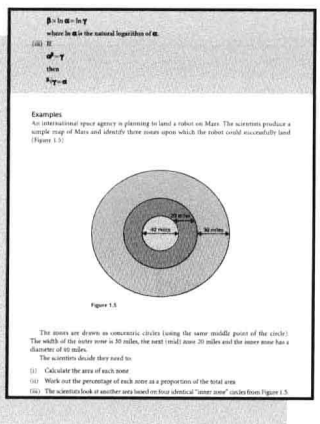
Learning outcomes

Each chapter opens with a set of learning outcomes, summarizing the mathematical knowledge, skills and understanding you should acquire from each chapter.


Student note

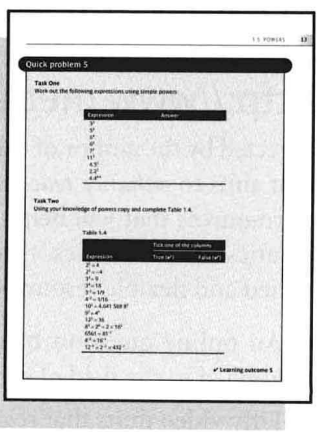
Student notes

Student notes are provided in boxes within each chapter. These are a useful quick reference tool, summarizing key terms and providing tips to help understanding.


Examples
Figure 1.5

Examples

Each chapter provides worked examples, to consolidate learning and demonstrate the mathematical principles as applied in an economic context.


Quick problem 5
Task One
Work out the following expressions using simple powers
Task Two
Using your knowledge of powers copy and complete Table 1.4.
Table 1.4
✔ Learning outcome 5

Quick problem boxes

Each chapter includes quick problem boxes to test your understanding and application of the mathematical principles taught in that chapter. Answers are provided at the end of each chapter so you can check your progress and see where you may need to refer back to the chapter to fill in any gaps in understanding.

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Online Learning Centre (OLC)

After completing each chapter, log on to the supporting Online Learning Centre website. Take advantage of the study tools offered to reinforce the material you have read in the text, and to develop your knowledge of mathematics for economics and business in a fun and effective way.

Resources for students include:

- *Self-test questions*
- *Weblinks*
- *Excel-based exercises*
- *Practice examination questions*

Also available for lecturers:

- *PowerPoint slides*
- *Seminar exercises*
- *Additional questions*
- *Teaching solutions*

Further resources to support the teaching and learning of mathematics for economics and business courses are available via the METAL project website: http://www.metalproject.co.uk

Directed by the author of this text, Dr Rebecca Taylor, METAL is a HEFCE funded FDTL5 project that aims to *enhance teaching and learning* by providing lecturers and students with a selection of resources that will help to engage Level 1 students more fully and enthusiastically in mathematics for economics, through the provision of an accessible and fully interactive toolkit of varied and flexible resources. These resources include:

- An online question bank of mathematics teaching and assessment materials specifically applied to the field of economics.
- Fifty video units that relate mathematical concepts to the field of economics.

- Ten teaching and learning guides that provide an extensive bank of teaching activities (large and small groups) covering all aspects of Level 1 Mathematics for Economics.
- Fifleen case studies for use in structions/tutorials.
- An interactive website to present the teaching and learning resources, to facilitate distance learning and to foster students' autonomy and ownership of the learning process.

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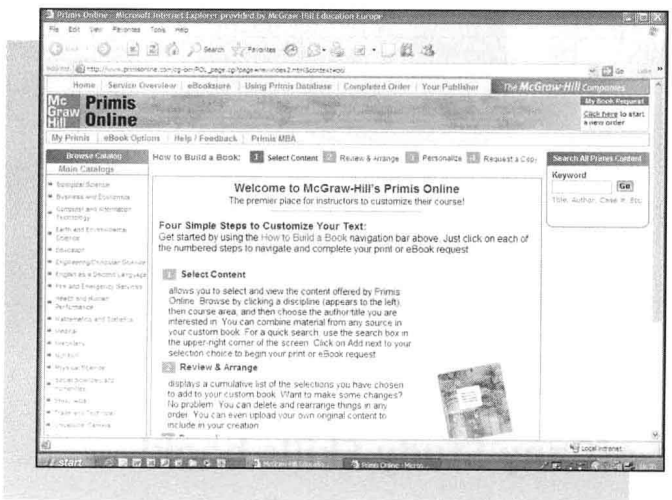

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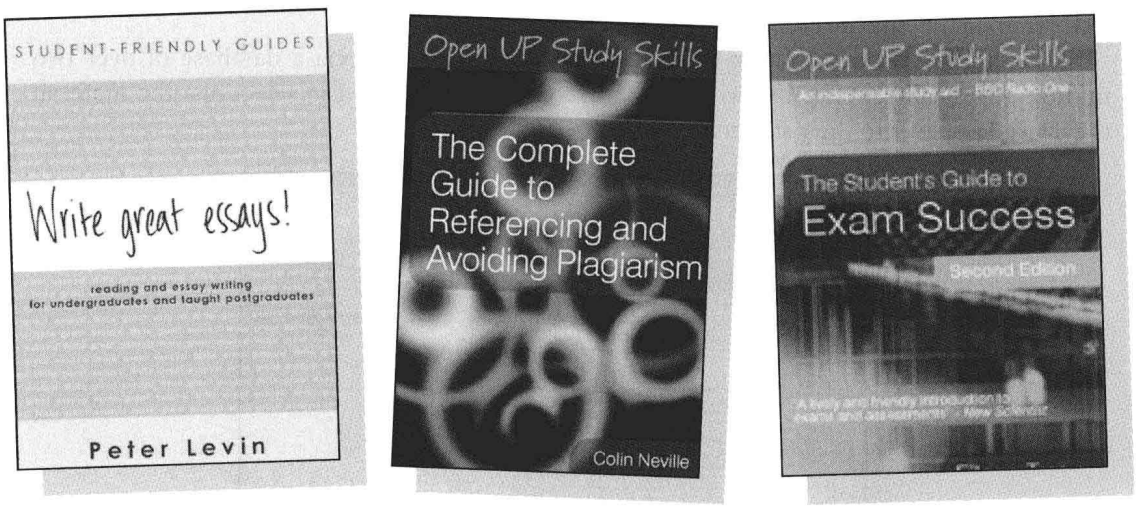

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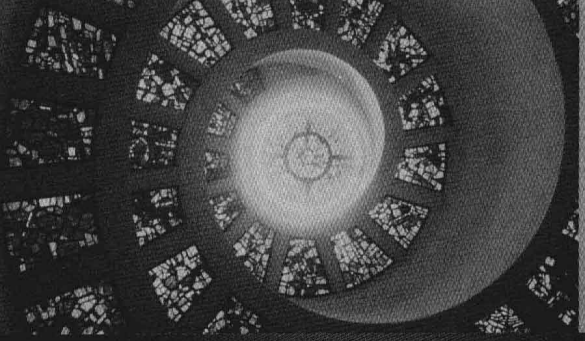
Detailed Table of Contents



Chapter 1

Mathematical review

Topics covered

LEARNING OUTCOMES

By the end of this chapter, you should understand:

1. The meaning of the four arithmetic operators and how to use them
2. How to calculate fractions and express them using common denominators
3. How to express simple probabilities using fractions
4. The calculation of percentages, percentage changes and their use
5. How to use simple calculations using powers
6. The basic rules governing powers and roots and how to rearrange and solve expressions
7. How to calculate logarithms and use logarithmic expressions
8. The rules of algebra and how to use them.

1.1 Introduction

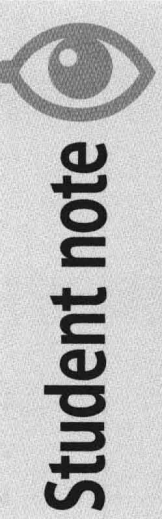
Business and economics is all around us and shapes our everyday lives. We are confronted by decisions, choices and issues which at their core are often to do with business and economics issues. How many hours do we choose to work each week? How much money are we prepared to accept for our services to an employer? Should we save some of our salary or should we spend it all?

Many of our choices are informed by mathematics, although we may not even know it.[1] The more complex or advanced the problem so we often find that the mathematics becomes more overt or explicit. The point here is that to really understand and solve business and economics issues we need to have a confident and secure understanding of the underlying mathematics. This needs to be put into a practical context: we should use mathematics to help us understand what is going on, not to confuse or dazzle people; it is a tool to illuminate and solve business and economic problems.

This first section focuses on some of the 'mathematical fundamentals' which you will need to understand business and economics. As your mathematical proficiency grows, so too will your capacity to think independently and critically about these concepts and your understanding of which concepts and methods to apply.

1.2 The arithmetic operators

'Arithmetic operators' simply refers to the nut and bolts of all basic mathematics: addition (+), subtraction (−), multiplication (×) and division (÷). You will recognize these operators and understand their function, and will probably be able to use them confidently.



You will need to be aware that different words or 'synonyms' can be used to describe the same mathematical operator.

Look at the list in Table 1.1 which summarizes some of the most common alternative words or expressions.

Table 1.1

Addition (+)	Subtraction (−)	Multiplication (×)	Division (÷)
Adding	Taking away	Times	Share
Adding-up	Minus	Product	Apportion
Summing (shown by Σ)	Deduction		Attribute
Summation	Take-off		Allocate
Totalling	Less		
Accumulating			

[1] Consider for example when a pedestrian chooses to cross a road. The pedestrian will be unaware that their decision to cross will be determined by their brain's continuous assessment of risk, taking into the likelihood of being hit by a vehicle against the need to cross the road and make good progress with their journey. It is only once these two factors have been balanced out that the pedestrian crosses the road.

Note that multiplication can also be denoted by a '·', i.e. $a \cdot b$, and simply as ab. These are equivalent to $a \times b$

Multiplication and division are closely linked: you can divide by multiplying by the reciprocal. Put another way:

$a/b = a \times (1/b) = (1/b) \times (a/1)$

Example:
You are asked to divide 7 by $^1/_4$. This could be solved by either:

$7/1 \div {}^1/_4$ or,
$7/1 \times 1/4$

Quick problem 1 gives you an opportunity to practise using the four operators.

Quick problem 1

Using your knowledge and understanding of the four operators complete each line by filling in the missing item(s) in Table 1.2. The first line has been done for you

✔ Learning outcome 1

Table 1.2

Figure	Operator (+, −, ×, ÷)	Figure	Equals	Answer
5	×	31	=	155
	+	17	=	125
	÷	57.5	=	230
−4	−		=	−10
13	÷	2	=	
16		5	=	80
	÷	4	=	169
169	+		=	150
346	÷		=	173
567		383	=	

Although the four arithmetic operators are basic mathematical tools, it is because they are so fundamental that you need to be able to use them with confidence and fluency. Simple calculations involving addition, subtraction, multiplication and division are the foundations of higher and more advanced calculations. Look at the table in the student note below to get a feel for the types of simple business calculations that you might encounter.

Many business and economics problems can be simply and routinely solved using the four arithmetic operators. The quadrants below provide brief lists of typical day-to-day calculations.

Addition Adding the incomes of a husband and wife to calculate their joint household income. Adding together the spending of government departments to calculate total government spending. Summing the tax payments of all workers in an economy to calculate the tax revenue which a government receives over a period of time.	**Subtraction** Subtracting tax and National Insurance payments from a worker's salary to calculate their net income. Subtracting inflation from a pay rise to calculate the value of the real pay award. Taking away the amount of money paid by a customer to a supplier to arrive at the final amount outstanding.
Multiplication Calculating the revenue a firm receives from the sales of a particular product by multiplying the price of the product by the quantity sold. Calculating the tax liability of a corporation by multiplying the taxable profits by the tax rate. Calculating a worker's daily wage by multiplying their hourly wage rate by the number of hours worked.	**Division** Calculating a salaried worker's hourly wage by dividing their total annual salary by the number of hours worked in a year. Calculating a company's solvency by dividing current assets by current liabilities. Calculating the dividend per share by dividing total profits by the number of shares in circulation.

1.3 Fractions

Fractions are a simple but powerful concept which can be used to express numbers as parts or elements of another number. For example the fraction '$^1/_4$' literally means 'one out of four identically sized parts'. In effect, the middle line which divides the top number from the bottom represents a division.

Fractions are usually expressed as 'proper fractions' with the top number (or numerator) smaller than the number underneath (or denominator). For example, $^1/_4$, $^1/_2$ and $^3/_4$ are all proper fractions. Sometimes, though, fractions are written or expressed in a way that can look a little odd: the numerator can be larger than the denominator. Fractions which are represented in this way are called 'improper fractions'. e.g. $^7/_3$ or $^9/_{16}$.