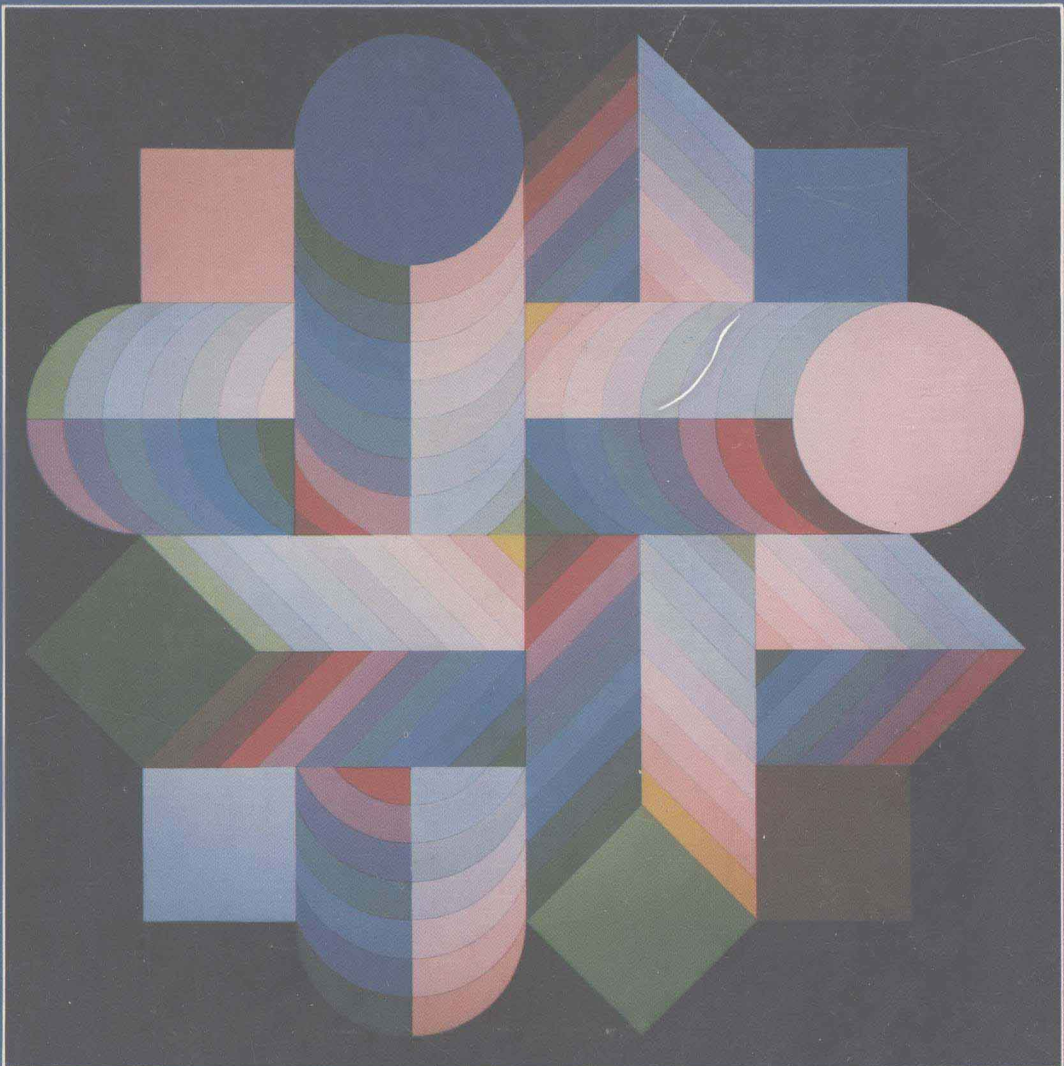
College Algebra

BERNARD KOLMAN & ARNOLD SHAPIRO



COLLEGE ALGEBRA

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To the memory of my mother Eva

B.K.

To my mother Helen

A.S.

PREFACE

This book is a complete and self-contained presentation of the fundamentals of algebra. Our objective has been *to write a textbook that will help the instructor teach the material and that the student will find readable.* Moreover, we believe that it is almost impossible to oversimplify an idea in the fundamentals of mathematics. To this end we have provided the following helpful features.

FEATURES

- **Presentation.** We have adopted an informal, supportive style to encourage the student to read the text and to develop confidence under its guidance. Concepts are introduced gradually with accompanying diagrams and illustrations that aid the student to grasp intuitively the "reasonableness" of results. Many algebraic procedures are described with the aid of a "split-screen" that simultaneously displays both the steps of an algorithm and a worked out example.

- **Warnings.** To help eliminate misconceptions and bad mathematical habits, we have inserted numerous **Warnings** (indicated by the symbol shown in the margin) that point out those incorrect practices most commonly found in homework and exam papers.
- **Progress Checks.** At carefully selected places, problems similar to those worked in the text have been inserted (with answers) to enable the student to test his or her understanding of the material just discussed.
- **End-of-Chapter.** Every chapter contains a summary that includes the following:

Terms and Symbols with appropriate page references.
Key Ideas for Review to stress the concepts.
Review Exercises to provide additional practice.
Progress Tests to provide self-evaluation and reinforcement.
The answers to all **Review Exercises** and **Progress Tests** appear in the back of the book.

- **Exercises.** Over 3,000 carefully graded exercises provide practice in both the mechanical and conceptual aspects of algebra. Exercises requiring the use of a calculator are indicated by the symbol shown in the margin. Answers to all odd-numbered exercises appear at the back of the book. Answers to all even-numbered exercises appear in the Instructor's Manual that is available to instructors upon request. The Instructor's Manual also contains an extensive Test Bank with solutions.

- **Supplement.** A Study Guide containing solutions to about one-third of the exercises as well as additional chapter tests can be purchased from the publisher at a nominal price.
- **Pedagogic Approach**
    - Section 1.1 presents an axiomatic development of the foundations of algebra. If desired, the material in this section can be omitted from classroom presentation and the student can be asked to read the section.
    - Set theory is not stressed. A brief introduction to set theory is presented in Chapter One and set notation is used throughout the book.
    - Complex numbers are defined in Chapter One and are used in Chapter Two to find the roots of *any* quadratic equation. Advanced concepts involving complex numbers are presented later in the chapter on roots of polynomials.
    - Functions and function notation are introduced in Chapter Three and are used as a unifying concept throughout the rest of the book.
    - Computations with logarithms are treated briefly in Section 4.4 and is an optional topic.
    - An entire chapter is devoted to analytic geometry and the conic sections. The use of a coordinate system to prove theorems from plane geometry is demonstrated and the equations of the conics are derived by applying the methods of analytic geometry.
    - The material on matrices includes matrix operations and matrix inverses. For a "softer" approach, Sections 2 and 3 of the chapter "Matrices and Determinants" as well as the Gauss–Jordan method can be omitted.

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Our thanks to Susan Pagano for typing much of the manuscript. We are grateful to Harry Spector for his contributions as copy editor. We know of no other copy editor who checks the worked-out examples!

TO THE STUDENT

This book was written for you. It gives you every possible chance to succeed—if you use it properly.

We would like to have you think of mathematics as a challenging game—but not as a spectator sport. Which leads to our primary rule: *Read this textbook with pencil and paper handy.* Every new idea or technique is illustrated by fully worked examples. As you read the text, carefully follow the worked-out examples and then do the **Progress Checks.** The key to success in a math course is working problems, and the **Progress Checks** are there to provide immediate practice with the material you have just learned.

Your instructor will assign homework from the extensive selection of exercises that follow each section in the book. *Do the assignments regularly, thoroughly, and independently.* By doing lots of problems you will develop the necessary skills in algebra and your confidence will grow. Since algebraic techniques and concepts build upon previous results, you can't afford to skip any of the work.

To help you eliminate improper habits and to help you avoid those errors that we see each semester as we grade papers, we have interspersed **Warnings** throughout the book. The **Warnings** point out common errors and show you the proper method.

There is other important review material at the end of each chapter. The **Terms and Symbols** should all be familiar by the time you reach them. If your understanding of a term or symbol is hazy, use the page reference to find the place in the text where it is introduced. Go back and read the definition.

It is possible to become so involved with the details of techniques that you may lose track of the broader concepts. The list of **Key Ideas for Review** at the end of each chapter will help you focus on the principal ideas.

The **Review Exercises** at the end of each chapter can be used as part of your preparation for examinations. You are then ready to try **Progress Test A.** You will soon pinpoint your weak spots and can go back for further review and more exercises in those areas. Then, and only then, should you proceed to **Progress Test B.**

The authors believe that the eventual "payoff" in studying mathematics is an improved ability to tackle practical problems in your chosen field of interest. To that end, this book places special emphasis on word problems, which recent surveys show are often troublesome to students. Since algebra is the basic language of most mathematical techniques as used in virtually all fields, the mastery of algebra is well worth your effort.

COLLEGE ALGEBRA

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CHAPTER ONE THE FOUNDATIONS OF ALGEBRA

No one would debate that "2 + 2 = 4" or that "5 + 3 = 3 + 5." The significance of the statement "2 + 2 = 4" lies in the recognition that it is true whether the objects under discussion are apples or ants, cradles or cars. Further, the statement "5 + 3 = 3 + 5" indicates that the order of addition is immaterial, and this is true for any pair of integers.

These simple examples illustrate the fundamental task of algebra: to abstract those properties that apply to a number system. Of course, the properties will depend upon the type of numbers we choose to deal with. We therefore begin with a discussion of the *real number system* and its properties since much of our work in algebra will involve this number system. We will then indicate a correspondence between the real numbers and the points on a real number line that permits a graphical presentation of many of our results.

The remainder of this chapter is devoted to a review of some fundamentals of algebra: the meaning and use of variables; algebraic expressions and polynomial forms; factoring; and operations with rational expressions or algebraic fractions.

1.1 THE REAL NUMBER SYSTEM

SETS

We will need to use the notation and terminology of sets from time to time. Recall that a set is a collection of objects or numbers that are called the **elements** or **members** of the set. The elements of a set are written within braces, so that

$$A = \{4, 5, 6\}$$

tells us that the set A consists of the numbers 4, 5, and 6. The set

$$B = \{\text{Exxon, Ford, Honeywell}\}$$

consists of the names of these three corporations. We also write $4 \in A$, which we read as "4 is a member of the set A." Similarly, Ford $\in B$ is read as "Ford is a member of the set B," and IBM $\notin B$ is read as "IBM is not a member of the set B."

If every element of a set A is also a member of a set B, then A is a **subset** of B. For example, the set of all robins is a subset of the set of all birds.

EXAMPLE 1

The set C consists of the names of all coins whose denominations are less than 50 cents. We may write C in set notation as

$$C = \{\text{penny, nickel, dime, quarter}\}$$

We see that dime $\in C$ but half dollar $\notin C$. Further, the set $H = \{\text{nickel, dime}\}$ is a subset of C.

PROGRESS CHECK

The set V consists of the vowels in the English alphabet.

(a) Write V in set notation.
(b) Is the letter k a member of V?
(c) Is the letter u a member of V?
(d) List the subsets of V having four elements.

Answers

(a) $V = \{a, e, i, o, u\}$ (b) No (c) Yes (d) $\{a, e, i, o\}$, $\{e, i, o, u\}$, $\{a, i, o, u\}$, $\{a, e, o, u\}$, $\{a, e, i, u\}$

THE REAL NUMBER SYSTEM

Much of our work in algebra deals with the real number system and we now review the composition of this number system.

The numbers 1, 2, 3, . . . used for counting form the set of **natural numbers.** If we had only these numbers to use to show the profit earned by a company, we would have no way to indicate that the company has no profit or has a loss. To indicate no profit we introduce 0 and for losses we need to introduce negative numbers. The numbers

$$\ldots, -2, -1, 0, 1, 2, \ldots$$

form the set of **integers.** Thus, every natural number is an integer, and the set of natural numbers is seen to be a subset of the set of integers.

When we try to divide two apples equally among four people we find no number in the set of integers that will express how many apples each person should get. We need to introduce the set of **rational**

numbers, which are numbers that can be written as a ratio of two integers,

$$\frac{p}{q}, \quad \text{with } q \text{ not equal to zero}$$

Examples of rational numbers are

$$0, \quad \frac{2}{3}, \quad -4, \quad \frac{7}{5}, \quad \frac{-3}{4}$$

Thus, when we divide two apples equally among four people, each person gets $\frac{1}{2}$ apple. Since every integer n can be written as $n/1$, we see that every integer is a rational number. The decimal number 1.3 is also a rational number since $1.3 = 13/10$.

We have now seen three fundamental number systems: the natural number system, the system of integers, and the rational number system. Each later system includes the previous system(s) and is more complicated. However, the rational number system is still inadequate for mature uses of mathematics since there exist numbers which are not rational, that is, numbers that cannot be written as the ratio of two integers. It can be shown that the number a that satisfies $a \cdot a = 2$ is such a number. The number π, which is the ratio of the circumference of a circle to its diameter, is also such a number. These are called **irrational numbers.** The decimal form of a rational number will either terminate, as

$$\frac{3}{4} = 0.75; \qquad -\frac{4}{5} = -0.8$$

or will form a repeating pattern, as

$$\frac{2}{3} = 0.666\ldots; \qquad \frac{1}{11} = 0.090909\ldots; \qquad \frac{1}{7} = 0.1428571\ldots$$

Remarkably, the decimal form of an irrational number *never* forms a repeating pattern.

The rational and irrational numbers together comprise the **real number system** (Figure 1).

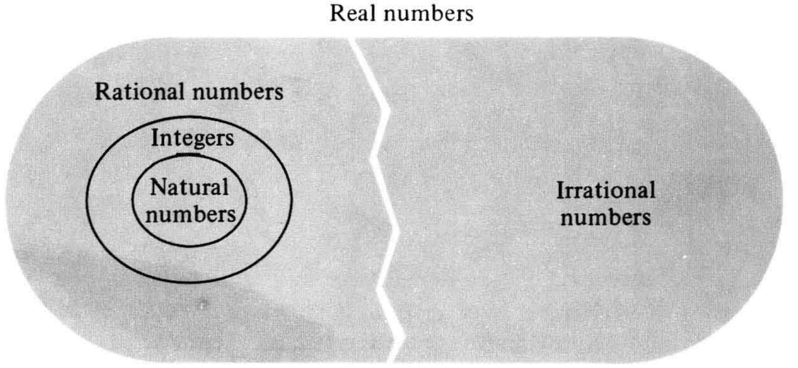

FIGURE 1

With respect to the operations of addition and multiplication, the real number system has properties that are fundamental to algebra. The letters a, b, and c will denote real numbers.

Closure

Property 1. The sum of a and b, denoted by $a + b$, is a real number.

Property 2. The product of a and b, denoted by $a \cdot b$ or ab, is a real number.

We say that the set of real numbers is **closed** with respect to the operations of addition and multiplication since the sum and product of two real numbers are also real numbers and are therefore members of the set.

Commutativity

Property 3. $a + b = b + a$ **Commutative law of addition**

Property 4. $ab = ba$ **Commutative law of multiplication**

That is, we may add or multiply real numbers in any order.

Associativity

Property 5. $(a + b) + c = a + (b + c)$ **Associative law of addition**

Property 6. $(ab)c = a(bc)$ **Associative law of multiplication**

That is, when adding or multiplying real numbers we may group them in any order.

Identities

Property 7. There is a unique real number, denoted by 0, such that $a + 0 = 0 + a = a$ for every real number a.

Property 8. There is a unique real number, denoted by 1, such that $a \cdot 1 = 1 \cdot a = a$ for every real number a.

The real number 0 of Property 7 is called the **additive identity;** the real number 1 of Property 8 is called the **multiplicative identity.**

Inverses

Property 9. For every real number a, there is a unique real number, denoted by $-a$, such that

$$a + (-a) = (-a) + a = 0$$

Property 10. For every real number $a \neq 0$, there is a unique real number, denoted by $1/a$, such that

$$a\left(\frac{1}{a}\right) = \left(\frac{1}{a}\right)a = 1$$

The number $-a$ of Property 9 is called the **negative** or **additive inverse** of a. The number $1/a$ of Property 10 is called the **reciprocal** or **multiplicative inverse** of a.