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The background of the cover features a dark, textured gradient transitioning from black at the top to deep red and orange at the bottom. Overlaid on this are several line graphs. A prominent green line with a jagged, irregular path runs horizontally across the upper half. Below it, a red line follows a similar but smoother path. At the bottom, a blue line curves upwards from the left, and a green line follows a similar path. The overall aesthetic is technical and data-driven.

Financial risk forecasting

*The Theory and Practice of Forecasting Market Risk,
with Implementation in R and MATLAB®*

JÓN DANÍELSSON

Financial Risk Forecasting

The Theory and Practice of Forecasting Market Risk,
with Implementation in R and Matlab

Jón Daniélsson



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Preface

The focus in this book is on the study of market risk from a quantitative point of view. The emphasis is on presenting commonly used state-of-the-art quantitative techniques used in finance for the management of market risk and demonstrate their use employing the principal two mathematical programming languages, R and Matlab. All the code in the book can be downloaded from the book's website at www.financialriskforecasting.com

The book brings together three essential fields: finance, statistics and computer programming. It is assumed that the reader has a basic understanding of statistics and finance; however, no prior knowledge of computer programming is required. The book takes a hands-on approach to the issue of financial risk, with the reading material intermixed between finance, statistics and computer programs.

I have used the material in this book for some years, both for a final year undergraduate course in quantitative methods and for master level courses in risk forecasting. In most cases, the students taking this course have no prior knowledge of computer programming, but emerge after the course with the ability to independently implement the models and code in this book. All of the material in the book can be covered in about 10 weeks, or 20 lecture hours.

Most chapters demonstrate the way in which the various techniques discussed are implemented by both R and Matlab. We start by downloading a sample of stock prices, which are then used for model estimation and evaluation.

The outline of the book is as follows. Chapter 1 begins with an introduction to financial markets and market prices. The chapter gives a foretaste of what is to come, discussing market indices and stock prices, the forecasting of risk and prices, and concludes with the main features of market prices from the point of view of risk. The main focus of the chapter is introduction of the three stylized facts regarding returns on financial assets: volatility clusters, fat tails and nonlinear dependence.

Chapters 2 and 3 focus on volatility forecasting: the former on univariate volatility and the latter on multivariate volatility. The aim is to survey all the methods used for volatility forecasting, while discussing several models from the GARCH family in considerable detail. We discuss the models from a theoretical point of view and demonstrate their implementation and evaluation.

This is followed by two chapters on risk models and risk forecasting: Chapter 4 addresses the theoretical aspects of risk forecasting—in particular, volatility, value-

at-risk (VaR) and expected shortfall; Chapter 5 addresses the implementation of risk models.

We then turn to risk analysis in options and bonds; Chapter 6 demonstrates such analytical methods as delta-normal VaR and duration-normal VaR, while Chapter 7 addresses Monte Carlo simulation methods for derivative pricing and risk forecasting.

After developing risk models their quality needs to be evaluated—this is the topic of Chapter 8. This chapter demonstrates how backtesting and a number of methodologies can be used to evaluate and compare the risk forecast methods presented earlier in the book. The chapter concludes with a comprehensive discussion of stress testing.

The risk forecast methods discussed up to this point in the book are focused on relatively common events, but in special cases it is necessary to forecast the risk of very large, yet uncommon events (e.g., the probability of events that happen, say, every 10 years or every 100 years). To do this, we need to employ extreme value theory—the topic of Chapter 9.

In Chapter 10, the last chapter in the book, we take a step back and consider the underlying assumptions behind almost every risk model in practical use and discuss what happens when these assumptions are violated. Because financial risk is fundamentally *endogenous*, financial risk models have the annoying habit of failing when needed the most. How and why this happens is the topic of this chapter.

There are four appendices: Appendix A introduces the basic concepts in statistics and the financial time series referred to throughout the book. We give an introduction to R and Matlab in Appendices B and C, respectively, providing a discussion of the basic implementation of the software packages. Finally, Appendix D is focused on maximum likelihood, concept, implementation and testing. A list of the most commonly used abbreviations in the book can be found on p. xvii. This is followed by a table of the notation used in the book on p. xix.

Jón Danielsson

Acknowledgments

This book is based on my years of teaching risk forecasting, both at undergraduate and master level, at the London School of Economics (LSE) and other universities, and in various executive education courses. I am very grateful to all the students and practitioners who took my courses for all the feedback I have received over the years.

I was fortunate to be able to employ an exemplary student, Jacqueline Li, to work with me on developing the lecture material. Jacqueline's assistance was invaluable; she made significant contributions to the book. Her ability to master all the statistical and computational aspects of the book was impressive, as was the apparent ease with which she mastered the technicalities. She survived the process and has emerged as a very good friend.

A brilliant mathematician and another very good friend, Maite Naranjo at the Centre de Recerca Matemàtica, Bellaterra in Barcelona, agreed to read the mathematics and saved me from several embarrassing mistakes.

Two colleagues at the LSE, Stéphane Guibaud and Jean-Pierre Zigrand, read parts of the book and verified some of the mathematical derivations.

My PhD student, Ilknur Zer, who used an earlier version of this book while a masters student at LSE and who currently teaches a course based on this book, kindly agreed to review the new version of the book and came up with very good suggestions on both content and presentation.

Kyle T. Moore and Pengfei Sun, both at Erasmus University, agreed to read the book, with a special focus on extreme value theory. They corrected many mistakes and made good suggestions on better presentation of the material.

I am very grateful to all of them for their assistance; without their contribution this book would not have seen the light of day.

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Abbreviations

ACF	Autocorrelation function
AR	Autoregressive
ARCH	Autoregressive conditional heteroskedasticity
ARMA	Autoregressive moving average
CCC	Constant conditional correlations
CDF	Cumulative distribution function
CLT	Central limit theorem
DCC	Dynamic conditional correlations
DJIA	Dow Jones Industrial Average
ES	Expected shortfall
EVT	Extreme value theory
EWMA	Exponentially weighted moving average
GARCH	Generalized autoregressive conditional heteroskedasticity
GEV	Generalized extreme value
GPD	Generalized Pareto distribution
HS	Historical simulation
IID	Identically and independently distributed
JB test	Jarque–Bera test
KS test	Kolmogorov–Smirnov test
LB test	Ljung–Box test
LCG	Linear congruential generator
LM	Lagrange multiplier
LR	Likelihood ratio
MA	Moving average
MC	Monte Carlo
ML	Maximum likelihood
MLE	Maximum likelihood estimation
MVGARCH	Multivariate GARCH
NaN	Not a number
NLD	Nonlinear dependence
OGARCH	Orthogonal GARCH
P/L	Profit and loss
PC	Principal component

PCA	Principal components analysis
PDF	Probability density function
POT	Peaks over thresholds
QML	Quasi-maximum likelihood
QQ plot	Quantile–quantile plot
RN	Random number
RNG	Random number generator
RV	Random variable
SV	Stochastic volatility
VaR	Value-at-risk
VR	Violation ratio

Notation

Chapter 1: Financial markets, prices and risk

T	Sample size
$t = 1, \dots, T$	A particular observation period (e.g., a day)
P_t	Price at time t
$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$	Simple return
$Y_t = \log \frac{P_t}{P_{t-1}}$	Continuously compounded return
y_t	A sample realization of Y_t
σ	Unconditional volatility
σ_t	Conditional volatility
K	Number of assets
ν	Degrees of freedom of the Student- t
ι	Tail index

Chapter 2: Univariate volatility modeling

W_E	Estimation window
λ	Decay factor in EWMA
Z_t	Residuals
α, β	Main model parameters
ζ, δ	Other model parameters
L_1, L_2	Lags in volatility models

Chapter 3: Multivariate volatility models

Σ_t	Conditional covariance matrix
$Y_{t,k}$	Return on asset k at time t
$y_{t,k}$	Sample return on asset k at time t
$y_t = \{y_{t,k}\}$	Vector of sample returns on all assets at time t
$y = \{y_t\}$	Matrix of sample returns on all assets and dates
A and B	Matrices of parameters
R	Correlation matrix

Chapter 4: Risk measures

p	Probability
Q	Profit and loss
q	Observed profit and loss
w	Vector of portfolio weights
X and Y	Refers to two different assets
$\varphi(\cdot)$	Risk measure
ϑ	Portfolio value

Chapter 5: Implementing risk forecasts

$\gamma(p)$	Significance level as a function of probability
μ	Mean

Chapter 6: Analytical value-at-risk for options and bonds

T	Delivery time/maturity
r	Annual interest rate
σ_r	Volatility of daily interest rate increments
σ_a	Annual volatility of an underlying asset
σ_d	Daily volatility of an underlying asset
τ	Cash flow
D^*	Modified duration
C	Convexity
Δ	Option delta
Γ	Option gamma
$g(\cdot)$	Generic function name for pricing equation

Chapter 7: Simulation methods for VaR for options and bonds

F	Futures price
g	Derivative price
S	Number of simulations
x^b	Portfolio holdings (basic assets)
x^o	Portfolio holdings (derivatives)

Chapter 8: Backtesting and stress testing

W_T	Testing window size
$T = W_E + W_T$	Number of observations in a sample
$\eta_t = 0, 1$	Indicates whether a VaR violation occurs (i.e., $\eta_t = 1$)
$v_i, i = 0, 1$	Number of violations ($i = 1$) and no violations ($i = 0$) observed in $\{\eta_t\}$
v_{ij}	Number of instances where j follows i in $\{\eta_t\}$

Chapter 9: Extreme value theory

ι	Tail index
$\xi = 1/\iota$	Shape parameter
M_T	Maximum of X
C_T	Number of observations in the tail
u	Threshold value
ψ	Extremal index

Contents

Preface	xiii
Acknowledgments	xv
Abbreviations	xvii
Notation	xix
1 Financial markets, prices and risk	1
1.1 Prices, returns and stock indices	2
1.1.1 Stock indices	2
1.1.2 Prices and returns	2
1.2 S&P 500 returns	5
1.2.1 S&P 500 statistics	6
1.2.2 S&P 500 statistics in R and Matlab	7
1.3 The stylized facts of financial returns	9
1.4 Volatility	9
1.4.1 Volatility clusters	11
1.4.2 Volatility clusters and the ACF	12
1.5 Nonnormality and fat tails	14
1.6 Identification of fat tails	16
1.6.1 Statistical tests for fat tails	16
1.6.2 Graphical methods for fat tail analysis	17
1.6.3 Implications of fat tails in finance	20
1.7 Nonlinear dependence	21
1.7.1 Sample evidence of nonlinear dependence	22
1.7.2 Exceedance correlations	23
1.8 Copulas	25
1.8.1 The Gaussian copula	25
1.8.2 The theory of copulas	25
1.8.3 An application of copulas	27
1.8.4 Some challenges in using copulas	28
1.9 Summary	29

2	Univariate volatility modeling	31
2.1	Modeling volatility	31
2.2	Simple volatility models	32
2.2.1	Moving average models	32
2.2.2	EWMA model	33
2.3	GARCH and conditional volatility	35
2.3.1	ARCH	36
2.3.2	GARCH	38
2.3.3	The “memory” of a GARCH model	39
2.3.4	Normal GARCH	40
2.3.5	Student- t GARCH	40
2.3.6	(G)ARCH in mean	41
2.4	Maximum likelihood estimation of volatility models	41
2.4.1	The ARCH(1) likelihood function	42
2.4.2	The GARCH(1,1) likelihood function	42
2.4.3	On the importance of σ_1	43
2.4.4	Issues in estimation	43
2.5	Diagnosing volatility models	44
2.5.1	Likelihood ratio tests and parameter significance	44
2.5.2	Analysis of model residuals	45
2.5.3	Statistical goodness-of-fit measures	45
2.6	Application of ARCH and GARCH	46
2.6.1	Estimation results	46
2.6.2	Likelihood ratio tests	47
2.6.3	Residual analysis	47
2.6.4	Graphical analysis	48
2.6.5	Implementation	48
2.7	Other GARCH-type models	51
2.7.1	Leverage effects and asymmetry	51
2.7.2	Power models	52
2.7.3	APARCH	52
2.7.4	Application of APARCH models	52
2.7.5	Estimation of APARCH	53
2.8	Alternative volatility models	54
2.8.1	Implied volatility	54
2.8.2	Realized volatility	55
2.8.3	Stochastic volatility	55
2.9	Summary	56
3	Multivariate volatility models	57
3.1	Multivariate volatility forecasting	57
3.1.1	Application	58
3.2	EWMA	59
3.3	Orthogonal GARCH	62
3.3.1	Orthogonalizing covariance	62
3.3.2	Implementation	62
3.3.3	Large-scale implementations	63

3.4	CCC and DCC models	63
3.4.1	Constant conditional correlations (CCC)	64
3.4.2	Dynamic conditional correlations (DCC)	64
3.4.3	Implementation	65
3.5	Estimation comparison	65
3.6	Multivariate extensions of GARCH	67
3.6.1	Numerical problems	69
3.6.2	The BEKK model	69
3.7	Summary	70
4	Risk measures	73
4.1	Defining and measuring risk	73
4.2	Volatility	75
4.3	Value-at-risk	76
4.3.1	Is VaR a negative or positive number?	77
4.3.2	The three steps in VaR calculations	78
4.3.3	Interpreting and analyzing VaR	78
4.3.4	VaR and normality	79
4.3.5	Sign of VaR	79
4.4	Issues in applying VaR	80
4.4.1	VaR is only a quantile	80
4.4.2	Coherence	81
4.4.3	Does VaR really violate subadditivity?	83
4.4.4	Manipulating VaR	84
4.5	Expected shortfall	85
4.6	Holding periods, scaling and the square root of time	89
4.6.1	Length of holding periods	89
4.6.2	Square-root-of-time scaling	90
4.7	Summary	90
5	Implementing risk forecasts	93
5.1	Application	93
5.2	Historical simulation	95
5.2.1	Expected shortfall estimation	97
5.2.2	Importance of window size	97
5.3	Risk measures and parametric methods	98
5.3.1	Deriving VaR	99
5.3.2	VaR when returns are normally distributed	101
5.3.3	VaR under the Student- t distribution	102
5.3.4	Expected shortfall under normality	103
5.4	What about expected returns?	104
5.5	VaR with time-dependent volatility	106
5.5.1	Moving average	106
5.5.2	EWMA	107
5.5.3	GARCH normal	108
5.5.4	Other GARCH models	109
5.6	Summary	109

6	Analytical value-at-risk for options and bonds	111
6.1	Bonds	112
6.1.1	Duration-normal VaR	112
6.1.2	Accuracy of duration-normal VaR	114
6.1.3	Convexity and VaR	114
6.2	Options	115
6.2.1	Implementation	117
6.2.2	Delta-normal VaR	119
6.2.3	Delta and gamma	120
6.3	Summary	120
7	Simulation methods for VaR for options and bonds	121
7.1	Pseudo random number generators	122
7.1.1	Linear congruential generators	122
7.1.2	Nonuniform RNGs and transformation methods	123
7.2	Simulation pricing	124
7.2.1	Bonds	125
7.2.2	Options	129
7.3	Simulation of VaR for one asset	132
7.3.1	Monte Carlo VaR with one basic asset	133
7.3.2	VaR of an option on a basic asset	134
7.3.3	Options and a stock	136
7.4	Simulation of portfolio VaR	137
7.4.1	Simulation of portfolio VaR for basic assets	137
7.4.2	Portfolio VaR for options	139
7.4.3	Richer versions	139
7.5	Issues in simulation estimation	140
7.5.1	The quality of the RNG	140
7.5.2	Number of simulations	140
7.6	Summary	142
8	Backtesting and stress testing	143
8.1	Backtesting	143
8.1.1	Market risk regulations	146
8.1.2	Estimation window length	146
8.1.3	Testing window length	147
8.1.4	Violation ratios	147
8.2	Backtesting the S&P 500	147
8.2.1	Analysis	150
8.3	Significance of backtests	153
8.3.1	Bernoulli coverage test	154
8.3.2	Testing the independence of violations	155
8.3.3	Testing VaR for the S&P 500	157
8.3.4	Joint test	159
8.3.5	Loss-function-based backtests	159
8.4	Expected shortfall backtesting	160
8.5	Problems with backtesting	162

8.6	Stress testing	163
8.6.1	Scenario analysis	163
8.6.2	Issues in scenario analysis	165
8.6.3	Scenario analysis and risk models	165
8.7	Summary	166
9	Extreme value theory	167
9.1	Extreme value theory	168
9.1.1	Types of tails	168
9.1.2	Generalized extreme value distribution	169
9.2	Asset returns and fat tails	170
9.3	Applying EVT	172
9.3.1	Generalized Pareto distribution	172
9.3.2	Hill method	173
9.3.3	Finding the threshold	174
9.3.4	Application to the S&P 500 index	175
9.4	Aggregation and convolution	176
9.5	Time dependence	179
9.5.1	Extremal index	179
9.5.2	Dependence in ARCH	180
9.5.3	When does dependence matter?	180
9.6	Summary	181
10	Endogenous risk	183
10.1	The Millennium Bridge	184
10.2	Implications for financial risk management	184
10.2.1	The 2007–2010 crisis	185
10.3	Endogenous market prices	188
10.4	Dual role of prices	190
10.4.1	Dynamic trading strategies	191
10.4.2	Delta hedging	192
10.4.3	Simulation of feedback	194
10.4.4	Endogenous risk and the 1987 crash	195
10.5	Summary	195
APPENDICES		
A	Financial time series	197
A.1	Random variables and probability density functions	197
A.1.1	Distributions and densities	197
A.1.2	Quantiles	198
A.1.3	The normal distribution	198
A.1.4	Joint distributions	200
A.1.5	Multivariate normal distribution	200
A.1.6	Conditional distribution	200

A.1.7	Independence	201
A.2	Expectations and variance	201
A.2.1	Properties of expectation and variance	202
A.2.2	Covariance and independence	203
A.3	Higher order moments	203
A.3.1	Skewness and kurtosis	204
A.4	Examples of distributions	206
A.4.1	Chi-squared (χ^2)	206
A.4.2	Student- t	206
A.4.3	Bernoulli and binomial distributions	208
A.5	Basic time series concepts	208
A.5.1	Autocovariances and autocorrelations	209
A.5.2	Stationarity	209
A.5.3	White noise	210
A.6	Simple time series models	210
A.6.1	The moving average model	210
A.6.2	The autoregressive model	211
A.6.3	ARMA model	212
A.6.4	Random walk	212
A.7	Statistical hypothesis testing	212
A.7.1	Central limit theorem	213
A.7.2	p -values	213
A.7.3	Type 1 and type 2 errors and the power of the test	214
A.7.4	Testing for normality	214
A.7.5	Graphical methods: QQ plots	215
A.7.6	Testing for autocorrelation	215
A.7.7	Engle LM test for volatility clusters	216
B	An introduction to R	217
B.1	Inputting data	217
B.2	Simple operations	219
B.2.1	Matrix computation	220
B.3	Distributions	222
B.3.1	Normality tests	223
B.4	Time series	224
B.5	Writing functions in R	225
B.5.1	Loops and repeats	226
B.6	Maximum likelihood estimation	228
B.7	Graphics	229
C	An introduction to Matlab	231
C.1	Inputting data	231
C.2	Simple operations	233
C.2.1	Matrix algebra	234
C.3	Distributions	235
C.3.1	Normality tests	237
C.4	Time series	237