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The Theory and Practice of Forecasting Market Risk, with Implementation in R and MATLAB®

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Preface

The focus in this book is on the study of market risk from a quantitative point of view. The emphasis is on presenting commonly used state-of-the-art quantitative techniques used in finance for the management of market risk and demonstrate their use employing the principal two mathematical programming languages, R and Matlab. All the code in the book can be downloaded from the book's website at `www.financialriskforecasting.com`

The book brings together three essential fields: finance, statistics and computer programming. It is assumed that the reader has a basic understanding of statistics and finance; however, no prior knowledge of computer programming is required. The book takes a hands-on approach to the issue of financial risk, with the reading material intermixed between finance, statistics and computer programs.

I have used the material in this book for some years, both for a final year undergraduate course in quantitative methods and for master level courses in risk forecasting. In most cases, the students taking this course have no prior knowledge of computer programming, but emerge after the course with the ability to independently implement the models and code in this book. All of the material in the book can be covered in about 10 weeks, or 20 lecture hours.

Most chapters demonstrate the way in which the various techniques discussed are implemented by both R and Matlab. We start by downloading a sample of stock prices, which are then used for model estimation and evaluation.

The outline of the book is as follows. Chapter 1 begins with an introduction to financial markets and market prices. The chapter gives a foretaste of what is to come, discussing market indices and stock prices, the forecasting of risk and prices, and concludes with the main features of market prices from the point of view of risk. The main focus of the chapter is introduction of the three stylized facts regarding returns on financial assets: volatility clusters, fat tails and nonlinear dependence.

Chapters 2 and 3 focus on volatility forecasting: the former on univariate volatility and the latter on multivariate volatility. The aim is to survey all the methods used for volatility forecasting, while discussing several models from the GARCH family in considerable detail. We discuss the models from a theoretical point of view and demonstrate their implementation and evaluation.

This is followed by two chapters on risk models and risk forecasting: Chapter 4 addresses the theoretical aspects of risk forecasting—in particular, volatility, value-at-risk (VaR) and expected shortfall; Chapter 5 addresses the implementation of risk models.

We then turn to risk analysis in options and bonds; Chapter 6 demonstrates such analytical methods as delta-normal VaR and duration-normal VaR, while Chapter 7 addresses Monte Carlo simulation methods for derivative pricing and risk forecasting.

After developing risk models their quality needs to be evaluated—this is the topic of Chapter 8. This chapter demonstrates how backtesting and a number of methodologies can be used to evaluate and compare the risk forecast methods presented earlier in the book. The chapter concludes with a comprehensive discussion of stress testing.

The risk forecast methods discussed up to this point in the book are focused on relatively common events, but in special cases it is necessary to forecast the risk of very large, yet uncommon events (e.g., the probability of events that happen, say, every 10 years or every 100 years). To do this, we need to employee extreme value theory—the topic of Chapter 9.

In Chapter 10, the last chapter in the book, we take a step back and consider the underlying assumptions behind almost every risk model in practical use and discuss what happens when these assumptions are violated. Because financial risk is fundamentally *endogenous*, financial risk models have the annoying habit of failing when needed the most. How and why this happens is the topic of this chapter.

There are four appendices: Appendix A introduces the basic concepts in statistics and the financial time series referred to throughout the book. We give an introduction to R and Matlab in Appendices B and C, respectively, providing a discussion of the basic implementation of the software packages. Finally, Appendix D is focused on maximum likelihood, concept, implementation and testing. A list of the most commonly used abbreviations in the book can be found on p. xvii. This is followed by a table of the notation used in the book on p. xix.

Jón Daníelsson

Acknowledgments

This book is based on my years of teaching risk forecasting, both at undergraduate and master level, at the London School of Economics (LSE) and other universities, and in various executive education courses. I am very grateful to all the students and practitioners who took my courses for all the feedback I have received over the years.

I was fortunate to be able to employ an exemplary student, Jacqueline Li, to work with me on developing the lecture material. Jacqueline's assistance was invaluable; she made significant contributions to the book. Her ability to master all the statistical and computational aspects of the book was impressive, as was the apparent ease with which she mastered the technicalities. She survived the process and has emerged as a very good friend.

A brilliant mathematician and another very good friend, Maite Naranjo at the Centre de Recerca Matemàtica, Bellaterra in Barcelona, agreed to read the mathematics and saved me from several embarrassing mistakes.

Two colleagues at the LSE, Stéphane Guibaud and Jean-Pierre Zigrand, read parts of the book and verified some of the mathematical derivations.

My PhD student, Ilknur Zer, who used an earlier version of this book while a masters student at LSE and who currently teaches a course based on this book, kindly agreed to review the new version of the book and came up with very good suggestions on both content and presentation.

Kyle T. Moore and Pengfei Sun, both at Erasmus University, agreed to read the book, with a special focus on extreme value theory. They corrected many mistakes and made good suggestions on better presentation of the material.

I am very grateful to all of them for their assistance; without their contribution this book would not have seen the light of day.

Jón Danielsson

Abbreviations

ACF	Autocorrelation function
AR	Autoregressive
ARCH	Autoregressive conditional heteroskedasticity
ARMA	Autoregressive moving average
CCC	Constant conditional correlations
CDF	Cumulative distribution function
CLT	Central limit theorem
DCC	Dynamic conditional correlations
DJIA	Dow Jones Industrial Average
ES	Expected shortfall
EVT	Extreme value theory
EWMA	Exponentially weighted moving average
GARCH	Generalized autoregressive conditional heteroskedasticity
GEV	Generalized extreme value
GPD	Generalized Pareto distribution
HS	Historical simulation
IID	Identically and independently distributed
JB test	Jarque–Bera test
KS test	Kolmogorov–Smirnov test
LB test	Ljung–Box test
LCG	Linear congruental generator
LM	Lagrange multiplier
LR	Likelihood ratio
MA	Moving average
MC	Monte Carlo
ML	Maximum likelihood
MLE	Maximum likelihood estimation
MVGARCH	Multivariate GARCH
NaN	Not a number
NLD	Nonlinear dependence
OGARCH	Orthogonal GARCH
P/L	Profit and loss
PC	Principal component

PCA	Principal components analysis
PDF	Probability density function
POT	Peaks over thresholds
QML	Quasi-maximum likelihood
QQ plot	Quantile–quantile plot
RN	Random number
RNG	Random number generator
RV	Random variable
SV	Stochastic volatility
VaR	Value-at-risk
VR	Violation ratio

Notation

Chapter 1: Financial markets, prices and risk

T	Sample size
$t = 1, \ldots, T$	A particular observation period (e.g., a day)
P_t	Price at time t
$R_t = \frac{P_t - P_{t-1}}{P_{t-1}}$	Simple return
$Y_t = \log \frac{P_t}{P_{t-1}}$	Continuously compounded return
y_t	A sample realization of Y_t
σ	Unconditional volatility
σ_t	Conditional volatility
K	Number of assets
ν	Degrees of freedom of the Student-t
ι	Tail index

Chapter 2: Univariate volatility modeling

W_E	Estimation window
λ	Decay factor in EWMA
Z_t	Residuals
α, β	Main model parameters
ζ, δ	Other model parameters
L_1, L_2	Lags in volatility models

Chapter 3: Multivariate volatility models

Σ_t	Conditional covariance matrix
$Y_{t,k}$	Return on asset k at time t
$y_{t,k}$	Sample return on asset k at time t
$y_t = \{y_{t,k}\}$	Vector of sample returns on all assets at time t
$y = \{y_t\}$	Matrix of sample returns on all assets and dates
A and B	Matrices of parameters
R	Correlation matrix

Chapter 4: Risk measures

p	Probability
Q	Profit and loss
q	Observed profit and loss
w	Vector of portfolio weights
X and Y	Refers to two different assets
$\varphi(\cdot)$	Risk measure
ϑ	Portfolio value

Chapter 5: Implementing risk forecasts

$\gamma(p)$	Significance level as a function of probability
μ	Mean

Chapter 6: Analytical value-at-risk for options and bonds

T	Delivery time/maturity
r	Annual interest rate
σ_r	Volatility of daily interest rate increments
σ_a	Annual volatility of an underlying asset
σ_d	Daily volatility of an underlying asset
τ	Cash flow
D^*	Modified duration
C	Convexity
Δ	Option delta
Γ	Option gamma
$g(\cdot)$	Generic function name for pricing equation

Chapter 7: Simulation methods for VaR for options and bonds

F	Futures price
g	Derivative price
S	Number of simulations
x^b	Portfolio holdings (basic assets)
x^o	Portfolio holdings (derivatives)

Chapter 8: Backtesting and stress testing

W_T	Testing window size
$T = W_E + W_T$	Number of observations in a sample
$\eta_t = 0, 1$	Indicates whether a VaR violation occurs (i.e., $\eta_t = 1$)
$v_i,\ i = 0, 1$	Number of violations ($i = 1$) and no violations ($i = 0$) observed in $\{\eta_t\}$
v_{ij}	Number of instances where j follows i in $\{\eta_t\}$

Chapter 9: Extreme value theory

ι	Tail index
$\xi = 1/\iota$	Shape parameter
M_T	Maximum of X
C_T	Number of observations in the tail
u	Threshold value
ψ	Extremal index

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