ENCYCLOPEDIA OF FINANCIAL MODELS

III

FRANK J. FABOZZI, EDITOR

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About the Editor

Frank J. Fabozzi is Professor of Finance at EDHEC Business School and a member of the EDHEC Risk Institute. Prior to joining EDHEC in August 2011, he held various professorial positions in finance at Yale University's School of Management from 1994 to 2011 and from 1986 to 1992 was a visiting professor of finance and accounting at MIT's Sloan School of Management. From 2008 to 2011, he was an affiliated professor in the Institute of Statistics, Econometrics, and Mathematical Finance at the University of Karlsruhe in Germany. Prior to 1986 he held professorial positions at Lafayette College, Fordham University, Queens College (CUNY), and Hofstra University. From 2003 to 2011, he served on Princeton University's Advisory Council for the Department of Operations Research and Financial Engineering and since then has been a visiting fellow in that department.

Professor Fabozzi is the editor of the *Journal of Portfolio Management*, as well as on the editorial board of the *Journal of Fixed Income*, *Journal of Asset Management*, *Quantitative Finance*, *Review of Futures Markets*, *Journal of Mathematical Finance*, *Journal of Structured Finance*, *Annals of Financial Economics*, and *Theoretical Economic Letters*.

He has authored and edited a number of books in asset management and quantitative finance. His coauthored books in quantitative finance include *A Probability Metrics Approach to Financial Risk Measures* (2011), *Financial Modeling with Lévy Processes and Volatility Clustering* (2011), *Quantitative Equity Investing: Techniques and Strategies* (2010), *Probability and Statistics for Finance* (2010), *Simulation and Optimization Modeling in Finance* (2010), *Bayesian Methods in Finance* (2008), *Advanced Stochastic Models, Risk Assessment, and Portfolio Optimization: The Ideal Risk* (2008), *Financial Econometrics: From Basics to Advanced Modeling Techniques* (2007), *Robust Portfolio Optimization and Management* (2007), and *Mathematics of Financial Modeling and Investment Management* (2004). His books in applied mathematics include *The Methods of Distances in the Theory of Probability and Statistics* (2013) and *Robust and Non-Robust Models in Statistics* (2009). He coauthored three monographs for the Research Foundation of the CFA Institute: *The Impact of the Financial Crisis on the Asset Management Industry* (2010), *Challenges in Quantitative Equity Management* (2008), and *Trends in Quantitative Finance* (2006).

Professor Fabozzi's research papers have appeared in numerous journals, including *Journal of Finance*, *Journal of Finance and Quantitative Analysis*, *Econometric Theory*, *Operations Research*, *Journal of Banking and Finance*, *Journal of Economic Dynamics and Control*, *Studies in Nonlinear Dynamics and Econometrics*, *European Journal of Operational Research*, *Annals of Operations Research*, *Quantitative Finance*, *European Financial Management*, and *The Econometric Journal*. His 2010 article published in *European Financial Management* with Professors Robert Shiller, and Radu Tunaru, "Property Derivatives for Managing European Real-Estate Risk," received the Best Paper Award and his paper with the same coauthors entitled "A Pricing Framework for Real Estate Derivatives" was awarded

Best Research Paper at the 10th Research Conference Campus for Finance held annually at WHU Otto Beisheim School of Management, Vallendar, Germany. An article coauthored with Dr. Sergio Focardi, "An Autoregressive Conditional Duration Model of Credit Risk Contagion," published in 2005 in *Journal of Risk Finance* was the winner of the 2006 Outstanding Paper by Emerald Literati Network.

He has received several awards and honors for his body of work. In 1994 he was awarded an Honorary Doctorate of Humane Letters from Nova Southeastern University. In 2002 he was inducted into the Fixed Income Analysts Society's Hall of Fame, established by the society "to recognize the lifetime achievements of outstanding practitioners in the advancement of the analysis of fixed-income securities and portfolios." In 2007 he was the recipient of the C. Stewart Sheppard Award given by the CFA Institute "in recognition of outstanding contribution to continuing education in the CFA profession." He was the cover story in the July 1999 issue of *Bloomberg Magazine* entitled "The Boswell of Bonds."

Professor Fabozzi was the co-founder of Information Management Network (now a subsidiary of Euromoney), a conference company specializing in financial topics. He is a trustee for the BlackRock family of closed-end funds where he is the chair of the performance committee and a member of the audit committee. He was a director of Guardian Mutual Funds and Guardian Annuity Funds.

He earned both an M.A. and B.A. in economics and statistics in June 1970 from the City College of New York and elected to Phi Beta Kappa in 1969. He earned a Ph.D. in Economics in September 1972 from the City University of New York. Professor Fabozzi holds two professional designations: Chartered Financial Analyst (1977) and Certified Public Accountant (1982).

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Preface

It is often said that investment management is an art, not a science. However, since the early 1990s the market has witnessed a progressive shift toward a more industrial view of the investment management process. There are several reasons for this change. First, with globalization the universe of investable assets has grown many times over. Asset managers might have to choose from among several thousand possible investments from around the globe. Second, institutional investors, often together with their consultants, have encouraged asset management firms to adopt an increasingly structured process with documented steps and measurable results. Pressure from regulators and the media is another factor. Finally, the sheer size of the markets makes it imperative to adopt safe and repeatable methodologies.

In its modern sense, financial modeling is the design (or engineering) of financial instruments and portfolios of financial instruments that result in predetermined cash flows contingent upon different events. Broadly speaking, financial models are employed to manage investment portfolios and risk. The objective is the transfer of risk from one entity to another via appropriate financial arrangements. Though the aggregate risk is a quantity that cannot be altered, risk can be transferred if there is a willing counterparty.

Financial modeling came to the forefront of finance in the 1980s, with the broad diffusion of derivative instruments. However, the concept and practice of financial modeling are quite old. The notion of the diversification of risk (central to modern risk management) and the quantification of insurance risk (a requisite for pricing insurance policies) were already understood, at least in practical terms, in the 14th century. The rich epistolary of Francesco Datini, a 14th-century merchant, banker, and insurer from Prato (Tuscany, Italy), contains detailed instructions to his agents on how to diversify risk and insure cargo.

What is specific to modern financial modeling is the quantitative management of risk. Both the pricing of contracts and the optimization of investments require some basic capabilities of statistical modeling of financial contingencies. It is the size, diversity, and efficiency of modern competitive markets that makes the use of financial modeling imperative.

This three-volume encyclopedia offers not only coverage of the fundamentals and advances in financial modeling but provides the mathematical and statistical techniques needed to develop and test financial models, as well as the practical issues associated with implementation. The encyclopedia offers the following unique features:

- The entries for the encyclopedia were written by experts from around the world. This diverse collection of expertise has created the most definitive coverage of established and cutting-edge financial models, applications, and tools in this ever-evolving field.

- The series emphasizes both technical and managerial issues. This approach provides researchers, educators, students, and practitioners with a balanced understanding of the topics and the necessary background to deal with issues related to financial modeling.
- Each entry follows a format that includes the author, entry abstract, introduction, body, listing of key points, notes, and references. This enables readers to pick and choose among various sections of an entry, and creates consistency throughout the entire encyclopedia.
- The numerous illustrations and tables throughout the work highlight complex topics and assist further understanding.
- Each volume includes a complete table of contents and index for easy access to various parts of the encyclopedia.

TOPIC CATEGORIES

As is the practice in the creation of an encyclopedia, the topic categories are presented alphabetically. The topic categories and a brief description of each topic follow.

VOLUME I

Asset Allocation

A major activity in the investment management process is establishing policy guidelines to satisfy the investment objectives. Setting policy begins with the asset allocation decision. That is, a decision must be made as to how the funds to be invested should be distributed among the major asset classes (e.g., equities, fixed income, and alternative asset classes). The term "asset allocation" includes (1) policy asset allocation, (2) dynamic asset allocation, and (3) tactical asset allocation. Policy asset allocation decisions can loosely be characterized as long-term asset allocation decisions, in which the investor seeks to assess an appropriate long-term "normal" asset mix that represents an ideal blend of controlled risk and enhanced return. In dynamic asset allocation the asset mix (i.e., the allocation among the asset classes) is mechanistically shifted in response to changing market conditions. Once the policy asset allocation has been established, the investor can turn his or her attention to the possibility of active departures from the normal asset mix established by policy. If a decision to deviate from this mix is based upon rigorous objective measures of value, it is often called tactical asset allocation. The fundamental model used in establishing the policy asset allocation is the mean-variance portfolio model formulated by Harry Markowitz in 1952, popularly referred to as the theory of portfolio selection and modern portfolio theory.

Asset Pricing Models

Asset pricing models seek to formalize the relationship that should exist between asset returns and risk if investors behave in a hypothesized manner. At its most basic level, asset pricing is mainly about transforming asset payoffs into prices. The two most well-known asset pricing models are the arbitrage pricing theory and the capital asset pricing model. The fundamental theorem of asset pricing asserts the equivalence of three key issues in finance: (1) absence of arbitrage; (2) existence of a positive linear pricing rule; and (3) existence of an investor who prefers more to less and who has maximized his or her utility. There are two types of arbitrage opportunities. The first is paying nothing today and obtaining something in the future, and the second is obtaining something today and with no future obligations. Although the principle of absence of arbitrage is fundamental for understanding asset valuation in a competitive market, there are well-known limits to arbitrage resulting from restrictions imposed on rational traders, and, as a result, pricing inefficiencies may exist for a period of time.

Bayesian Analysis and Financial Modeling Applications

Financial models describe in mathematical terms the relationships between financial random variables through time and/or across assets. The fundamental assumption is that the model relationship is valid independent of the time period or the asset class under consideration. Financial data contain both meaningful information and random noise. An adequate financial model not only extracts optimally the relevant information from the historical data but also performs well when tested with new data. The uncertainty brought about by the presence of data noise makes imperative the use of statistical analysis as part of the process of financial model building, model evaluation, and model testing. Statistical analysis is employed from the vantage point of either of the two main statistical philosophical traditions—frequentist and Bayesian. An important difference between the two lies with the interpretation of the concept of probability. As the name suggests, advocates of the frequentist approach interpret the probability of an event as the limit of its long-run relative frequency (i.e., the frequency with which it occurs as the amount of data increases without bound). Since the time financial models became a mainstream tool to aid in understanding financial markets and formulating investment strategies, the framework applied in finance has been the frequentist approach. However, strict adherence to this interpretation is not always possible in practice. When studying rare events, for instance, large samples of data may not be available, and in such cases proponents of frequentist statistics resort to theoretical results. The Bayesian view of the world is based on the subjectivist interpretation of probability: Probability is subjective, a degree of belief that is updated as information or data are acquired. Only in the last two decades has Bayesian statistics started to gain greater acceptance in financial modeling, despite its introduction about 250 years ago. It has been the advancements of computing power and the development of new computational methods that have fostered the growing use of Bayesian statistics in financial modeling.

Bond Valuation

The value of any financial asset is the present value of its expected future cash flows. To value a bond (also referred to as a fixed-income security), one must be able to estimate the bond's remaining cash flows and identify the appropriate discount rate(s) at which to discount the cash flows. The traditional approach to bond valuation is to discount every cash flow with the same discount rate. Simply put, the relevant term structure of interest rate used in valuation is assumed to be flat. This approach, however, permits opportunities for arbitrage. Alternatively, the arbitrage-free valuation approach starts with the premise that a bond should be viewed as a portfolio or package of zero-coupon bonds. Moreover, each of the bond's cash flows is valued using a unique discount rate that depends on the term structure of interest rates and when in time the cash flow is. The relevant set of discount rates (that is, spot rates) is derived from an appropriate term structure of interest rates and when used to value risky bonds augmented with a suitable risk spread or premium. Rather than modeling to calculate the fair value of its price, the market price can be taken as given so as to compute a yield measure or a spread measure. Popular yield measures are the yield to maturity, yield to call, yield to put, and cash flow yield. Nominal spread, static (or zero-volatility) spread, and option-adjusted spread are popular relative value measures quoted in the bond market. Complications in bond valuation arise when a bond has one or more embedded options such as call, put, or conversion features. For bonds with embedded options, the financial modeling draws from options theory, more specifically, the use of the lattice model to value a bond with embedded options.

Credit Risk Modeling

Credit risk is a broad term used to refer to three types of risk: default risk, credit spread risk, and downgrade risk. Default risk is the risk that the counterparty to a transaction will fail to satisfy the terms of the obligation with respect to the timely payment of interest and repayment of the amount borrowed. The counterparty could be the issuer of a debt obligation or an entity on the other side of a private transaction such as a derivative trade or a collateralized loan agreement (i.e., a repurchase agreement or a securities lending agreement). The default risk of a counterparty is often initially gauged by the credit rating assigned by one of the three rating companies—Standard & Poor's, Moody's Investors Service, and Fitch Ratings. Although default risk is the one that most market participants think of when reference is made to credit risk, even in the absence of default, investors are concerned about the decline in the market value of their portfolio bond holdings due to a change in credit spread or the price performance of their holdings relative to a bond index. This risk is due to an adverse change in credit spreads, referred to as credit spread risk, or when it is attributed solely to the downgrade of the credit rating of an entity, it is called downgrade risk. Financial modeling of credit risk is used (1) to measure, monitor, and control a portfolio's credit risk, and (2) to price credit risky debt instruments. There are two general categories of credit risk models: structural models and reduced-form models. There is considerable debate as to which type of model is the best to employ.

Derivatives Valuation

A derivative instrument is a contract whose value depends on some underlying asset. The term "derivative" is used to describe this product because its value is derived from the value of the underlying asset. The underlying asset, simply referred to as the "underlying," can be either a commodity, a financial instrument, or some reference entity such as an interest rate or stock index, leading to the classification of commodity derivatives and financial derivatives. Although there are close conceptual relations between derivative instruments and cash market instruments such as debt and equity, the two classes of instruments are used differently: Debt and equity are used primarily for raising funds from investors, while derivatives are primarily used for dividing up and trading risks. Moreover, debt and equity are direct claims against a firm's assets, while derivative instruments are usually claims on a third party. A derivative's value depends on the value of the underlying, but the derivative instrument itself represents a claim on the "counterparty" to the trade. Derivatives instruments are classified in terms of their payoff characteristics: linear and nonlinear payoffs. The former, also referred to as symmetric payoff derivatives, includes forward, futures, and swap contracts while the latter include options. Basically, a linear payoff derivative is a risk-sharing arrangement between the counterparties since both are sharing the risk regarding the price of the underlying. In contrast, nonlinear payoff derivative instruments (also referred to as asymmetric payoff derivatives) are insurance arrangements because one party to the trade is willing to insure the counterparty of a minimum or maximum (depending on the contract) price. The amount received by the insuring party is referred to as the contract price or premium. Derivative instruments are used for controlling risk exposure with respect to the underlying. Hedging is a special case of risk control where a party seeks to eliminate the risk exposure. Derivative valuation or pricing is developed based on no-arbitrage price relations, relying on the assumption that two perfect substitutes must have the same price.

VOLUME II

Difference Equations and Differential Equations

The tools of linear difference equations and differential equations have found many applications in finance. A difference equation is an equation that involves differences between successive values of a function of a discrete variable. A function of such a variable is one that provides a rule for assigning values in sequences to it. The theory of linear difference equations covers three areas: solving difference equations, describing the behavior of difference equations, and identifying the equilibrium (or critical value) and stability of difference equations. Linear difference equations are important in the context of dynamic econometric models. Stochastic models in finance are expressed as linear difference equations with random disturbances added. Understanding the behavior of solutions of linear difference equations helps develop intuition for the behavior of these models. In nontechnical terms, differential equations are equations that express a relationship between a function and one or more derivatives (or differentials) of that function. The relationship between difference equations and differential equations is that the latter are invaluable for modeling situations in finance where there is a continually changing value. The problem is that not all changes in value occur continuously. If the change in value occurs incrementally rather than continuously, then differential equations have their limitations. Instead, a financial modeler can use difference equations, which are recursively defined sequences. It would be difficult to overemphasize the importance of differential equations in financial modeling where they are used to express laws that govern the evolution of price probability distributions, the solution of economic variational problems (such as intertemporal optimization), and conditions for continuous hedging (such as in the Black-Scholes option pricing model). The two broad types of differential equations are ordinary differential equations and partial differential equations. The former are equations or systems of equations involving only one independent variable. Another way of saying this is that ordinary differential equations involve only total derivatives. Partial differential equations are differential equations or systems of equations involving partial derivatives. When one or more of the variables is a stochastic process, we have the case of stochastic differential equations and the solution is also a stochastic process. An assumption must be made about what is driving noise in a stochastic differential equation. In most applications, it is assumed that the noise term follows a Gaussian random variable, although other types of random variables can be assumed.

Equity Models and Valuation

Traditional fundamental equity analysis involves the analysis of a company's operations for the purpose of assessing its economic prospects. The analysis begins with the financial statements of the company in order to investigate the earnings, cash flow, profitability, and debt burden. The fundamental analyst will look at the major product lines, the economic outlook for the products (including existing and potential competitors), and the industries in which the company operates. The result of this analysis will be the growth prospects of earnings. Based on the growth prospects of earnings, a fundamental analyst attempts to determine the fair value of the stock using one or more equity valuation models. The two most commonly used approaches for valuing a firm's equity are based on discounted cash flow and relative valuation models. The principal idea underlying discounted cash flow models is that what an investor pays for a share of stock should reflect what is expected to be received from it—return on the investor's investment. What an investor receives are cash dividends in the future. Therefore, the value of a share of stock should be equal to the present value of all the future cash flows an investor expects to receive from that share. To value stock, therefore, an investor must project future cash flows, which, in turn, means projecting future dividends. Popular discounted cash flow models include the basic dividend discount model, which assumes a constant dividend growth, and the multiple-phase models, which include the two-stage dividend growth model and the stochastic dividend discount models. Relative valuation methods use multiples or ratios—such as price/earnings, price/book, or price/free cash flow—to determine whether a stock is trading at higher or lower multiples than its peers.