ALGEBRA and TRIGONOMETRY

A PROBLEM SOLVING APPROACH

FOURTH EDITION

WALTER FLEMING
DALE VARBERG
HERBERT KASUBE

Fourth Edition

Algebra and Trigonometry

A PROBLEM SOLVING APPROACH

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Prentice Hall
Englewood Cliffs, New Jersey 07632

Library of Congress Cataloging-in-Publication Data
Fleming, Walter.
Algebra and trigonometry / Walter Fleming, Dale Varberg, Herbert Kasube.—4th ed.
p. cm.
Includes bibliographical references and index.
ISBN 0-13-028911-6
1. Algebra. 2. Trigonometry. I. Varberg, Dale E. II. Kasube, Herbert. III. Title.
QA154.2.F52 1992
512'.13—dc20 91-24122
CIP

Acquisition Editor: Priscilla McGeehon
Editor-in-Chief: Tim Bozik
Development Editor: Leo Gaffney
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Designer: Judith A. Matz-Coniglio
Cover Designer: Marjory Dressler
Prepress Buyer: Paula Massenaro
Manufacturing Buyer: Lori Bulwin
Supplements Editor: Susan Black
Editorial Assistant: Marisol L. Torres
Page Layout: Meryl Poweski

A Simon & Schuster Company
Englewood Cliffs, New Jersey 07632

The material in this book has been previously published in portions of Fleming and Varberg's *Algebra and Trigonometry*, 3rd Ed., *College Algebra*, 3rd Ed., and *Precalculus Mathematics*, 2nd Ed.

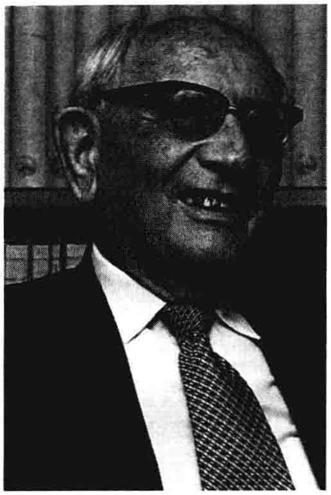
Parts of Sections 12-8 and 12-9 are taken from A. Wayne Roberts and Dale E. Varberg: *Faces of Mathematics* (1978), Sections 6-1 and 6-2, and used by permission of Harper & Row, Publishers, Inc.

Credits for quotations used in text appear on page 657.

Printed in the United States of America
10 9 8 7 6 5 4 3

ISBN 0-13-028911-6

Prentice-Hall International (UK) Limited, *London*
Prentice-Hall of Australia Pty. Limited, *Sydney*
Prentice-Hall Canada Inc., *Toronto*
Prentice-Hall Hispanoamericana, S.A., *Mexico*
Prentice-Hall of India Private Limited, *New Delhi*
Prentice-Hall of Japan, Inc., *Tokyo*
Simon & Schuster Asia Pte. Ltd., *Singapore*
Editora Prentice-Hall do Brasil, Ltda., *Rio de Janeiro*



George Polya

A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest; but if it challenges your curiosities and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. Such experiences at a susceptible age may create a taste for mental work and leave their imprint on the mind and character for a life time.

—How to Solve It (p. v)

Solving a problem is similar to building a house. We must collect the right material, but collecting the material is not enough; a heap of stones is not yet a house. To construct the house or the solution, we must put together the parts and organize them into a purposeful whole.

—Mathematical Discovery (vol. 1, p. 66)

You turn the problem over and over in your mind; try to turn it so it appears simpler. The aspect of the problem you are facing at this moment may not be the most favorable. Is the problem as simply, as clearly, as suggestively expressed as possible? Could you restate the problem?

—Mathematical Discovery (vol. 2, p. 80)

We can scarcely imagine a problem absolutely new, unlike and unrelated to any formerly solved problem; but, if such a problem could exist, it would be insoluable. In fact, when solving a problem, we should always profit from previously solved problems, using their result, or their method, or the experience we acquired solving them. . . . Have you seen it before? Or have you seen the same problem in slightly different form?

—How to Solve It (p. 98)

An insect tries to escape through the windowpane, tries the same hopeless thing again and again, and does not try the next window which is open and through which it came into the room. A mouse may act more intelligently; caught in a trap, he tries to squeeze between two bars, then between the next two bars, then between other bars; he varies his trials, he explores various possibilities. A man is able, or should be able, to vary his trials more intelligently, to explore the various possibilities with more understanding, to learn by his errors and shortcomings. "Try, try again" is popular advice. It is good advice. The insect, the mouse, and the man follow it; but if one follows it with more success than the others it is because he varies his problem more intelligently.

—How to Solve It (p. 209)

These quotations are taken from George Polya, *How to Solve It,* Second Edition (Garden City, NY: Doubleday & Company, Inc., 1957) and George Polya, *Mathematical Discovery,* vols. 1 and 2 (New York: John Wiley & Sons, Inc., 1962).

Preface

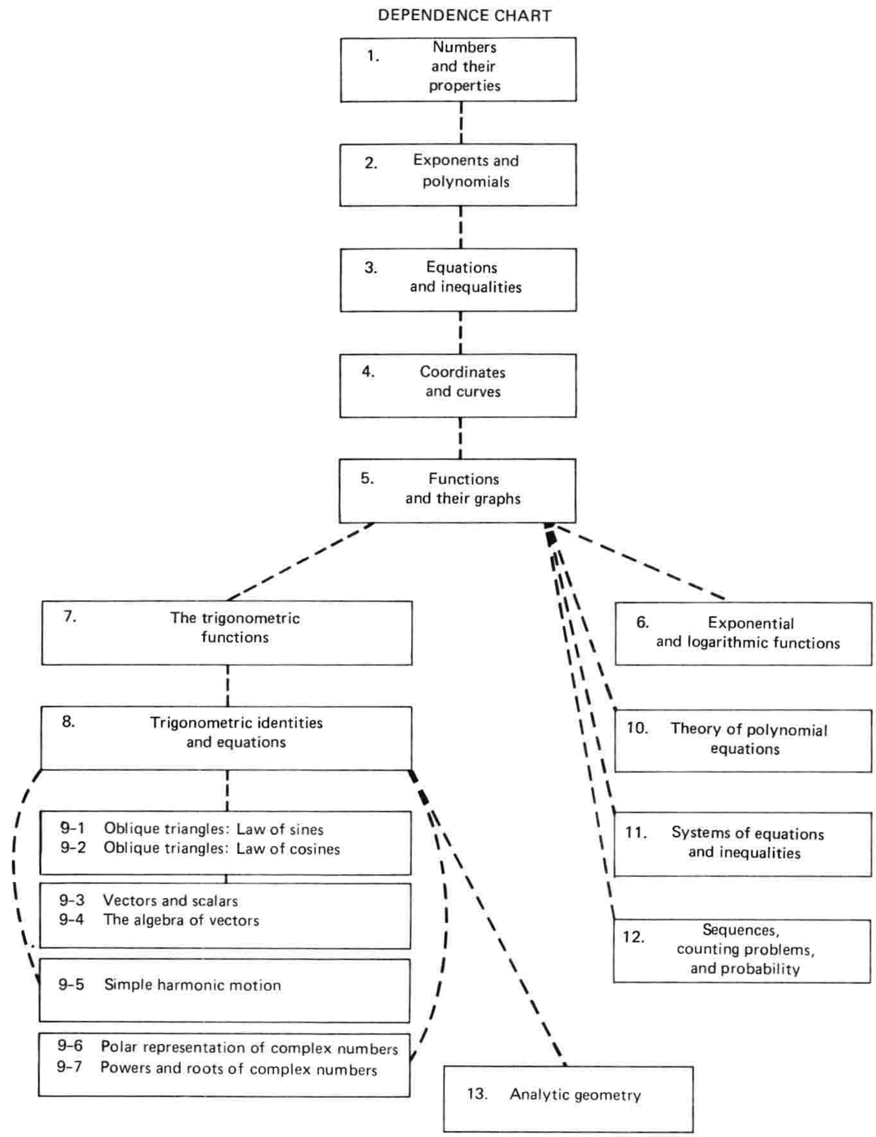
This edition of *Algebra and Trigonometry* builds on the strengths of its predecessors.

- **Writing style:** informal but not sloppy
- **Section openers:** an anecdote, quotation, cartoon, or problem
- **Cautions:** warning students of common errors
- **Problem-solving emphasis:** in the spirit of George Polya
- **Carefully graded problem sets:** culminating in a TEASER Problem
- **Chapter reviews:** to help students prepare for tests
- **Attractive design:** featuring open format and use of color
- **Formula card:** a memory aid that can be removed

New to this edition. Following the advice of many reviewers, we have placed all examples in the text proper rather than in the problem sets as in earlier editions. Since this required major changes, we took the opportunity to rewrite every section, adding examples and figures and thereby improving clarity. We have also reworked the section problems sets, which are now divided into two parts. Part A: Skills and Techniques is closely tied to the examples of the section. Part B: Applications and Extensions asks the student to integrate the various techniques, to apply them in real-life situations, and to extend them in novel and challenging ways. Answers to odd-numbered problems can be found at the end of the book. We have also reworked and greatly expanded the chapter review problem sets. Our goal for them is to provide students with a versatile tool to use in preparing for tests. Note that there are answers to all the review problems in the answer key at the back of the book.

The day of the hand-held calculator is here. We expect students to use a scientific calculator freely and no longer bother to identify those problems that require their use. But there is a new development. The day of the graphing calculator (the Casio fx series, the Sharp EL-5200, the HP-28S, and the TI-81) has also arrived. Although a simple scientific calculator (costing under $20) is adequate for this course, the availability of graphing calculators (costing much more but going down in price) allows us to explore many ideas in more depth than in the past. For those who have their own graphing calculator (or for classes where they are required tools), we have written a section (Section 5-3) on how to use a graphing calculator and have thereafter added graphing calculator problems (each identified with a symbol) at the ends of most sections. Graphing calculators do not diminish the need to understand hand-graphing strategies. Rather, they enrich the subject by allowing the analysis of more complicated functions and the asking of deeper questions. Besides, students will find graphing calculator problems to be great fun. However, we emphasize that they are supplementary material; the course is complete without them.

Flexibility This book has plenty of material for a two-semester course. It is flexible in that syllabi for many quite different courses can be based on the book. The dependence chart below will help in keeping track of prerequisites. Here are three items worthy of special note.



1. The first three chapters are a review of basic algebra. In some classes, they can be omitted or covered rapidly.
2. Complex numbers are introduced early (Section 1-6). However, this section can be postponed until just before Section 9-6 if desired. To facilitate this, the few problems in the early part of the book that use complex numbers are marked with the symbol [i] and can be omitted without loss of continuity.
3. An instructor who plans to cover Chapter 13 (Analytic Geometry) may wish to omit the optional section on ellipses and hyperbolas that occurs early in the book (Section 4-5) since this material is handled in much more detail in Chapter 13.

SUPPLEMENTARY MATERIALS

Instructor's Resource Manual was prepared by the authors. It contains the following items:

(a) Teaching Outlines for every section of each chapter.
(b) Complete solutions to all even-numbered problems, including **TEASER** problems.
(c) A Test Item File of more than 1200 problems with answers, designed to be used in conjunction with the computerized Prentice Hall TESTMANAGER.

Prentice Hall TESTMANAGER is a test bank of more than 1200 problems on disk for the IBM PC. This allows the instructor to generate examinations by choosing individual problems, and either editing them or creating completely new problems.

Tutorial Software includes tutorial and drill problems for IBM and MacIntosh.

Student Solutions Manual has worked-out solutions to odd-numbered problems and solutions to all Chapter Review Problem Sets.

ACKNOWLEDGMENTS

This and previous editions have profited from the warm praise and constructive criticism of many reviewers. We offer our thanks to the following people who gave helpful suggestions.

JoAnne B. Brooks, *Blue Mountain College*
Natalie M. Creed, *Belmont Abbey College*
James Daly, *University of Colorado, Boulder*
Milton P. Eisner, *Mount Vernon College*
George T. Fix, *University of Texas, Arlington*

Juan Gatica, *University of Iowa, Iowa City*
Wojciech Komornicki, *Hamline University*
Bruce Lecher, *State University of New York, Binghamton*
Fred Liss, *University of Wisconsin, Rock City*
Carroll Matthews, *Montgomery College, Rockville*
Phil Miles, *University of Wisconsin, Madison*
Michael Montano, *Riverside Community College*
Jim Newsom, *Tidewater Community College, Virginia Beach*
Marvin Papenfuss, *Loras College*
Margot Pullman, *Maryville College*
Cheryl Roberts, *James Madison University*
Richard Semmler, *Northern Virginia Community College*
Cynthia Siegal, *University of Missouri, St. Louis*
John Smashy, *Southwest Baptist University*
Diane Spresser, *James Madison University*
Henry Waldman, *South Dakota School of Mines and Technology*

We thank Ann Phipps of Texas Instruments for granting permission to reproduce a picture of the TI-81 and four diagrams from the TI-81 guidebook, and also for preparing several calculator-generated graphs for inclusion in this text.

The staff at Prentice Hall is to be congratulated on another fine production job. The authors wish to express appreciation especially to Priscilla McGeehon (mathematics editor), Leo Gaffney (development editor), Edward Thomas (production editor), and Judith Matz-Coniglio (designer) for their exceptional contributions.

Walter Fleming
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Herbert Kasube

Algebra and Trigonometry

Contents

7 The Trigonometric Functions 260

8 Trigonometric Identities and Equations 302

9 Applications of Trigonometry 337

10 Theory of Polynomial Equations 386

11 Systems of Equations and Inequalities 417

12 Sequences, Counting Problems, and Probability 473

13 Analytic Geometry 539

CHAPTER 1

Numbers and Their Properties

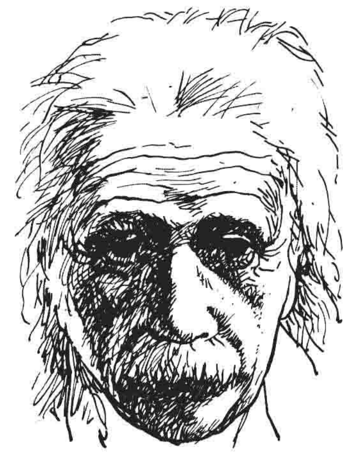
■ Numbers are an indispensable tool of civilization, serving to whip its activities into some sort of order . . . The complexity of a civilization is mirrored in the complexity of its numbers.

Philip J. Davis

1-1 WHAT IS ALGEBRA?

"Algebra is a merry science," Uncle Jakob would say. "We go hunting for a little animal whose name we don't know, so we call it x. When we bag our game we pounce on it and give it its right name."

Albert Einstein



Sometimes the simplest questions seem the hardest to answer. One frustrated ninth grader responded, "Algebra is all about x and y, but nobody knows what they are." Albert Einstein was fond of his Uncle Jakob's definition, which is quoted above. A contemporary mathematician, Morris Kline, refers to algebra as generalized arithmetic. There is some truth in all of these statements, but perhaps Kline's statement is closest to the heart of the matter. What does he mean?

In arithmetic we are concerned with numbers and the four operations of addition, subtraction, multiplication, and division. We learn to understand and manipulate expressions like

$$16 - 11 \qquad \frac{3}{24} \qquad (13)(29)$$

In algebra we do the same thing, but we are more likely to write

$$a - b \qquad \frac{x}{y} \qquad mn$$

without specifying precisely what numbers these letters represent. This determination to stay uncommitted (not to know what x and y are) offers some tremendous advantages. Here are two of them.

Generality and Conciseness

All of us know that $3 + 4$ is the same as $4 + 3$ and that $7 + 9$ equals $9 + 7$. We could fill pages and books, even libraries, with the corresponding facts about other numbers. All of them would be correct and all would be important. But we can achieve the same effect much more economically by writing

$$a + b = b + a$$

The simple formula says all there is to be said about adding two numbers in opposite order. It states a general law and does it on one-fourth of a line.

Or take the well-known facts that if I drive 30 miles per hour for 2 hours, I will travel 60 miles, and that if I fly 120 miles per hour for 3 hours, I will cover 360 miles. These and all other similar facts are summarized in the general formula

$$D = RT$$

which is an abbreviation for

$$\text{Distance} = \text{rate} \times \text{time}$$

Formulas

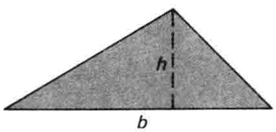


Figure 1

The formula $D = RT$ is just one of many that scientists use almost without thinking. Among these formulas are those of area and volume, which have been known since the time of the Greeks. As a premier example, we mention the formula for the area of a triangle (Figure 1), namely,

$$A = \frac{1}{2}bh$$

a formula that we will have occasion to use innumerable times in this book. Here b denotes the length of the base and h stands for the height (or altitude) of the triangle. Thus a triangle with base $b = 24$ and height $h = 10$ has area

$$A = \frac{1}{2}bh = \frac{1}{2}(24)(10) = 120$$

Of course, we must be careful about units. If the base and height are given in inches, then the area is in square inches.

A more interesting formula is the familiar one

$$A = \pi r^2$$

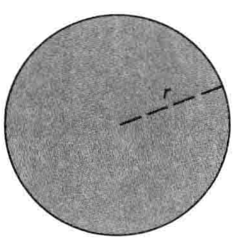


Figure 2

for the area of a circle of radius r (Figure 2). It is interesting because of the appearance of the number π. Perhaps you have learned to approximate π by the fraction 22/7, actually, a rather poor approximation. In this course, we suggest that you use the decimal approximation 3.14159 or the even better approximation that your calculator gives (it should have a π button). Thus a circle of radius 10 centimeters has area

$$A = \pi r^2 = (3.14159)(10)(10) = 314.159$$

The area is about 314 square centimeters.

We are confident that you once learned all the important area and volume formulas but, because your memory may need jogging, we have listed those we will need most often in Figures 3 and 4, which accompany the first problem set.

Problem Solving

Typically, the problems of the real world come to us in words. If we are to use algebra to solve such problems, we must first be able to translate word phrases into algebraic symbols. Here are two simple illustrations.