SECOND EDITION

General Mathematics for Technicians

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General mathematics for technicians

Second edition

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In memory of Linda

Preface

This book, a revised edition of our previous title *General mathematics for technical colleges*, has been written and planned to meet the requirements of the mathematics Standard Unit Level One of the Technician Education Council's programmes in Engineering and Science.

At the beginning of each chapter a list of objectives is included. These objectives are the items of mathematics that a student is expected to understand and use after working through the chapter.

At the end of each chapter is an assessment test composed of short answer and multi-choice questions. These tests can be used to monitor a student's progress. Since these questions are more difficult to construct than the traditional questions, any comments regarding them will be greatly appreciated.

Assessment questions of this type only test one or two items of information. But a technician applying mathematics to technological problems must be able to handle several items of mathematical knowledge at the same time. To this end, short answer or multi-choice questions are not suitable. For this reason many traditional questions have also been included, both within each chapter and as a revision exercise at the end of the book. This revision exercise has been divided into sections.

The questions in the revision exercise have been selected from past Technician examination papers of the Regional Examining Boards. All such questions are used by kind permission of the following Boards:

East Midland Educational Union
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Extracts have been included from Frank Castle's *Logarithm and other tables*, by kind permission of Macmillan Co. Ltd.

During the preparatory stages of the manuscript we were involved with the Dyfed Mathematics panel, which was convened to consider the standard units prepared by the Technician Education Council. It is a pleasure to record our indebtedness to the following members of the panel for many useful discussions: D. G. Hazelby, J. K. Jones, J. G. Thomas, and R. E. Warlow.

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H. G. Davies
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Contents

1. Arithmetic 1

Objectives

After working through this chapter you should be able to

1. Apply the rules of precedence to numbers and fractions.
2. Apply the commutative, associative, and distributive laws.
3. Find the prime factors of numbers.
4. Calculate the square and cube roots of numbers using prime factors.
5. Determine the Highest Common Factor of two or more numbers.
6. Determine the Lowest Common Multiple of two or more numbers.
7. Simplify, add, subtract, multiply, and divide vulgar fractions.
8. Recognize proper, improper, and mixed fractions.
9. Multiply decimal numbers by multiples of 10.
10. Divide decimal numbers by multiples of 10.
11. Convert vulgar fractions to decimals.
12. Convert decimals to vulgar fractions.
13. Distinguish between a terminating and non-terminating decimal.
14. Define a recurring decimal.
15. Reduce a number to a given number of decimal places.
16. Reduce a number to a given number of significant figures.
17. Add, subtract, multiply, and divide decimal numbers.

1.1 Mixed operations and the rules of precedence

The word 'operation' means $+$, $-$, $\times$ or $\div$. When a mixture of operations are used in a calculation it is necessary to decide in which order they are carried out. The rules which determine this order are called the 'rules of precedence'. In the following simple calculation

$$3+4\times5-10$$

the order in which it is to be worked out must be decided. The order is decided in accordance with Rule 1.

Rule 1. $\times$ *and* $\div$ *is carried out before* $+$ *and* $-$.

Hence in the above calculation, we have

$$3+20-10 = 13.$$

Again in

$$63 \div 7 - 4 \times 2$$

the $\div$ and $\times$ are carried out first to give

$$9 - 8 = 1.$$

Sometimes a calculation contains a bracket. In this case Rule 2 is used.

Rule 2. *Calculations inside the bracket are always carried out first.*

For example, in the calculation

$$63 \div (7 - 4) \times 2$$

the inside of the bracket must be worked out first, to give

$$63 \div 3 \times 2 = 21 \times 2 = 42$$

Rules 1 and 2 may be summarized by the word BODMAS, which identifies the order of preference for *B*rackets, *O*f, *D*ivision, *M*ultiplication, *A*ddition, *S*ubtraction.

EXAMPLE 1.1 Evaluate

(a) $30 \div (10 - 4) + 2$
(b) $30 \div 10 - 4 + 2$

(a) $30 \div (10-4)+2 = 30 \div 6 + 2$, using Rule 2
$= 5 + 2$, using Rule 1
$= 7$

(b) $30 \div 10 - 4 + 2 = 3 - 4 + 2$, using Rule 1
$= 1$

EXERCISE 1.1

Evaluate the following:

1. $6 \times 2 - 3$
2. $27 \div 3 - 27 \div 9$
3. $14 + 2 \times 3 - 28 \div 2$
4. $6 - 2 \times 3 + 10$
5. $10 \times 5 - 3 \times 5$
6. $6 \times (4 - 2)$
7. $(27 - 20) \times (27 + 3)$
8. $(14 + 2 \times 3 - 10) \div 2$
9. $(6 - 2) \times 3 + 10$
10. $(3 + 5) \times \{3 + (6 - 4)\}$

Note: The two rules of precedence apply to vulgar and decimal fractions, as well as simple numbers.

1.2 The three laws of arithmetic

(a) Commutative law

Consider the operations

$$6+4 = 4+6$$

and

$$3 \times 5 = 5 \times 3$$

It does not matter which of the figures is written down first, the answer is the same. These two results constitute the commutative law, which states that in the addition or multiplication of two numbers the order in which they are written down is immaterial.

(b) Associative law

In arithmetic the addition or multiplication of three numbers is independent of the order in which the operation is carried out. For example, it can easily be seen that

$$4+(7+3) = (4+7)+3$$

that is

$$4+\ 10 = 11\ +3$$

Again

$$3 \times (4 \times 5) = (3 \times 4) \times 5$$

since

$$3 \times\ 20 = 12\ \times 5$$

The two results constitute the associative law.

(c) Distributive law

The following calculation is an example of the distributive law

$$5 \times (2+4) = 5 \times 2+5 \times 4$$

The result is readily seen to be true since

$$5 \times 6 = 10 + 20$$

Again

$$7 \times (9-4) = 7 \times 9-7 \times 4$$

since

$$7 \times\ 5 = 63 - 28$$

EXERCISE 1.2

Verify the following, and state which law each obeys.

1. $5+3=3+5$
2. $4\times 7=7\times 4$
3. $8+7=7+8$
4. $3\times 1=1\times 3$
5. $8+(2+3)=(8+2)+3$
6. $4\times(6\times 2)=(4\times 6)\times 2$
7. $7+(3+2)=(7+3)+2$
8. $9\times(4\times 5)=(9\times 4)\times 5$
9. $3\times(8+2)=3\times 8+3\times 2$
10. $5\times(5+6)=5\times 5+5\times 6$
11. $8\times(8-3)=8\times 8-8\times 3$
12. $6\times(7-5)=6\times 7-6\times 5$

1.3 Factors and prime factors of numbers

Consider the number 42. The number 6 will divide exactly into it, that is:

$$42=6\times 7$$

6 and 7 are called **factors** of 42. A factor of any number is any other number which will divide exactly into it.

Some factors can themselves have factors. For example, 6 has two factors, 3 and 2. Other factors, such as 7 do not have factors. Such factors are called **prime factors.** A prime factor cannot be expressed in further factors. In the example above, 42 has two factors 6 and 7, but its prime factors are 2, 3, and 7. The **prime numbers** therefore are those numbers which have no factors; they are:

$$1, 2, 3, 5, 7, 11, \ldots, \text{etc.}$$

The prime factors of any number may be obtained by

(a) dividing repeatedly by 2 until 2 ceases to be a factor,

(b) dividing repeatedly by 3 until 3 ceases to be a factor and so on with 5, 7, 11, etc. The method is shown in Example 1.2.

EXAMPLE 1.2 Determine the prime factors of 720.

2	720	divide repeatedly by 2
2	360	
2	180	
2	90	
3	45	2 ceases to be a factor—try 3
3	15	
5	5	3 ceases to be a factor—try 5
	1	

Therefore the prime factors of 720 are $2\times 2\times 2\times 2\times 3\times 3\times 5$.

It is worth knowing the following facts when finding prime factors.

(a) A number is exactly divisible by 2 if the last digit on the right is 0 or an even number, e.g., $13\overset{*}{6}$ or $72\overset{*}{0}$.

(b) A number is exactly divisible by 3 if the sum of the digits is divisible by 3; e.g., 51 is divisible by 3 since $5+1 = 6$ which is divisible by 3.

(c) A number is exactly divisible by 5 if the last digit on the right is 0 or 5, e.g., $42\overset{*}{0}$ or $14\overset{*}{5}$.

EXERCISE 1.3

1. State whether 2, 3 or 5 is a factor of the following:
   (a) 202 (b) 550 (c) 363 (d) 300 (e) 729
2. State which of the following are prime numbers:
   3, 4, 5, 9, 10, 11, 13, 15, 16, 17
3. Find the prime factors of the following:
   (a) 48 (b) 76 (c) 128 (d) 920 (e) 108 (f) 210 (g) 525
4. Find the prime factors of:
   (a) 18 816 (b) 111 111 (c) 131 313

1.4 Square roots and cube roots using prime factors

(a) Square roots

Consider a number such as 9. Its square root is defined as that number, which multiplied by itself, gives an answer of 9. Such a number is 3, since $3 \times 3 = 9$. Therefore, the square root of 9 is 3, and is written as

$$\sqrt{9} = 3$$

Again

$$25 = 5 \times 5$$

so that

$$\sqrt{25} = 5$$

Thus whenever a number can be expressed in terms of a pair of identical factors, the square root will be one of these factors.

Most numbers do not have exact square roots, numbers such as 2, 5, 7, etc. Numbers such as 4, 9, 25, 36, which have exact square roots are called **perfect squares**.

The square roots of perfect squares can be found using *prime* factors, as follows.

Step 1. Express the number in prime factors.

Step 2. Select one from each pair of factors.

Step 3. Multiply out the prime factors selected to obtain the square root.

EXAMPLE 1.3 Find the square root of 400.

Step 1. $400 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{5 \times 5}$

Step 2. $\quad 2 \quad 2 \quad 5$

Step 3. $\sqrt{400} = 2 \times 2 \times 5 = 20$

A number such as 800 is seen *not* to be a perfect square if it is expressed in prime factors.

$$800 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \overset{*}{2} \times \underbrace{5 \times 5}$$

This can be deduced since there is one factor 2 (shown with an asterisk) which is not paired. In order to make 800 into a perfect square it must be multiplied by 2 to make up a pair. Therefore to convert 800 into a perfect square it must be multiplied by 2.

$$\text{Perfect square} = 800 \times 2 = 1600$$

$$\text{Square root} = 2 \times 2 \times 2 \times 5 = 40$$

(b) Cube roots

The cube root of a number may be found using a similar method. If the number can be expressed as a trio of identical factors the cube root will be one of these factors, for example,

$$8 = \underbrace{2 \times 2 \times 2}$$

then

$$\sqrt[3]{8} = 2$$

A number such as 8 is called a **perfect cube**. The cube roots of such numbers can be obtained as follows.

Step 1. Write the numbers in prime factors.

Step 2. Select one from each trio of factors.

Step 3. Multiply out the factors selected to obtain the cube root.

EXAMPLE 1.4 Determine the cube root of 3375

Step 1. Prime factors: $3375 = \underbrace{3 \times 3 \times 3} \times \underbrace{5 \times 5 \times 5}$

Step 2. Select one from each trio: 3×5

Step 3. Multiply: $\sqrt[3]{3375} = 15$