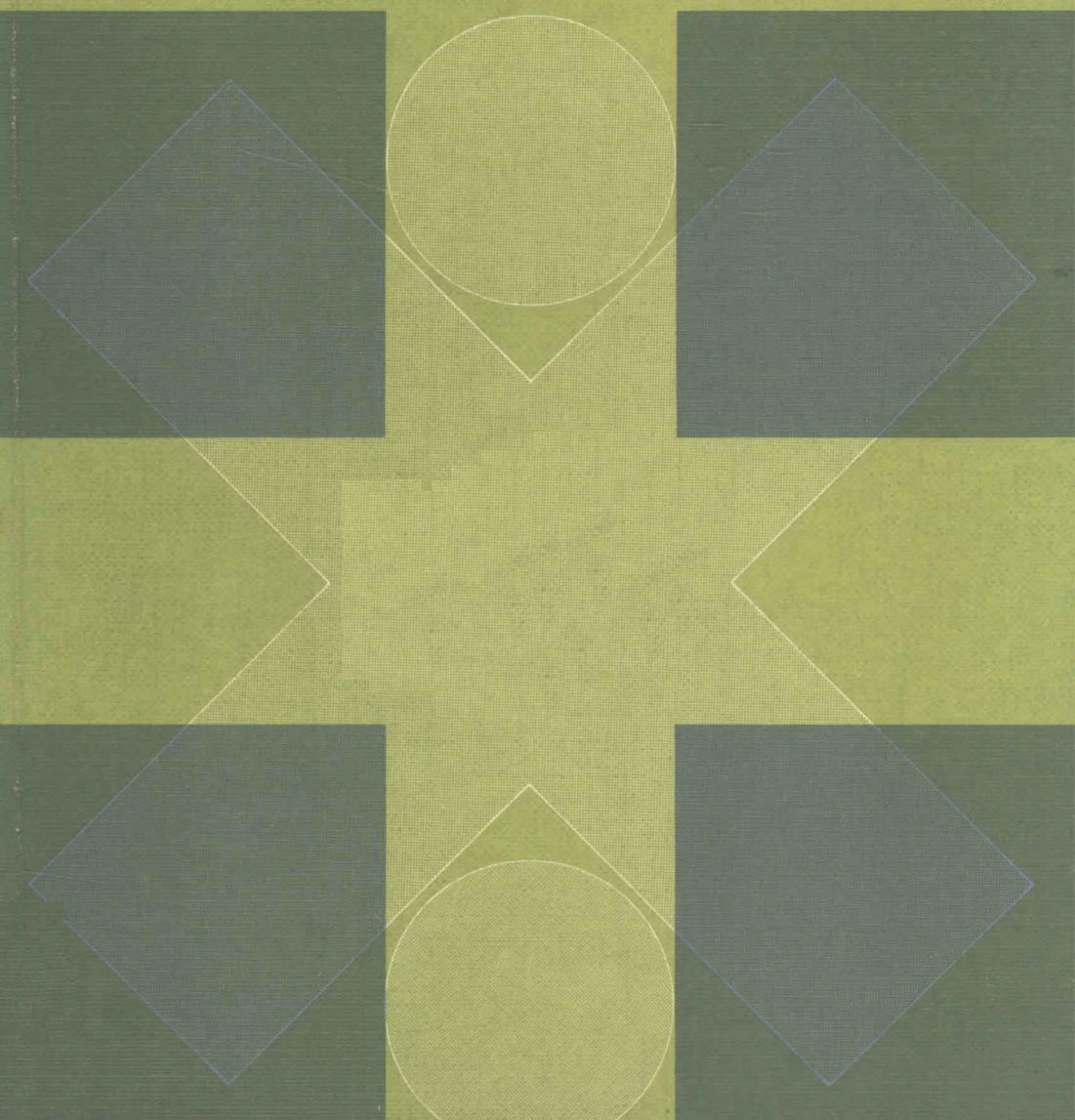


General Mathematics for Technicians

SECOND EDITION

H.G.Davies and G.A.Hicks



General mathematics for technicians

Second edition

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General mathematics for technicians

Second edition

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In memory of Linda

Preface

This book, a revised edition of our previous title *General mathematics for technical colleges*, has been written and planned to meet the requirements of the mathematics Standard Unit Level One of the Technician Education Council's programmes in Engineering and Science.

At the beginning of each chapter a list of objectives is included. These objectives are the items of mathematics that a student is expected to understand and use after working through the chapter.

At the end of each chapter is an assessment test composed of short answer and multi-choice questions. These tests can be used to monitor a student's progress. Since these questions are more difficult to construct than the traditional questions, any comments regarding them will be greatly appreciated.

Assessment questions of this type only test one or two items of information. But a technician applying mathematics to technological problems must be able to handle several items of mathematical knowledge at the same time. To this end, short answer or multi-choice questions are not suitable. For this reason many traditional questions have also been included, both within each chapter and as a revision exercise at the end of the book. This revision exercise has been divided into sections.

The questions in the revision exercise have been selected from past Technician examination papers of the Regional Examining Boards. All such questions are used by kind permission of the following Boards:

East Midland Educational Union

Northern Counties Technical Examination Council

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Extracts have been included from Frank Castle's *Logarithm and other tables*, by kind permission of Macmillan Co. Ltd.

During the preparatory stages of the manuscript we were involved with the Dyfed Mathematics panel, which was convened to consider the standard units prepared by the Technician Education Council. It is a pleasure to record our indebtedness to the following members of the panel for many useful discussions: D. G. Hazelby, J. K. Jones, J. G. Thomas, and R. E. Warlow.

Finally, we would like to thank the editorial staff of McGraw-Hill for their forbearance and assistance over the whole period of the book's preparation.

H. G. Davies
G. A. Hicks

Contents

Preface

1. ARITHMETIC 1	
1.1 Mixed operations and the rules of precedence	1
1.2 The three laws of arithmetic	3
1.3 Factors and prime factors of numbers	4
1.4 Square roots and cube roots using prime factors	5
1.5 Highest Common Factor of two or more numbers (HCF)	7
1.6 Lowest Common Multiple (LCM) of two or more numbers	8
1.7 Vulgar fractions	8
1.8 Comparison of fractions and addition and subtraction	10
1.9 Multiplication and division of fractions	13
1.10 Fractions involving a mixture of $+$, $-$, \times , \div	15
1.11 Decimal fractions	16
1.12 Multiplying and dividing by 10, 100, 1000	17
1.13 Conversions of fractions	18
1.14 Non-terminating decimal fractions and their reduction to a number of places	20
1.15 Significant figures	21
1.16 Basic operations with decimals	22
2. ARITHMETIC 2	
2.1 Ratio	29
2.2 Proportion	32
2.3 Percentages	35
2.4 Indices	39
2.5 Numbers expressed in standard form	43
2.6 Binary numbers	45
3. CALCULATIONS	
3.1 Introduction	52
3.2 Errors and accuracy	53
3.3 Approximate values	54
3.4 Aids to numerical calculations	55
3.5 Evaluation of square roots, squares, and reciprocals by tables	55
3.6 Logarithms	59
3.7 Multiplication and division using logarithms	63
3.8 Logarithms of numbers less than 1	65

3.9	Logarithm of unity	67
3.10	Reciprocals using logarithm tables	67
3.11	Powers and roots	68
3.12	Powers and roots of numbers less than 1	69
3.13	The slide rule	71
3.14	Calculators	72
3.15	Comparison of aids	73
4.	ALGEBRA 1—Basic operations	
4.1	Letters, and their addition and subtraction	78
4.2	Three laws of algebra	80
4.3	Simple multiplication and division	82
4.4	The rules of precedence	83
4.5	Simple substitution	83
4.6	Directed numbers	84
4.7	Indices	90
4.8	Brackets	95
4.9	Multiplication of binomials	97
4.10	Factors	97
5.	ALGEBRA 2	
5.1	Expressions and equations	105
5.2	Simple equations	106
5.3	More difficult simple equations	108
5.4	Construction and solution of equations in engineering and science	110
5.5	Simultaneous equations	112
5.6	Evaluation of formulae	114
5.7	Transposition of formulae	116
6.	DIAGRAMS AND GRAPHS	
6.1	Conversion of data into a related system of units	124
6.2	Representation of related values using two parallel axes	125
6.3	Mappings	129
6.4	Two axes at right angles	129
6.5	Scales on the axes	130
6.6	Plotting a point given its co-ordinates	132
6.7	Straight-line graphs	133
6.8	Gradient of a straight-line graph	134
6.9	Values from a straight-line graph	134

7. STATISTICS	
7.1 Introduction	140
7.2 Display of data	140
7.3 Frequency table	141
7.4 Pictorial displays	143
8. GEOMETRY 1	
8.1 Angles	156
8.2 Properties of angles	157
8.3 Angles of a triangle	162
8.4 Pythagoras' theorem	166
8.5 Construction of a right angle	168
8.6 Congruent triangles	170
8.7 Similar triangles	173
8.8 Construction of triangles	176
9. GEOMETRY 2	
9.1 The circle	187
9.2 Angles subtended by an arc of a circle	190
9.3 Properties of tangents	193
9.4 Property of a chord	194
9.5 Radian measure and length of arc	196
9.6 Quadrilaterals	200
9.7 Regular hexagon	202
10. AREA AND VOLUME	
10.1 Perimeter and area	209
10.2 Areas of plane rectilinear figures	210
10.3 Areas of circular figures	217
10.4 Volumes of prisms	221
10.5 Volumes and surface areas of prisms	222
10.6 Volume and surface area of a cylinder	226
10.7 Volume of a sphere	229
10.8 Volume of a pyramid and a cone	231
11. TRIGONOMETRY	
11.1 Introduction	241
11.2 Trigonometrical ratios of acute angles	242
11.3 Use of trigonometrical tables	243
11.4 Trigonometrical ratios for 30° , 45° , and 60°	246
11.5 Sine and cosine of complementary angles	248
11.6 Solution of right-angled triangles	248

11.7 Trigonometrical graphs	254
11.8 Sine and cosine waves	256
Revision exercise	262
Answers	281
Index	293

1. Arithmetic 1

Objectives

After working through this chapter you should be able to

1. Apply the rules of precedence to numbers and fractions.
2. Apply the commutative, associative, and distributive laws.
3. Find the prime factors of numbers.
4. Calculate the square and cube roots of numbers using prime factors.
5. Determine the Highest Common Factor of two or more numbers.
6. Determine the Lowest Common Multiple of two or more numbers.
7. Simplify, add, subtract, multiply, and divide vulgar fractions.
8. Recognize proper, improper, and mixed fractions.
9. Multiply decimal numbers by multiples of 10.
10. Divide decimal numbers by multiples of 10.
11. Convert vulgar fractions to decimals.
12. Convert decimals to vulgar fractions.
13. Distinguish between a terminating and non-terminating decimal.
14. Define a recurring decimal.
15. Reduce a number to a given number of decimal places.
16. Reduce a number to a given number of significant figures.
17. Add, subtract, multiply, and divide decimal numbers.

1.1 Mixed operations and the rules of precedence

The word 'operation' means $+$, $-$, \times or \div . When a mixture of operations are used in a calculation it is necessary to decide in which order they are carried out. The rules which determine this order are called the 'rules of precedence'. In the following simple calculation

$$3 + 4 \times 5 - 10$$

the order in which it is to be worked out must be decided. The order is decided in accordance with Rule 1.

Rule 1. \times and \div is carried out before $+$ and $-$.

Hence in the above calculation, we have

$$3 + 20 - 10 = 13.$$

Again in

$$63 \div 7 - 4 \times 2$$

the \div and \times are carried out first to give

$$9 - 8 = 1.$$

Sometimes a calculation contains a bracket. In this case Rule 2 is used.

Rule 2. *Calculations inside the bracket are always carried out first.*

For example, in the calculation

$$63 \div (7 - 4) \times 2$$

the inside of the bracket must be worked out first, to give

$$63 \div 3 \times 2 = 21 \times 2 = 42$$

Rules 1 and 2 may be summarized by the word BODMAS, which identifies the order of preference for *B*rackets, *O*f, *D*ivision, *M*ultiplication, *A*ddition, *S*ubtraction.

EXAMPLE 1.1 Evaluate

$$(a) \quad 30 \div (10 - 4) + 2$$

$$(b) \quad 30 \div 10 - 4 + 2$$

$$\begin{aligned}(a) \quad 30 \div (10 - 4) + 2 &= 30 \div 6 + 2, \text{ using Rule 2} \\ &= 5 + 2, \text{ using Rule 1} \\ &= 7\end{aligned}$$

$$\begin{aligned}(b) \quad 30 \div 10 - 4 + 2 &= 3 - 4 + 2, \text{ using Rule 1} \\ &= 1\end{aligned}$$

EXERCISE 1.1

Evaluate the following:

$$1. \quad 6 \times 2 - 3$$

$$6. \quad 6 \times (4 - 2)$$

$$2. \quad 27 \div 3 - 27 \div 9$$

$$7. \quad (27 - 20) \times (27 + 3)$$

$$3. \quad 14 + 2 \times 3 - 28 \div 2$$

$$8. \quad (14 + 2 \times 3 - 10) \div 2$$

$$4. \quad 6 - 2 \times 3 + 10$$

$$9. \quad (6 - 2) \times 3 + 10$$

$$5. \quad 10 \times 5 - 3 \times 5$$

$$10. \quad (3 + 5) \times \{3 + (6 - 4)\}$$

Note: The two rules of precedence apply to vulgar and decimal fractions, as well as simple numbers.

1.2 The three laws of arithmetic

(a) Commutative law

Consider the operations

$$6 + 4 = 4 + 6$$

and

$$3 \times 5 = 5 \times 3$$

It does not matter which of the figures is written down first, the answer is the same. These two results constitute the commutative law, which states that in the addition or multiplication of two numbers the order in which they are written down is immaterial.

(b) Associative law

In arithmetic the addition or multiplication of three numbers is independent of the order in which the operation is carried out. For example, it can easily be seen that

$$4 + (7 + 3) = (4 + 7) + 3$$

that is

$$4 + 10 = 11 + 3$$

Again

$$3 \times (4 \times 5) = (3 \times 4) \times 5$$

since

$$3 \times 20 = 12 \times 5$$

The two results constitute the associative law.

(c) Distributive law

The following calculation is an example of the distributive law

$$5 \times (2 + 4) = 5 \times 2 + 5 \times 4$$

The result is readily seen to be true since

$$5 \times 6 = 10 + 20$$

Again

$$7 \times (9 - 4) = 7 \times 9 - 7 \times 4$$

since

$$7 \times 5 = 63 - 28$$

EXERCISE 1.2

Verify the following, and state which law each obeys.

1. $5+3=3+5$
2. $4 \times 7 = 7 \times 4$
3. $8+7=7+8$
4. $3 \times 1 = 1 \times 3$
5. $8+(2+3)=(8+2)+3$
6. $4 \times (6 \times 2) = (4 \times 6) \times 2$
7. $7+(3+2)=(7+3)+2$
8. $9 \times (4 \times 5) = (9 \times 4) \times 5$
9. $3 \times (8+2) = 3 \times 8 + 3 \times 2$
10. $5 \times (5+6) = 5 \times 5 + 5 \times 6$
11. $8 \times (8-3) = 8 \times 8 - 8 \times 3$
12. $6 \times (7-5) = 6 \times 7 - 6 \times 5$

1.3 Factors and prime factors of numbers

Consider the number 42. The number 6 will divide exactly into it, that is:

$$42 = 6 \times 7$$

6 and 7 are called **factors** of 42. A factor of any number is any other number which will divide exactly into it.

Some factors can themselves have factors. For example, 6 has two factors, 3 and 2. Other factors, such as 7 do not have factors. Such factors are called **prime factors**. A prime factor cannot be expressed in further factors. In the example above, 42 has two factors 6 and 7, but its prime factors are 2, 3, and 7. The **prime numbers** therefore are those numbers which have no factors; they are:

$$1, 2, 3, 5, 7, 11, \dots, \text{etc.}$$

The prime factors of any number may be obtained by

- (a) dividing repeatedly by 2 until 2 ceases to be a factor,
- (b) dividing repeatedly by 3 until 3 ceases to be a factor and so on with 5, 7, 11, etc. The method is shown in Example 1.2.

EXAMPLE 1.2 Determine the prime factors of 720.

2	720] divide repeatedly by 2
2	360	
2	180	
2	90	
3	45	
3	15	2 ceases to be a factor—try 3
5	5	3 ceases to be a factor—try 5
	1	

Therefore the prime factors of 720 are $2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$.

It is worth knowing the following facts when finding prime factors.

- (a) A number is exactly divisible by 2 if the last digit on the right is 0 or an even number, e.g., 136^* or 720^* .
- (b) A number is exactly divisible by 3 if the sum of the digits is divisible by 3; e.g., 51 is divisible by 3 since $5 + 1 = 6$ which is divisible by 3.
- (c) A number is exactly divisible by 5 if the last digit on the right is 0 or 5, e.g., 420^* or 145^* .

EXERCISE 1.3

- 1. State whether 2, 3 or 5 is a factor of the following:
(a) 202 (b) 550 (c) 363 (d) 300 (e) 729
- 2. State which of the following are prime numbers:
3, 4, 5, 9, 10, 11, 13, 15, 16, 17
- 3. Find the prime factors of the following:
(a) 48 (b) 76 (c) 128 (d) 920 (e) ~~108~~ (f) 210 (g) 525
- 4. Find the prime factors of:
(a) 18 816 (b) 111 111 (c) 131 313

1.4 Square roots and cube roots using prime factors

(a) Square roots

Consider a number such as 9. Its square root is defined as that number, which multiplied by itself, gives an answer of 9. Such a number is 3, since $3 \times 3 = 9$. Therefore, the square root of 9 is 3, and is written as

$$\sqrt{9} = 3$$

Again

$$25 = 5 \times 5$$

so that

$$\sqrt{25} = 5$$

Thus whenever a number can be expressed in terms of a pair of identical factors, the square root will be one of these factors.

Most numbers do not have exact square roots, numbers such as 2, 5, 7, etc. Numbers such as 4, 9, 25, 36, which have exact square roots are called **perfect squares**.

The square roots of perfect squares can be found using *prime* factors, as follows.

Step 1. Express the number in prime factors.

Step 2. Select one from each pair of factors.

Step 3. Multiply out the prime factors selected to obtain the square root.

EXAMPLE 1.3 Find the square root of 400.

Step 1. $400 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{5 \times 5}$

Step 2. $\quad \quad \quad 2 \quad \quad 2 \quad \quad 5$

Step 3. $\sqrt{400} = 2 \times 2 \times 5 = 20$

A number such as 800 is seen *not* to be a perfect square if it is expressed in prime factors.

$$800 = \underbrace{2 \times 2} \times \underbrace{2 \times 2} \times \underbrace{2 \times 5}_{*} \times 5$$

This can be deduced since there is one factor 2 (shown with an asterisk) which is not paired. In order to make 800 into a perfect square it must be multiplied by 2 to make up a pair. Therefore to convert 800 into a perfect square it must be multiplied by 2.

$$\text{Perfect square} = 800 \times 2 = 1600$$

$$\text{Square root} = 2 \times 2 \times 2 \times 5 = 40$$

(b) Cube roots

The cube root of a number may be found using a similar method. If the number can be expressed as a trio of identical factors the cube root will be one of these factors, for example,

$$8 = \underbrace{2 \times 2 \times 2}$$

then

$$\sqrt[3]{8} = 2$$

A number such as 8 is called a **perfect cube**. The cube roots of such numbers can be obtained as follows.

Step 1. Write the numbers in prime factors.

Step 2. Select one from each trio of factors.

Step 3. Multiply out the factors selected to obtain the cube root.

EXAMPLE 1.4 Determine the cube root of 3375

Step 1. Prime factors: $3375 = \underbrace{3 \times 3 \times 3} \times \underbrace{5 \times 5 \times 5}$

Step 2. Select one from each trio: 3×5

Step 3. Multiply: $\sqrt[3]{3375} = 15$