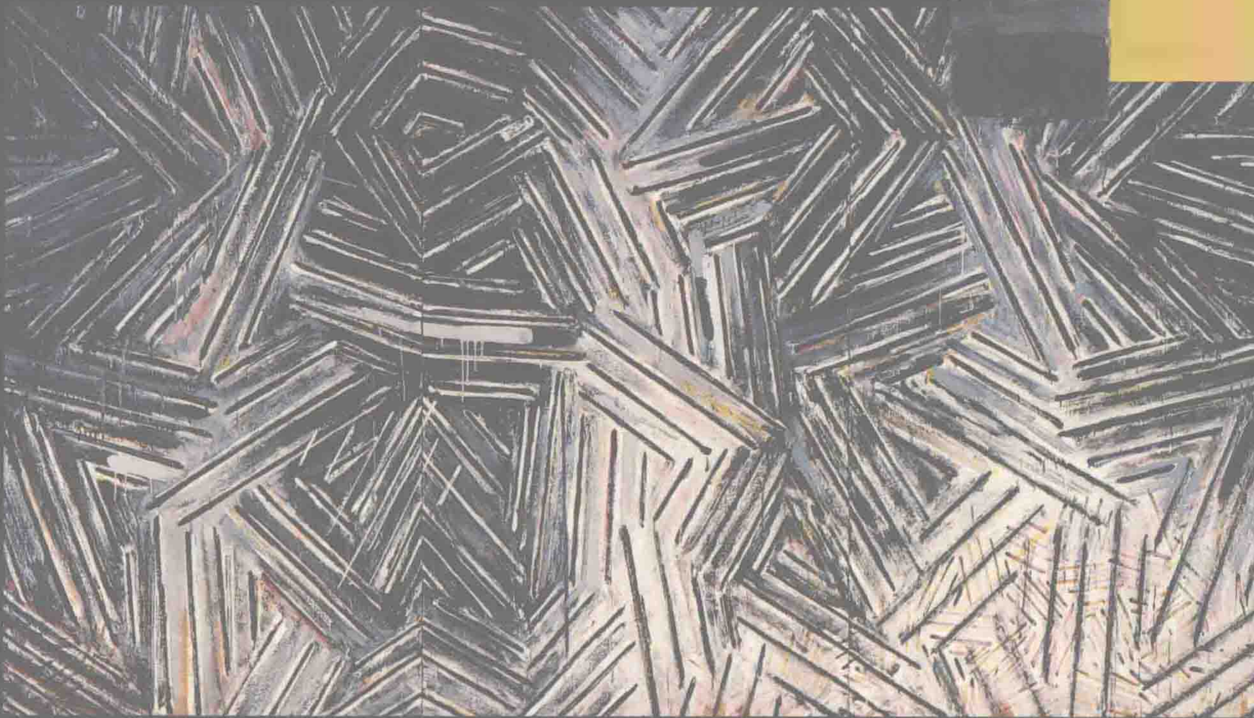

DISCRETE MATHEMATICS AND ITS APPLICATIONS

Second Edition

Kenneth H. Rosen

Discrete Mathematics and Its Applications

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AT&T Bell Laboratories

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Preface

In writing the first edition of this book and revising it for the second edition, I have been guided by two purposes that have resulted from my longstanding experience and interest in teaching discrete mathematics. For the student, my purpose was to write in a precise, readable manner, with the concepts and techniques of discrete mathematics clearly presented and demonstrated. For the instructor, my purpose was carefully to design a flexible, comprehensive teaching tool that uses proven pedagogical techniques in mathematics.

I have been extremely gratified by the tremendous success of the first edition. The many improvements in the second edition have been made possible by the generous feedback of scores of instructors and hundreds of students at many of the more than 200 schools where this book has been successfully used.

This text is designed for a one- or two-term introductory discrete mathematics course to be taken by students in a wide variety of majors, including mathematics, computer science, and engineering. College algebra is the only prerequisite.

Goals of a Discrete Mathematics Course

A discrete mathematics course has more than one purpose. Students should learn a particular set of mathematical facts and how to apply them; but more importantly, such a course should teach students how to think mathematically. To achieve these goals, this text stresses mathematical reasoning and the different ways problems are solved. Five important themes are interwoven in this text: mathematical reasoning, combinatorial analysis, discrete structures, applications and modeling, and algorithmic thinking. A successful discrete mathematics course should blend and carefully balance all five of these themes.

1. *Mathematical Reasoning:* Students must understand mathematical reasoning in order to read, comprehend, and construct mathematical arguments. This

text starts with a discussion of mathematical logic, which serves as the foundation for the subsequent discussions of methods of proof. The technique of mathematical induction is stressed through many different types of examples of such proofs and a careful explanation of why mathematical induction is a valid proof technique.

2. *Combinatorial Analysis:* An important problem-solving skill is the ability to count or enumerate objects. The discussion of enumeration in this book begins with the basic techniques of counting. The stress is on performing combinatorial analysis to solve counting problems, not on applying formulae.
3. *Discrete Structures:* A course in discrete mathematics should teach students how to work with discrete structures, which are the abstract mathematical structures used to represent discrete objects and relationships between these objects. These discrete structures include sets, permutations, relations, graphs, trees, and finite-state machines.
4. *Applications and Modeling:* Discrete mathematics has applications to almost every conceivable area of study. There are many applications to computer science in this text, as well as applications to such diverse areas as chemistry, botany, zoology, linguistics, geography, and business. These applications are natural and important uses of discrete mathematics and are not contrived. Modeling with discrete mathematics is an extremely important problem-solving skill, which students have the opportunity to develop by constructing their own models in some of the exercises in the book.
5. *Algorithmic Thinking:* Certain classes of problems are solved by the specification of an algorithm. After an algorithm has been described, a computer program can be constructed implementing it. The mathematical portions of this activity, which include the specification of the algorithm, the verification that it works properly, and the analysis of the computer memory and time required to perform it, are all covered in this text. Algorithms are described using both English and an easily understood form of pseudocode.

Features

ACCESSIBILITY: There are no mathematical prerequisites beyond college algebra for this text. The few places in the book where calculus is referred to are explicitly noted. Most students should easily understand the pseudocode used in the text to express algorithms, regardless of whether they have formally studied programming languages. There is no formal computer science prerequisite.

Each chapter begins at an easily understood and accessible level. Once basic mathematical concepts have been carefully developed, more difficult material and applications to other areas of study are presented.

FLEXIBILITY: This text has been carefully designed for flexible use. The dependence of chapters on previous material has been minimized. Each chapter is divided into sections of approximately the same length, and each section is divided into subsections that form natural blocks of material for teaching. Instructor's can easily pace their lectures using these blocks.

WRITING STYLE: The writing style in this book is direct and pragmatic. Precise mathematical language is used without excessive formalism and abstraction. Notation is introduced and used when appropriate. Care has been taken to balance the mix of notation and words in mathematical statements.

EXTENSIVE CLASSROOM USE: This book has been used at over 250 schools and more than 200 have used it more than once. The feedback from instructors and students at many of the schools has helped make the second edition an even more successful teaching tool than the first edition.

MATHEMATICAL RIGOR AND PRECISION: All definitions and theorems in this text are stated extremely carefully so that students will appreciate the precision of language and rigor needed in mathematics. Proofs are motivated and developed slowly; their steps are all carefully justified. Recursive definitions are explained and used extensively.

FIGURES AND TABLES: This text contains more than 500 figures. The figures are designed to illustrate key concepts and steps of proofs. Color has been carefully used in figures to illustrate important points. Whenever possible, tables have been used to summarize key points and illuminate quantitative relationships.

WORKED EXAMPLES: Over 600 examples are used to illustrate concepts, relate different topics, and introduce applications. In the examples, a question is first posed, then its solution is presented with the appropriate amount of detail.

APPLICATIONS: The applications included in this text demonstrate the utility of discrete mathematics in the solution of real-world problems. Applications to a wide variety of areas including computer science, psychology, chemistry, engineering, linguistics, biology, business, and many other areas are included in this text.

ALGORITHMS: Results in discrete mathematics are often expressed in terms of algorithms; hence, key algorithms are introduced in each chapter of the book. These algorithms are expressed in words and in an easily understood form of structured pseudocode, which is described and specified in Appendix 2. The computational complexity of the algorithms in the text are also analyzed at an elementary level.

HISTORICAL INFORMATION: The background of many topics is succinctly described in the text. Brief biographies of nearly 50 mathematicians and computer scientists are included as footnotes. These biographies include information about the lives, careers, and accomplishments of these important contributors to discrete mathematics. In addition, numerous historical footnotes are included that supplement the historical information in the main body of the text.

KEY TERMS AND RESULTS: A list of key terms and results follows each chapter. The key terms include only the most important that students should learn, not every term defined in the chapter.

EXERCISES: There are over 2400 exercises in the text. There are many different types of questions posed. There is an ample supply of straightforward exercises that develop basic skills, a large number of intermediate exercises, and a good supply of challenging exercises. Exercises are stated clearly and unambiguously, and all are carefully graded for level of difficulty. Exercise sets contain special discussions, with exercises, that develop new concepts not covered in the text, permitting students to discover new ideas through their own work. Exercises that are somewhat more difficult than average are marked with a single star; those that are much more challenging are marked with two stars. Exercises whose solutions require calculus are explicitly noted. Exercises that develop results used in the text are clearly identified with the symbol ☞. Solutions to all odd-numbered exercises are provided at the back of the text. The solutions include proofs in which most of the steps are clearly spelled out.

REVIEW QUESTIONS: A set of review questions is provided at the end of each chapter. These questions are designed to help students focus their study on the most important concepts and techniques of that chapter. To answer these questions students need to write long answers, rather than just perform calculations or give short replies.

SUPPLEMENTARY EXERCISE SETS: Each chapter is followed by a rich and varied set of supplementary exercises. These exercises are generally more difficult than those in the exercise sets following the sections. The supplementary exercises reinforce the concepts of the chapter and integrate different topics more effectively.

COMPUTER PROJECTS: Each chapter is followed by a set of computer projects. The 135 computer projects tie together what students may have learned in computing and in discrete mathematics. Computer projects that are more difficult than average, from both a mathematical and a programming point of view, are marked with a star, and those that are extremely challenging are marked with two stars.

APPENDIXES: There are three appendixes to the text. The first covers exponential and logarithmic functions, reviewing some basic material used heavily in the course; the second specifies the pseudocode used to describe algorithms in this text; and the third discusses generating functions.

SUGGESTED READING: A list of suggested readings for each chapter is provided in a section at the end of the text. These suggested readings include books at or below the level of this text, more difficult books, expository articles, and articles in which discoveries in discrete mathematics were originally published.

Changes in the Second Edition

This new edition includes a wide range of improvements and additions that make the book more readable, teachable, flexible, and interesting. However, the basic features of the first edition, as well as the tone and emphases, have been retained. The following are the most noteworthy changes in the second edition.

ENHANCED COVERAGE OF KEY TOPICS: Many topics troublesome to students have been given expanded coverage. For example, the material on quantifiers has been expanded, including the addition of examples from Lewis Carroll; material on big-O estimates has been expanded and clarified; the treatment of mathematical induction has been enhanced; the material on the recursive definition of sets has been improved; and the coverage of graph theory, including bipartite graphs and graph isomorphism, has been enhanced.

NEW TOPICS: The second edition of the book contains two new optional sections requested by many instructors. A section on probability theory has been added to Chapter 4, supplementing the material on discrete probability. This section covers probability functions, expected values, conditional probability, and Bernoulli trials; it also applies this material to computing the average-case complexity of algorithms. A section on number theory has been added to Chapter 2; it covers such topics as the Chinese Remainder Theorem, arithmetic with large integers, Fermat's Little Theorem, pseudoprimes, and public key cryptography.

NEW APPLICATIONS: Many applications have been added. These include pseudorandom number generation, hashing functions, computer arithmetic with large integers, public key cryptography, and interconnection networks for parallel processors.

NEW EXAMPLES: More than 50 examples have been added, including many that help clarify difficult points. Examples in the first edition identified as less than effective have been replaced with better examples.

NEW AND IMPROVED EXERCISES: More than 500 new exercises have been added. These include routine exercises (especially where lacking in the first edition), many intermediate-level exercises, and selected challenging exercises. Exercises that were unclear or ambiguous have been clarified or deleted. In the second edition exercises of a particular type occur both as odd-numbered and even-numbered exercises. The grading of exercises has been reviewed and revised.

REVIEW QUESTIONS: Review questions are now provided at the end of each chapter. These questions are designed to help students think through the most important ideas in each chapter. To answer these questions students must write out complete answers, rather than just provide short responses.

BIOGRAPHICAL AND HISTORICAL FOOTNOTES: More than 50 brief biographies covering ancient, classical, and modern mathematicians and computer scientists have been added as footnotes. In addition, interesting historical notes are included throughout the book as footnotes, supplementing the historical information included in the main body of the text.

IMPROVED DESIGN: The design of the book has been enhanced for easier reading. For example, all definitions are now screened in color so that they are clearly highlighted.

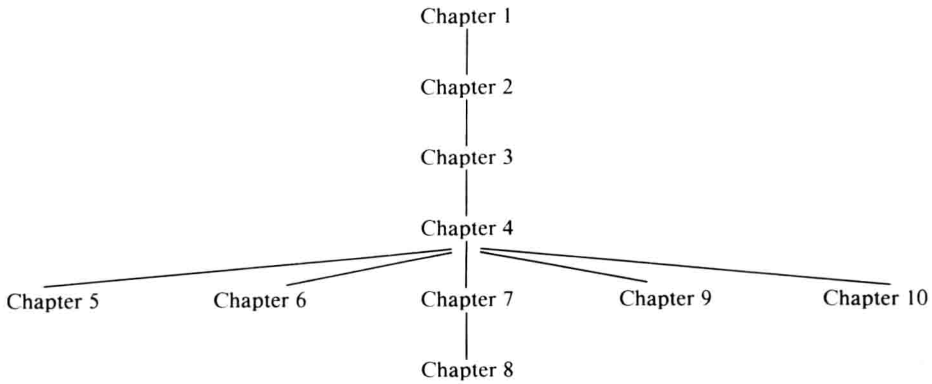
NEW AND IMPROVED ANCILLARIES: A supplemental volume *Applications of Discrete Mathematics*, is now available for use by instructors in conjunction with the text. The **Student Solutions Guide** for the second edition contains material that helps students answer review questions and provides sample tests and crib sheets for each chapter to help students prepare for exams. Material designed to help instructors teaching a writing-across-the-curriculum-style course using this text and an accompanying Student Workbook will also be available.

How to Use This Book

This text has been carefully written and constructed to support discrete mathematics courses at several levels. The following table identifies the core and optional sections. An introductory one-term course in discrete mathematics at the sophomore level can be based on the core sections of the text, with other sections covered at the discretion of the instructor. A two-term introductory course could include all the optional mathematics sections in addition to the core sections. A course with a strong computer science emphasis can be taught by covering some or all of the optional computer science sections.

Chapter	*Core Sections*	*Optional Computer Science Sections*	*Optional Mathematics Sections*
1	1.1–1.9 (as needed)		
2	2.1–2.3, 2.6 (as needed)	2.4	2.5
3	3.1–3.3	3.4, 3.5	
4	4.1-4.4	4.7	4.5, 4.6
5	5.1, 5.4	5.3	5.2, 5.5
6	6.1, 6.3, 6.5	6.2	6.4, 6.6
7	7.1–7.5		7.6–7.8
8	8.1	8.2, 8.3, 8.4	8.5, 8.6
9		9.1–9.4	
10		10.1–10.4	

Instructors using this book can adjust the level of difficulty of their course by omitting the more challenging examples at the end of sections as well as the more challenging exercises. The dependence of chapters on earlier chapters is shown in the following chart.



Ancillaries

STUDENT SOLUTIONS GUIDE: This student manual, available separately, contains full solutions to all the odd-numbered problems in the exercise sets. These solutions explain why a particular method is used and why it works. For some exercises, one or two other possible approaches are described to show that a problem may be solved in several different ways. The guide also includes sample tests for each chapter and a sample crib sheet for each chapter, both designed to help students prepare for exams. Students have found this guide extremely useful.

INSTRUCTOR'S RESOURCE GUIDE: This manual contains full solutions to even-numbered exercises in the text. It also provides suggestions on how to teach the material in each chapter of the book, including the points to stress in each section and how to put the material into perspective. Furthermore, the manual contains sample examination questions for each chapter; the solutions to these sample questions are provided as well. Finally, sample syllabi are presented.

COMPUTER PROJECTS SOLUTIONS GUIDE: A manual containing solutions to the computer projects is available to instructors who adopt the text. This manual gives the code in Pascal for these projects, including sample input and output. The programs are available on a disc that will run on a DOS PC.

APPLICATIONS OF DISCRETE MATHEMATICS: This ancillary is a separate text that can be used either in conjunction with the text or independently. It contains more than 20 chapters (each with its own set of exercises) written by instructors who have used the text. Following a common format similar to that of the text, the chapters in this book can be used as a text for a separate course, for a student seminar, or for a student doing independent study. Subsequent editions of this ancillary are planned that will broaden the range of applications covered. Instructors are invited to submit additional applications for possible inclusion in later versions.

Acknowledgments

I would like to thank the many instructors and students at many different schools who have used this book and provided me with their valuable feedback and helpful suggestions. Their input has made this a much better book than it would have been otherwise. In particular, I would like to thank my students at Monmouth College for their help in the preparation of the second edition. I especially want to thank Jerrold Grossman and John Michaels for their technical reviews of the second edition and their "eagle eyes," which have helped ensure the accuracy of this book.

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Kenneth H. Rosen

To the Student

What is discrete mathematics? Discrete mathematics is the part of mathematics devoted to the study of discrete objects. (Here *discrete* means consisting of distinct or unconnected elements.) The kind of problems solved using discrete mathematics include: How many ways are there to choose a valid password on a computer system? What is the probability of winning a lottery? Is there a link between two computers in a network? What is the shortest path between two cities using a transportation system? How can a list of integers be sorted so that the integers are in increasing order? How many steps are required to do such a sorting? How can a circuit be designed that adds two integers? You will learn the discrete structures and techniques needed to solve problems such as these.

More generally, discrete mathematics is used whenever objects are counted, when relationships between finite sets are studied, and when processes involving a finite number of steps are analyzed. A key reason for the growth in the importance of discrete mathematics is that information is stored and manipulated by computing machines in a discrete fashion.

There are several important reasons for studying discrete mathematics. First, through this course you can develop your mathematical maturity, that is, your ability to understand and create mathematical arguments. You will not get very far in your studies in the mathematical sciences without these skills. Second, discrete mathematics is the gateway to more advanced courses in all parts of the mathematical sciences. Math courses based on the material studied in discrete mathematics include logic, set theory, number theory, linear algebra, abstract algebra, combinatorics, graph theory, and probability theory (the discrete part of the subject). Discrete mathematics provides the mathematical foundations for many computer science courses, including data structures, algorithms, data base theory, automata theory, formal languages, compiler theory, computer security, and operating systems. Students find these courses much more difficult when they have not had the appropriate mathematical foundations from discrete math. Also, discrete mathematics contains the necessary mathematical background for solving problems in operations research, including many discrete optimization techniques, chemistry, engineering, biology, and so on. In the text, we will study applications to some of these areas.

Finally, I would like to offer some helpful advice to students about how best to learn discrete mathematics. You will learn the most by working exercises. I suggest you do as many as you possibly can, including both the exercises at the end of each section of the text and the supplementary exercises at the end of each chapter. Always attempt exercises yourself before consulting the answers at the end of the book or the Student Solutions Guide. Only once you have put together a solution, or you find yourself at an impasse, should you look up the suggested solution. At that point you will find the discussions in the Student Solutions Guide most helpful. When doing exercises, keep in mind that the more difficult exercises are marked as described in the following table.

Key to the Exercises

No marking	A routine exercise
*	A difficult exercise
**	An extremely challenging exercise
☞	An exercise containing a result used in the text
(requires calculus)	An exercise whose solution requires the use of limits