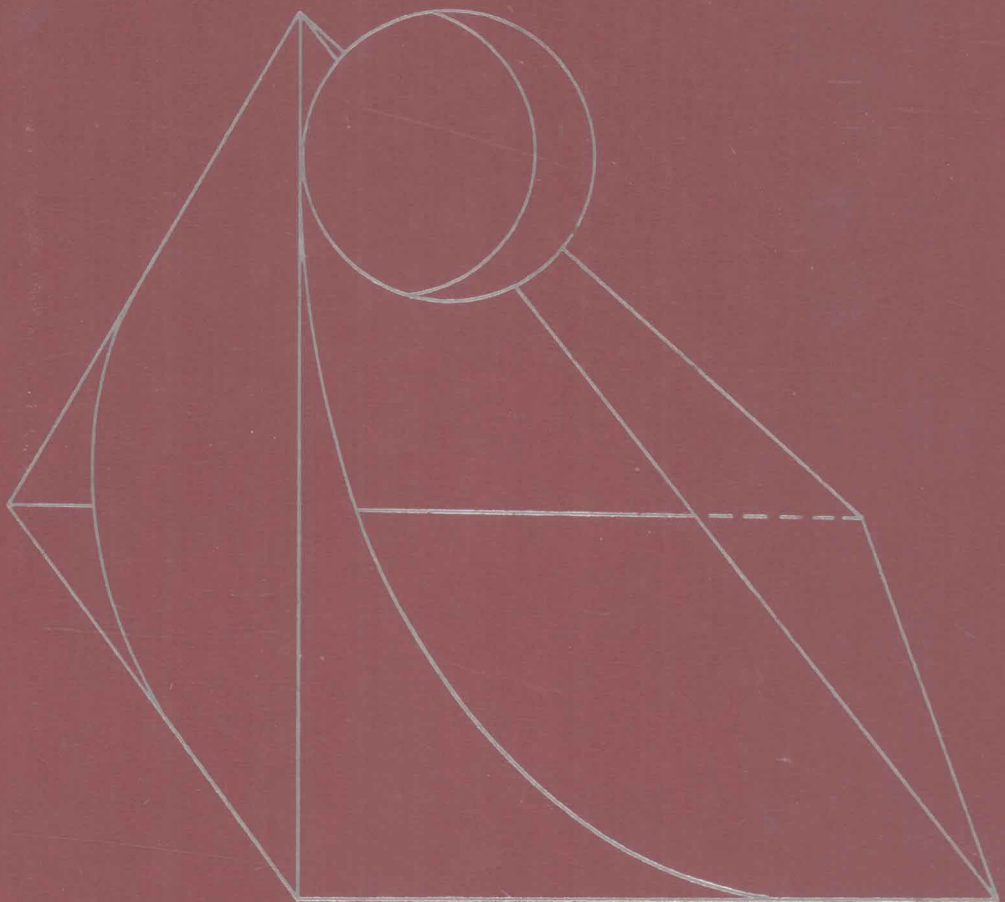
RICHARD S. PAUL
ERNEST F. HAEUSSLER

ALGEBRA AND TRIGONOMETRY

FOR COLLEGE STUDENTS



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For College Students

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PREFACE

This text is designed for students who need to develop manipulative skills in algebra and trigonometry. It can be used either by students taking a terminal course in mathematics or by those who need a background for more advanced work in mathematics. Included are functions and their applications as well as other topics that we feel are most basic to students' needs. We have purposely omitted certain topics so as not to forfeit our emphasis on basics.

Every attempt has been made to write in "down-to-earth" language with short paragraphs, simple vocabulary, and conversational style. Over 800 examples, in which all steps are shown, have been included. Where appropriate, diagrams appear in order to convey and reinforce ideas.

After most sections, two sets of exercises appear. First, there are Completion Sets, which include programmed-type problems. These are followed by Problem Sets, which include traditional-type problems. In each Problem Set the exercises are given in order of increasing difficulty. However, there are a sufficient number of problems that students can do rather easily. This allows students to gain confidence and skills and to fix firmly in their minds the basic concepts studied.

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At the end of each chapter there is a Review section that contains a list of important terms and symbols, together with page numbers that indicate where these items first appear. Following the list are numerous review problems. In total, there are more than 4100 exercises in the text. At the back of the book are answers to all Completion Sets and all odd-numbered exercises in the Problem Sets and Review Problems.

Numerous verbal problems (requiring no prior knowledge) are scattered throughout. Thus, students can see relevant ways in which the mathematics they are learning can be used.

A skull and crossbones symbol is employed to highlight errors commonly made by students. It is intended to alert them and cause them to "beware." We feel that it is just as important for students to know what they cannot do as to know what they can do.

The solution of basic equations occurs early in the book. Using equations as a springboard, the text proceeds to manipulative aspects of algebra that enable students to solve more complicated problems.

Chapter 1 contains selected topics in arithmetic. Although not necessarily part of a formal course, it nevertheless provides a convenient reference for students. In Chapter 3, positive integral exponents and radicals are introduced. A more complete treatment is given in Chapter 13 where zero, fractional, and negative exponents are considered along with further properties of radicals. By dividing exponents and radicals in this way, students are not overburdened at the beginning of the course with laborious calculations involving negative and fractional exponents.

Available from the publisher is an extensive instructor's manual that contains answers to all exercises in the Problem Sets and all Review Problems. Worked-out solutions to many of the problems, including all verbal problems, are also included. In addition, the manual offers examination questions and their answers.

We wish to express our sincere appreciation to the following people who contributed comments and suggestions for the improvement of the manuscript: Elmar Zemgalis of Highline Community College, Irene Buttery of Long Beach City College, Shirley Sorenson of the University of Maryland, Marie A. Ritten of the National College of Business, Harold Harutunian of Salem State College, and George W. Schultz of St. Petersburg Junior College.

Finally, we express our thanks to our typist Mary Walko who never failed us in meeting deadlines.

Richard S. Paul
Ernest F. Haeussler, Jr.

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CHAPTER 1

Arithmetic Refresher

Many things in algebra are done in the same way as in arithmetic. That's why a good understanding of arithmetic may be important to you in learning algebra. For this reason we'll begin with a brief review of some basic topics in arithmetic.†

1-1 BASIC OPERATIONS

There are four rules to keep in mind when you work with numbers. Two are:

1. **You can add numbers in any order.**
2. **You can multiply numbers in any order.**

For example, $5 + 4 = 9$ and $4 + 5 = 9$. So,

$$5 + 4 = 4 + 5 = 9.$$

†Note to instructor: In this chapter, only nonnegative rational numbers are considered.

Rule 2 involves multiplication. Some ways to show a multiplication, say 6 times 2, are (6)(2) and 6(2). Thus by Rule 2,

$$(6)(2) = (2)(6) = 12.$$

EXAMPLE 1

Order is not important in addition or multiplication.

a. $2 + 3 + 4 = 3 + 2 + 4 = 3 + 4 + 2 = 9.$

b. $(5)(2)(3) = (5)(3)(2) = (3)(5)(2) = 30.$

When you multiply numbers, the answer is called their **product**. Each of the numbers is called a **factor** of the product. For example,

$$(3)(2) = 6.$$

↑ ↑ ↑

These are *factors* of 6. This is the *product* of 3 and 2.

The next rules are:

3. **You must subtract numbers in the given order.**
4. **You must divide numbers in the given order.**

$5 - 2$ is **not** the same as $2 - 5$. Also, $6 \div 3$ is **not** the same as $3 \div 6$. When you subtract or divide, order **is** important. We point out that in division there are some words you should know:

$$\underset{\uparrow}{6} \quad \div \quad \underset{\uparrow}{3} \quad = \quad \underset{\uparrow}{2}$$

dividend *divided by* **divisor** *equals* **quotient**

In some problems with more than one operation, it may not be clear to you as to what operation to do first. *In these cases do any multiplications and divisions first.* Then do any additions and subtractions.

EXAMPLE 2

The Right Way	*The Wrong Way*
a. $2 + (3)(4) = 2 + 12 = 14.$	**Don't** add 2 to 3 and then multiply by 4. (This gives 20.)

b. $10 - (3)(2) = 10 - 6 = 4.$ **Don't** subtract 3 from 10 and then multiply by 2. (This gives 14.)

c. $(5)(7) - 6 = 35 - 6 = 29.$ **Don't** subtract 6 from 7 and then multiply by 5. (This gives 5.)

EXAMPLE 3

$$\begin{aligned} 16 + 4(5) \div 2 - 3(2) &= 16 + 20 \div 2 - 6 \\ &= 16 + 10 - 6 \\ &= 26 - 6 = 20. \end{aligned}$$

When operations are inside parentheses, do those first, as the next example shows.

EXAMPLE 4

a. $14 + (1 + 4) - 3(6 - 3)$ ← Do these first.

$= 14 + 5 - 3(3)$ [Next, do the multiplication.]

$= 14 + 5 - 9$ [Do the addition next.]

$= 19 - 9$ [Finally, do the subtraction.]

$= 10.$

b. $12 + (15 \div 3)(4 - 2)$ ← Do these first.

$= 12 + (5)(2)$ [Next, do the multiplication.]

$= 12 + 10$

$= 22.$

c. $(2 - 2)(5) = (0)(5) = 0.$ *Remember*: **0 times any number is 0.**

Don't mix up $2 + 5(6)$ *with* $(2 + 5)6$. *We have*

$$2 + 5(6) = 2 + 30 = 32,$$

but $$(2 + 5)(6) = (7)(6) = 42.$$

Problem Set 1–1

In Problems **1–32**, *compute the numbers.*

1. $5 + (2)(8)$ **2.** $17 - (3)(4)$ **3.** $(2)(4) - 7$

4. $(3)(2) - (2)(2)$ **5.** $(8 - 3)(4)$ **6.** $2(5 + 1)$

7. $15 - 3(4)$ **8.** $(6 - 4) + 3$ **9.** $3(4 + 6)$

10. $(6 - 6)8$ **11.** $(7 - 2)(3 + 1)$ **12.** $(4 + 5)(2 - 0)$

13. $0(8 - 5)$ **14.** $(3 - 3)(4 - 4)$ **15.** $(6 - 1) - (10 - 7)$

16. $(6 + 7) - (8 - 4)$ **17.** $(8 \div 4) + 3$ **18.** $(7 - 2)(3) + 2(3 + 1)$

19. $10 + (9 \div 3)$ **20.** $(3 \div 3) + (2)(3 - 1)$

21. $18 \div (6 + 3)$ **22.** $55 \div (17 - 6)$

23. $(6 + 2 - 3) \div (7 - 2)$ **24.** $(8 - 4)(7 - 3) + (16 \div 4)$

25. $13 - 2(5 - 2) + (2 + 3)$ **26.** $8 + (15 \div 3)(3 - 1)$

27. $(36 \div 6)(6 \div 2) - 6$ **28.** $16 + (12 \div 3)(4 - 3)$

29. $4(4 - 1) - 5(4 \div 2) + 9$ **30.** $2(3 + 4) + 5 - (2 - 1)(2 + 1)$

31. $3(6 \div 3) + 2 - 4(2 \div 2)$ **32.** $(4 + 3)(3) + 5(5 - 2)(2 - 2)$

33. In $8 \div 2 = 4$, what is the dividend? What is the divisor?

34. In $16 \div 8 = 2$, what is the quotient?

35. In $30 \div (3 + 2) = 6$, what number is the divisor?

36. The product of three factors is 30. Two of the factors are 2 and 3. What is the other factor?

37. The product of five factors is 32. Three of the factors are each 2. What is the *product* of the other factors?

38. The numbers 2 and 3 are factors of 6. Find two other whole numbers that are also factors of 6. (Whole numbers are the numbers 0, 1, 2, 3, etc.)

1-2 FRACTIONS

You can write the division $6 \div 3$ as the fraction $\frac{6}{3}$. In this fraction, 6 is the **numerator** and 3 is the **denominator**. The fraction is also called the **quotient** of 6 divided by 3, or simply the quotient $\frac{6}{3}$. Since $6 \div 3 = 2$, then $\frac{6}{3} = 2$. Here is

another way of looking at $\frac{6}{3}$:

$$\frac{6}{3} \text{ is the number which when multiplied by 3 gives 6.}$$

Since (3)(2) = 6, then $\frac{6}{3} = 2$.

In some fractions the numerator is *less* than the denominator, as in $\frac{3}{8}$. We call this a **proper fraction**. Its value is less than 1. Fractions where the numerator is *equal to or larger than* the denominator, as in $\frac{9}{2}$, are called **improper**. The value is at least 1.

$$\text{Proper fractions:} \quad \frac{1}{2}, \frac{12}{17}, \frac{100}{102}.$$

$$\text{Improper fractions:} \quad \frac{15}{8}, \frac{3}{3}, \frac{102}{100}.$$

It's possible to write an improper fraction as a whole number† plus a proper fraction. We call this a **mixed number**. To write $\frac{19}{5}$ as a mixed number, we divide 19 by 5.

$$\begin{array}{r} 3 \\ 5\overline{)19} \\ \underline{15} \\ 4 \end{array}$$

This gives 3 with a remainder of 4. So $\frac{19}{5}$ is 3 plus the fraction $\frac{4}{5}$.

$$\underset{\substack{\uparrow \\ \text{improper} \\ \text{fraction}}}{\frac{19}{5}} = 3 + \frac{4}{5} = \underset{\substack{\uparrow \\ \text{mixed} \\ \text{number}}}{3\frac{4}{5}}.$$

EXAMPLE 1

Writing improper fractions as mixed numbers.

a. $\frac{15}{11} = 1 + \frac{4}{11} = 1\frac{4}{11}$. b. $\frac{17}{3} = 5\frac{2}{3}$. c. $\frac{27}{4} = 6\frac{3}{4}$.

---

†Whole numbers are the numbers 0, 1, 2, 3, etc.

Fractions that have the same value are called **equivalent fractions**. Here's an important rule for getting equivalent fractions. It is called the **fundamental principle of fractions**.

> **You can multiply or divide both the numerator and denominator of a fraction by the same number (except 0). The result is equivalent to the original fraction.**

EXAMPLE 2

Using the fundamental principle of fractions.

a. *Write* $\frac{5}{8}$ *as an equivalent fraction whose denominator is* 24.

24 can be written as a product of 8 and 3. So we must multiply the denominator by 3 to get 24. Thus we must also multiply the numerator by 3.

$$\frac{5}{8} = \frac{(5)(3)}{(8)(3)} = \frac{15}{24}.$$

b. *Write* 7 *as an equivalent fraction with denominator* 2.

Since 7 is the same as the fraction $\frac{7}{1}$, we have

$$7 = \frac{7}{1} = \frac{(7)(2)}{(1)(2)} = \frac{14}{2}.$$

c. *Write* $\frac{12}{20}$ *as an equivalent fraction with numerator* 3.

The numerator will be 3 if we divide it by 4. So we also divide the denominator by 4.

$$\frac{12}{20} = \frac{12 \div 4}{20 \div 4} = \frac{3}{5}.$$

A fraction is in **lowest terms**, or is (completely) **reduced**, when its numerator and denominator have no whole-number factors in common except 1. For example, $\frac{18}{12}$ is not in lowest terms because 18 and 12 have a common factor of 6:

$$18 = (6)(3) \quad \text{and} \quad 12 = (6)(2).$$

To reduce $\frac{18}{12}$, let's use the fundamental principle by dividing the numerator and denominator by this common factor 6.

$$\frac{18}{12} = \frac{\frac{18}{6}}{\frac{12}{6}}.$$

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