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393

M. Remoissenet M. Peyrard (Eds.)

**Nonlinear Coherent
Structures
in Physics and Biology**

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Nonlinear Coherent Structures in Physics and Biology

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This Book is Dedicated to the Memory of Stephanos Pnevmatikos

PREFACE

This volume contains the text of most of the contributions presented at the 7th Interdisciplinary Workshop on "Nonlinear Coherent Structures in Physics and Biology", which was held on the campus of the Université de Bourgogne, Dijon, France, from June 4 to 6, 1991, with about 80 participants.

As with earlier workshops in this series, the purpose of this workshop was to bring together scientists concerned with recent developments and various aspects of nonlinear structures and to provide a forum to stimulate the exchange of ideas among scientists of different backgrounds, including physicists, mathematicians, biologists and engineers.

Nature provides many examples of coherent nonlinear structures and waves, and these have been observed and studied in various fields, ranging from fluids and plasmas through solid state physics to chemistry and biology. Among these beautiful nonlinear phenomena, localized wave packets, solitary waves and solitons, which propagate without dispersing, are the simplest structures, and these provide a continuing source of fascination for the student of nonlinearity. In fact, many real systems sharing the same underlying nonlinear phenomenon can be modeled by the same basic equations, leading to an understanding of their dynamic properties. This correctly indicates the importance of maintaining the interdisciplinary feature of nonlinear science.

The proceedings reflect the remarkable progress in understanding and modeling nonlinear phenomena in various systems, and these new developments show that the study of nonlinear coherent structures is in a state of healthy growth. Experimental, numerical and theoretical activities are interacting in various studies, which we present according to the following classification :

- magnetic and optical systems
- biosystems and molecular systems
- lattice excitations and localized modes
- two-dimensional structures
- theoretical physics
- mathematical methods

We gratefully acknowledge the *Centre National de la Recherche Scientifique*, the *Région Bourgogne* and the *Université de Bourgogne*, which contributed to the opportunity of gathering in Dijon leading scientists in both experimental and theoretical nonlinear science by providing the workshop with financial support .

We are grateful to Mrs A.Levy, D.Arnoux and Y.Boiteux for their active collaboration in the meeting, and to all our colleagues who helped us in many ways.

Dijon , June 1991

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PART I

MAGNETIC AND OPTICAL SYSTEMS

EQUATIONS OF MOTION FOR VORTICES IN 2-D EASY-PLANE MAGNETS

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The dynamics of individual and pairs of vortices in a classical easy-plane Heisenberg spin model is studied. There are two types of vortices possible: in-plane, with small out-of-plane spin components present only at nonzero velocity, and out-of-plane, with large out-of-plane spin components even when at rest. As a result, the two types are governed by different equations of motion when in the presence of neighboring vortices. We review the static spin configurations and the changes due to nonzero velocity. An equation of motion introduced by Thiele and used by Huber will be re-examined. However, that equation may be inadequate to describe vortices in the XY model, due to their zero gyrovector. An alternative dynamical equation is developed, and effective mass and dissipation tensors are defined. These are relevant for models with spatially anisotropic coupling in combination with easy-plane spin exchange.

INTRODUCTION

A model for the dynamic correlations of vortices in easy-plane two-dimensional magnets has been presented, that uses the idea of an ideal gas of weakly interacting vortices.¹ Assuming a Boltzmann velocity distribution, and if the velocity-dependent spin field of the vortices is known, then the dynamic structure function $S^{\alpha\alpha}(\vec{q},\omega)$ can be determined. At the microscopic level we would like to investigate the time-dependent motion of a single

vortex, to understand how the neighboring vortices cause forces and accelerations, and to have a clear picture of how equilibrium is achieved.

Huber^{2,3} has done such an analysis for diffusive motion of so-called "out-of-plane" vortices, ones that possess large out-of-easy-plane spin components. However, it is now realized that there are two type of vortices possible,^{4,5} depending on the strength of the easy-plane anisotropy.^{6,7} The stable vortices of the XY model, for example, are so-called "planar" vortices that only have small out-of-plane spin components. In that case the equation of motion that was used^{3,8} is found to be inapplicable because these planar vortices have a zero gyrovector, to be discussed below. Here we propose an alternative dynamic equation of motion that applies to both types of vortices.

We begin by summarizing the properties of the two types of vortices allowed in the easy-plane anisotropic ferromagnetic Heisenberg model. The derivation of the equation of motion introduced by Thiele,⁸ in terms of conserved force densities, will be sketched out, and the breakdown for planar vortices will be discussed. An alternative formalism using a canonical momentum for the vortex is developed. The new equation of motion includes the effects of vortex shape changes that are the result of acceleration. This leads to definition of an effective mass tensor, and, the gyrovector also re-appears. The new equation allows for a consistent description of both types of vortices.

Anisotropic Heisenberg Ferromagnet

The model system is the nearest neighbor 2D Heisenberg ferromagnet with easy-plane anisotropic exchange, characterized by a parameter $0 \leq \lambda < 1$; the Hamiltonian is