

**BASIC
MATHEMATICS
FOR TRADES
AND
TECHNOLOGIES**

BASIC MATHEMATICS FOR TRADES AND TECHNOLOGIES

CHERYL CLEAVES
MARGIE HOBBS
PAUL DUDENHEFER, PH.D.

*State Technical Institute at Memphis
Memphis, Tennessee*

PRENTICE-HALL, INC., Englewood Cliffs, New Jersey 07632

Library of Congress Cataloging in Publication Data

CLEAVES, CHERYL S. (date)

Basic mathematics for trades and technologies.

Includes index.

1. Mathematics—1961— I. Hobbs, Margie J.,
1943— II. Dudenhefer, Paul. III. Title.

QA39.2.C58 1983 512'.1 82-16492

ISBN 0-13-063032-2

Editorial/production supervision
and interior design: Karen Skrable

Cover design: Ray Lundgren

Manufacturing buyer: Anthony Caruso

Page layout: James M. Wall

© 1983 by Prentice-Hall, Inc., Englewood Cliffs, New Jersey 07632

*All rights reserved. No part of this book may be
reproduced, in any form or by any means,
without permission in writing from the publisher.*

Printed in the United States of America

10 9 8 7 6 5 4 3 2 1

ISBN 0-13-063032-2

Prentice-Hall International, Inc., London

Prentice-Hall of Australia Pty. Limited, Sydney

Editora Prentice-Hall do Brasil, Ltda., Rio de Janeiro

Prentice-Hall Canada Inc., Toronto

Prentice-Hall of India Private Limited, New Delhi

Prentice-Hall of Japan, Inc., Tokyo

Prentice-Hall of Southeast Asia Pte. Ltd., Singapore

Whitehall Books Limited, Wellington, New Zealand

Preface

Rapid growth of technology develops greater need for skilled personnel in business and industry. Competence in basic job-related mathematics is one of the most essential skills required for employment in the trades and technologies. *Basic Mathematics for Trades and Technologies* was developed to help meet this need for trained personnel.

Content

Our text provides an easy-to-understand approach to building skills in basic technical mathematics. We begin with a thorough review of arithmetic in Part I and then carefully guide the student through basic algebra in Part II. Using the foundation in basic algebra, we continue with fundamentals of geometry and elementary trigonometry in Parts III and IV, respectively.

Readability Level

To estimate the readability level of our text, we selected 100-word samples from every 50 pages and applied both the Fry and the Flesch readability formulas to the samples. We then averaged the results obtained from application of each formula to each sample passage. From this analysis we determined that the readability level of our text is what Flesch calls "standard," that is, comparable to the readability levels of daily newspapers and popular magazines. We believe this level makes our text quite suitable for just about any instructional use. It is easy enough so that students will not get bogged

down by the narrative and explanations, and yet is mature enough so that our narrative and explanations will not talk down to the students.

Flexibility of Use

Basic Mathematics for Trades and Technologies is written for use in the classroom, business and industrial training programs, or learning laboratories. Our text may be easily adapted to self-paced courses, individualized programs, or independent study.

The instructional method of the text uses simple, direct language; easy-to-find rules, definitions, and explanations; and over 600 number-coded examples with step-by-step directions for each skill presented. Using these examples, we show nearly 1000 different problems and their solutions. This frees the instructor from having to make unnecessary explanations and makes our text easy for both instructors and students to use in an unlimited number of instructional programs and learning situations.

The organization of our text also contributes to its flexibility of use. Each chapter is divided into sections that present one set of related topics at a time. Each section contains one or more sets of self-study exercises with answers and a set of assignment exercises for additional practice in the skills of the entire section. Our text contains more than 180 sets of self-study exercises, and over 100 sets of assignment exercises comprising over 4000 problems. Each chapter ends with a trial test that enables the student to review the major skills and concepts of the entire chapter. The twenty-four trial tests contain a total of over 600 problems and each problem is coded to the appropriate section in the text. With this organizational format we have achieved in our text a desirable variety of exercise types, practice materials, and review aids—making our text suitable for use with various instructional methods, teaching styles, and learning preferences.

The material is sufficient for two semesters or three quarters if a thorough presentation is intended. However, the material may be presented in one semester or two quarters if only a review or survey is intended or if chapters not needed in some programs or by some classes are omitted or treated in less depth than other chapters.

Applications

In *Basic Mathematics for Trades and Technologies*, we include an abundance of trade, technical, and other job-related applications. Included are applications in welding, bookkeeping, pipefitting, carpentry, finance, mechanics, air conditioning and refrigeration, economics, TV service, drafting, building construction, electronics, surveying, electricity, masonry, business operations, machinery, manufacturing, automotive technology, and other areas. We use a practical, learn-by-doing approach to basic technical mathematics without frills and with a minimum of theory.

Use of Calculators

Use of a basic calculator is optional for Parts I and II, Arithmetic and Algebra. In many cases it may be preferable for the student to perform the arithmetic calculations, particularly in Part I, without a calculator to assure a solid foundation in basic computational skills. In other cases it may be more desirable for the student to acquire speed and accuracy in the use of a calculator to assure efficient use later in the text and on the job.

For Parts III and IV, Geometry and Trigonometry, we recommend use of a basic calculator. In these latter parts, a basic calculator is used to take the square roots of numbers not included in the square root table and to perform the multiple calculations of geometry and trigonometry with precision and convenience. To make these computations without a calculator would be too time consuming both in the classroom and on the job.

In Parts I and II we show both noncalculator and calculator answers (in parentheses) if they differ because of precision lost through rounding. In Parts III and IV we show only the calculator answers. Here the major difference between calculator and noncalculator answers will be found in problems involving square roots of numbers not in the table. All other noncalculator answers will be approximately the same as the calculator answers.

To help the student, we include in Part I a handy explanation of how to use a basic calculator to perform a continuous series of operations in problem-solving. Throughout the text we highlight particular problems where a calculator is especially convenient and indicate the sequence of calculator operations necessary for solving the problem. We present the calculator as an aid to problem solving and not as a substitute for mathematical competence.

Field Testing

Basic Mathematics for Trades and Technologies was field tested at State Technical Institute at Memphis, Richards Manufacturing Company, and Ripley Industries, Inc., all located in or near Memphis, Tennessee. We wish to thank these institutions and the instructors for field testing our text. We especially thank Ted Davis and Jana Turner, who made valuable suggestions for improvement.

Acknowledgments

We thank Ara Sullenberger, Tarrant County Junior College; Gary Wright, State Technical Institute at Knoxville; Joseph Jordan, John Tyler Community College; and Frank Juszli, who reviewed the manuscript and offered valuable suggestions in the development stage of the project. We also thank the administration at State Technical Institute at Memphis for continuous en-

couragement and support throughout the project. We thank the personnel in the various business, computer, and engineering technologies and in the related studies at State Technical Institute at Memphis for offering suggestions and helpful comments during the development of the manuscript. We also thank the members of our department for their contributions to this project. We thank all the students who worked through the manuscript and offered many valuable comments for its improvement. Finally, we thank Sheree Holcomb and Docie Peden, who typed a significant portion of the manuscript.

CHERYL CLEAVES
MARGIE HOBBS
PAUL DUDENHEFER

Contents

Preface xiii

I
ARITHMETIC
1

- 1 Whole Numbers 3
 - 1-1 Reading Numbers 3
 - 1-2 Rounding Numbers 6
 - 1-3 Adding Whole Numbers 10
 - 1-4 Subtracting Whole Numbers 17
 - 1-5 Multiplying Whole Numbers 24
 - 1-6 Dividing Whole Numbers 32
 - 1-7 Order of Operations 41
- 2 Fractions 48
 - 2-1 Fraction Terminology 48
 - 2-2 Equivalent Fractions 52
 - 2-3 Improper Fractions and Mixed Numbers 58
 - 2-4 Comparing Fractions and Finding Common Denominators 61
 - 2-5 Adding Fractions and Mixed Numbers 64
 - 2-6 Subtracting Fractions and Mixed Numbers 69
 - 2-7 Multiplying Fractions and Mixed Numbers 73
 - 2-8 Dividing Fractions and Mixed Numbers 78
 - 2-9 Complex Fractions 83

	Decimals 86
	3-1 <i>Decimal Numbers and the Place-Value System</i> 86
	3-2 <i>Rounding Decimals</i> 93
	3-3 <i>Adding and Subtracting Decimals</i> 97
	3-4 <i>Multiplying Decimals</i> 101
	3-5 <i>Dividing Decimals</i> 105
	3-6 <i>Finding Averages and Estimating</i> 110
	3-7 <i>Changing Decimals to Fractions and Fractions to Decimals</i> 114
4	Percents 120
	4-1 <i>Changing Numbers to Percent Equivalents</i> 120
	4-2 <i>Changing Percents to Numerical Equivalents</i> 123
	4-3 <i>Common Equivalents</i> 129
	4-4 <i>Problems Involving Percent</i> 132
	4-5 <i>Increases and Decreases</i> 145
	4-6 <i>Business Applications</i> 153
5	Metric System of Measurement 162
	5-1 <i>Introduction to the Metric System</i> 162
	5-2 <i>Changing Metric Units</i> 167
	5-3 <i>Basic Operations with Metric Measures</i> 174
6	English System of Measurement and Metric-English Comparisons 180
	6-1 <i>Review of the English System</i> 180
	6-2 <i>Basic Operations with English Measures</i> 184
	6-3 <i>Dividing Measures by Measures; Rate Measures</i> 191
	6-4 <i>Reading the English Rule</i> 195
	6-5 <i>Metric-English Comparisons</i> 199
	6-6 <i>Temperature Conversions</i> 203

II ALGEBRA

209

7	Basic Operations with Signed Numbers 211
	7-1 <i>Signed Numbers</i> 211
	7-2 <i>Adding Signed Numbers</i> 215
	7-3 <i>Subtracting Signed Numbers</i> 221
	7-4 <i>Multiplying Signed Numbers</i> 225
	7-5 <i>Dividing Signed Numbers</i> 229

	Solving Equations	233
	8-1 Algebraic Terminology	233
	8-2 Solving Basic Equations	238
	8-3 Isolating the Variable in Solving Equations	244
	8-4 Applying the Distributive Principle in Solving Equations	249
	8-5 Expressing Verbal Statements as Equations	252
9	Fractional and Decimal Equations	259
	9-1 Solving Fractional Equations	259
	9-2 Combining Like Fractional Terms Under Addition	267
	9-3 Using Proportions to Solve Problems	276
	9-4 Solving Equations Containing Decimal Numbers	282
10	Powers	287
	10-1 Laws of Exponents and Basic Operations with Powers	287
	10-2 Basic Operations with Algebraic Expressions	294
	10-3 Powers of 10	299
	10-4 Scientific Notation	302
11	Square Roots and Square Root Radicals	310
	11-1 Square Roots	310
	11-2 Simplifying Square Root Radicals with Perfect Square Factors in the Radicand	317
	11-3 Simplifying Fractions with Square Root Radicals in the Denominators	320
	11-4 Equations with Squares and Square Roots	323
12	Formula Evaluation and Rearrangement	329
	12-1 Formula Evaluation	329
	12-2 Formula Rearrangement	334
13	Products and Factoring	340
	13-1 The Distributive Principle and Common Factors	340
	13-2 Multiplying Two Binomials	342
	13-3 Factoring Special Products	346
	13-4 Factoring General Trinomials	349
14	Solving Quadratic Equations	355
	14-1 Types of Quadratic Equations	355
	14-2 Solving Pure Quadratic Equations	356
	14-3 Solving Incomplete Quadratic Equations	358
	14-4 Solving Complete Quadratic Equations by Factoring	361
	14-5 Solving Quadratic Equations Using The Quadratic Formula	362

- 15 **Graphs and Linear Equations 370**
 15-1 *Reading Circle, Bar, and Line Graphs 370*
 15-2 *The Rectangular Coordinate System 376*
 15-3 *Graphing Linear Equations With Two Variables 383*
- 16 **Systems of Equations With Two Variables 392**
 16-1 *Solving Systems of Equations Graphically 392*
 16-2 *Solving Systems of Equations Using The Addition Method 395*
 16-3 *Solving Systems of Equations by Substitution 402*
 16-4 *Applied Problems 404*

III
GEOMETRY
409

- 17 **Angles and Angle Measure 411**
 17-1 *Points, Lines, and Planes 411*
 17-2 *Angles 415*
 17-3 *Angle Calculations 421*
- 18 **Triangles 428**
 18-1 *Classification of Triangles 428*
 18-2 *Congruent and Similar Triangles 435*
 18-3 *Right Triangles 441*
 18-4 *Special Right Triangles 447*
- 19 **Polygons and Their Areas and Perimeters 459**
 19-1 *Squares, Rectangles, and Parallelograms 459*
 19-2 *Trapezoids and Triangles 468*
 19-3 *Composite Shapes 474*
- 20 **Circles 490**
 20-1 *Area and Circumference of a Circle 490*
 20-2 *Area of Sectors and Segments 501*
 20-3 *Inscribed and Circumscribed Regular Polygons and Circles 508*
- 21 **Geometric Solids 524**
 21-1 *Prisms and Cylinders 524*
 21-2 *Spheres and Cones 530*

IV
TRIGONOMETRY
537

- 22 Introduction to Trigonometry 539
22-1 *Radians and Degrees* 539
22-2 *Trigonometric Functions* 547
22-3 *Reading Trigonometric Tables* 555
- 23 Right Triangle Trigonometry 565
23-1 *Sine, Cosine, and Tangent Functions* 565
23-2 *Applied Problems Using Right Triangle Trigonometry* 577
- 24 Oblique Triangles 588
24-1 *Vectors and Trigonometric Functions For Angles More Than 90°* 588
24-2 *Law of Sines* 596
24-3 *Law of Cosines* 608
24-4 *Area of Triangles* 616
- Appendices 627
- Index 631

Arithmetic

Arithmetic is a branch of mathematics that helps us solve problems involving numbers. When we calculate the gasoline mileage an automobile gets after an engine tune-up, we are using arithmetic. We are using arithmetic when we figure out how much mortar is needed to lay a brick wall or what combination of resistors is needed to complete an electronic circuit. We are also using arithmetic when we compute the cooling capacity of an air conditioner, estimate the amount of paint needed to redecorate a living room, take a temperature reading, or set the margins on a typewriter. Arithmetic gives us short-cuts for solving these and other job-related problems containing numbers.



Whole Numbers

When we study a subject for the first time or review in some detail a subject we studied some time ago, we need to begin with the basics. Often, as we examine the basics of a subject, we discover—or rediscover—many bits and pieces of useful information. In this sense arithmetic is no different from any other subject. We will begin arithmetic with a study of whole numbers and the basic operations we perform with them, including the order of these operations. It is whole numbers and their operations which form the foundation for our study of arithmetic. We will begin our study of whole numbers in this chapter with a review of how to read numbers.

1-1 READING NUMBERS

Our system of numbers uses 10 individual figures called *digits*: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. A *number* is made up of one or more digits. When a number contains two or more digits, each digit must be in the right place for the number to have the value we want it to have. If we mean “ninety-eight,” we must place the 9 first and the 8 second to represent 98. If we change the places of these two digits by putting the 8 first and the 9 second, we get a new value (eighty-nine) and a new number (89).

Place Value

Each place a digit can occupy in a number has a value called a *place value*. If we know the place value of each digit in a number, we can read the number and understand how much it means. Look at the chart of place values in

Billions			Millions			Thousands			Units		
Hundred Billions (100,000,000,000's)	Ten Billions (10,000,000,000's)	Billions (1,000,000,000's)	Hundred Millions (100,000,000's)	Ten Millions (10,000,000's)	Millions (1,000,000's)	Hundred Thousands (100,000's)	Ten Thousands (10,000's)	Thousands (1,000's)	Hundreds (100's)	Tens (10's)	Ones (1's)

Figure 1-1

Fig. 1-1. Notice that each place value *increases* as we move from *right to left* and that each increase is *10 times* the value of the place to the right. For example, the tens place is ten times the ones place, the hundreds place is ten times the tens place, and so on.

The place values are arranged in groups of three to make numbers easier to read. The first group of three is called *units*, the second group of three is called *thousands*, the third group is *millions*, and the fourth group is *billions*. Commas are used to mark off these groups in a number. In four-digit numbers the comma separating the units group from the thousands group is optional. Thus, both 4,575 and 4575 are correct.

Example In the number 2,472,694,500 identify the place value of the digit 7.
1-1.1 To solve this problem we will apply the following rule.

Rule 1-1.1. Identifying place values of digits:

1. Identify the place-value *group* in which the digit is located.

2,	472,	694,	500
Billions	Millions	Thousands	Units

The 7 is in the millions group.

2. In the millions group, name the place which the digit 7 occupies.

		ten millions place	
2,	472,	694,	500
Billions	Millions	Thousands	Units

Answer: 7 is in the ten millions place.

Self-Study Exercises 1-1.1

In the number 2,304,976,186 identify the place value of the following digits.

- | | | | | |
|------|------|------|------|------|
| 1. 3 | 2. 7 | 3. 1 | 4. 0 | 5. 2 |
|------|------|------|------|------|

In the number 8,972,069,143 identify the place value of the following digits.

- | | | | | |
|------|------|------|------|-------|
| 6. 0 | 7. 4 | 8. 7 | 9. 8 | 10. 6 |
|------|------|------|------|-------|

Answers

1. hundred millions 2. ten thousands 3. hundreds 4. ten millions 5. billions
6. hundred thousands 7. tens 8. ten millions 9. billions 10. ten thousands

Now that we have reviewed place values, we can read numbers without much difficulty. All we need to do is follow the three steps shown in the following examples.

Example 1-1.2 Read 7543026129 by writing it in words.

Rule 1-1.2. Reading numbers:

1. Starting at the right, separate the number into place-value groups with commas.

7,543,026,129

2. Name each place-value group.

7,	543,	026,	129
Billions	Millions	Thousands	Units

3. From the left, read the numbers in each group and the group name. (The group name *units* is not usually read.)

Seven *billion*, five hundred forty-three *million*, twenty-six *thousand*, one hundred twenty-nine

Note: The word "and" should *not* be used when reading whole numbers. Also, the numbers from 21 to 99 use a hyphen when they are written out as two words (forty-three, twenty-six, etc.).

Example 1-1.3 Show how 2000125 would be read by writing it in words.

1. 2,000,125 (*Separate with commas.*)

2. 2, 000, 125 (*Name groups.*)
 Millions Thousands Units

3. Two *million*, one hundred twenty-five (*Read from left.*)

Note: If a place-value group contains all zeros, it usually is *not* read.

Self-Study Exercises 1-1.2

Show how these numbers would be read by writing them out as words.

- | | | | |
|----------------|----------|--------------|------------|
| 1. 6704 | 2. 89021 | 3. 662900714 | 4. 3000101 |
| 5. 15407294376 | 6. 150 | | |

Write these words as numbers. Use commas when necessary.

7. Seven billion, four hundred
8. One million, six hundred twenty-seven thousand, one hundred six
9. Fifty-eight thousand, two hundred one
10. In a telephone conversation a contractor submitted the following bid for a job, "one thousand six dollars." Write this bid in numbers.