## SECURITIES MARKETS



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# SECURITIES MARKETS

#### Kenneth Garbade

Professor of Economics and Finance Graduate School of Business Administration New York University

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#### **SECURITIES MARKETS**

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### **PREFACE**

This text is the outgrowth of a set of lecture notes I used from 1973 to 1981 in teaching a course called "Financial Markets: Their Structure and Performance" at the Graduate School of Business Administration, New York University. The course, and this text, have two principal foci: (1) the pricing of securities and (2) the institutional characteristics of securities markets, including both the new issue and secondary markets.

The text is divided into eight parts. Part One provides a descriptive introduction to the major types of financial claims traded in American securities markets, including Treasury, municipal, and corporate debt and corporate stock. Part One also describes how issuers sell new issues in the primary markets through price auctions, subscription offerings, underwritten offerings, and tap offerings.

The analytical core of the text consists of the equilibrium valuation of common stock—separated into the analysis of investor demand (portfolio theory) in Part Three and the characterization of a stock market in equilibrium (the capital asset pricing model and the efficient markets hypothesis) in Part Four—and the equilibrium valuation of claims with a finite lifetime in Parts Five and Six. The latter includes the term structure of interest rates, the structure of settlement prices on futures contracts, the pricing of call option contracts, and the pricing of risky debt.

The text also includes, in Parts Seven and Eight, a description of the structure of a variety of secondary markets and an analysis of the behavior of transactions prices in those markets. These topics, which are given only cursory treatment in most finance texts, have become areas of active interest to researchers and regulators during the past decade. My own teaching experience suggests that the material in Parts Seven and Eight provides a useful point of intersection between the Walrasian auction markets assumed in most economic models (including those in Parts Four, Five, and Six of this text) and the acquaintance of students with real securities markets.

Despite its focus on securities markets, this text also discusses, in Part Two, the Federal Reserve System in order to establish the determinants of short-term interest rates in the Federal funds and repurchase agreement markets.

This reflects the crucial role played by those markets in determining the general level of yields on other financial instruments.

I should mention here that throughout the text the generic masculine pronoun has been used solely on account of the brevity it affords. "The investor...he," for example, is far less cumbersome than "the investor...he or she." Such use of the masculine pronoun should not be interpreted as a wish to exclude women from the use of this text or from the field in general.

In the course of teaching and writing about securities markets, I have had the good fortune to discuss analytical and institutional issues with many practitioners, including Kevin Baltazar, Steve Black, Dick Fisher, Bob Geiger, Irwin Guttag, Eric Gronningsater, David Harris, Joe Hunt, Kevin Kenny, Arlen Klinger, Martin Lipton, Frank McDermott, Andrew Melton III, Bill Melton, Jay Peake, Fred Siesel, Doug Skolnick, and Don Stone. I am particularly indebted to Richard Fieldhouse, Homer Kripke, Alan Lerner, Ken Marks, and Jay Pomrenze for numerous conversations over the past few years.

My greatest obligations are to my friend, coresearcher, and colleague, Bill Silber. His influence on this text, and on our research, is greatly appreciated and can hardly be overestimated, even if I do not heed his wise counsel as often as I should.

I thank Bob Kavesh and Larry Ritter for fostering a creative and enjoyable atmosphere in the Economics and Finance departments at the Graduate School of Business Administration.

I would also like to express my thanks for the many useful comments and suggestions provided by colleagues who reviewed this text during the course of its development, especially to J. Kimball Dietrich (University of Southern California) and Dale Osborne (Oklahoma State University).

Finally, I add the caveat that while the credit for this text must go to many, the errors are mine alone.

> Kenneth Garbade Hoboken, New Jersey

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### MARKETABLE FINANCIAL ASSETS

CHAPTER 1 Treasury and Federal Agency Securities

CHAPTER 2 Corporate Securities

CHAPTER 3 Municipal Securities

CHAPTER 4 Commercial Bank Liabilities

A financial asset represents a claim on the issuer for a stream of future payments. The claim may be for a fixed dollar amount, as with a debt security, or it may be a residual claim, as with common stock. The four chapters in this part present a descriptive introduction to some of the principal financial assets traded in American securities markets. These assets include issues of the United States Treasury, federal agency issues, municipal and corporate securities, and bank-related liabilities. One of the principal objectives of this part is to contrast the claims on future payment streams associated with each security.

New issues of financial assets are sold by issuers in what are called *primary markets*. It is in these markets that governments and corporations raise funds to finance their activities. Each of the chapters in Part One describes how issuers sell new issues. Along the way we note the differences among alternative selling methods, such as auction offerings, subscription offerings, and tap offerings.

Current holders of most of the financial assets described below can exchange their future claims for cash. These exchanges are accomplished by selling the assets in *secondary-market* transactions. Although such transactions do not yield any additional funds for issuers, the existence of secondary markets is quite important to both issuers and investors. The facility with which future claims can be converted to cash is one of the principal reasons investors are willing to buy and hold marketable financial assets.

The remaining chapters in this text are concerned with (1) the pricing of financial assets in secondary markets and (2) the organization of those markets. Part Two analyzes the determination of yields on a basic debt instrument: overnight credit. Equilibrium valuation of common stock is treated in Parts Three

and Four. Valuation of debt securities with a maturity of more than 1 day is treated in Part Five. Part Six analyzes the valuation of options on risky securities and uses that analysis to investigate credit risk on debt instruments. The basic objective in each of these parts is analysis of the relative yields on different financial assets. For example, why should one stock be expected to appreciate at a greater rate than another stock, and why should yields on debt instruments differ as a function of the maturities of those instruments?

The last two parts of the text describe and analyze more closely the actual functioning of secondary markets. The institutional structures of secondary markets are described in Part Seven. Part Eight analyzes the transactional characteristics of secondary-market purchases and sales. The objective of these two parts is to study the processes by which buyers and sellers trade securities.

In summary, this text is concerned with the nature of securities, their valuation, and their exchange. We begin with the securities themselves.

### TREASURY AND FEDERAL AGENCY SECURITIES

From time to time the federal government issues debt securities to finance expenditures in excess of tax receipts or to refinance maturing debt. The cumulative size of the federal deficits carried over from World War II and the large deficits of the 1970s has fostered a central role for Treasury securities in American financial markets. Investors find those securities attractive because they are free of the risk of default: Treasury securities do not bear what is conventionally known as credit risk, or the risk that a debtor will not pay according to his promises. In addition, Treasury securities are usually outstanding in issue sizes which run to \$5 billion or more. These large sizes ensure the existence of what is called a liquid secondary market, or one in which buyers and sellers can trade large quantities of a given security quickly without affecting the market price of the security. Many investors have a strong preference for Treasury securities because of their great liquidity. The first two sections of this chapter describe the characteristics of two types of Treasury debt: bills and coupon issues.

In addition to its regular tax and expenditure programs, the federal government also sponsors several financial intermediaries, commonly called federal agencies. These intermediaries issue debt securities and relend the proceeds of their borrowings for the benefit of selected interest groups, primarily farmers and home buyers. Although the activities of the federal credit agencies are not substantively different from those of private financial intermediaries like commercial banks and finance companies, discussions of federal agency debt are traditionally coupled with descriptions of Treasury debt. Moreover, the secondary markets for agency debt are closely associated with those for Treasury

debt, because the same investors are typically active in both markets. The third section of this chapter discusses some of the important characteristics of federal agency debt.

#### 1.1 TREASURY BILLS

Treasury bills are promises of the United States Treasury to pay a stipulated amount (called the *face value* of the bill) on a stated maturity date. There are no intermediate payments such as semiannual coupons associated with the obligation. All Treasury bills have a maturity date no more than 1 year from their issue date; that is, they have original maturities of less than a year. Most bills currently issued have original maturities of 91 days (13 weeks), 182 days (26 weeks), or 364 days (52 weeks).

#### **Price Conventions**

One might think that because the holder of a Treasury bill receives only one future payment in return for his current investment outlay, the rate of return on a Treasury bill should be unambiguous and easy to express. This is not quite the case. In fact, market participants use one type of interest rate (the discount rate) when they are buying and selling bills and a different interest rate (the bond equivalent yield) when they are measuring rates of return. Moreover, analysts frequently use a third expression (the continuously compounded yield) for comparing returns on bills with different maturities. We now examine each of these interest rates.

The Discount Rate Bills offered for sale and sought for purchase in the secondary market are quoted on a discount rate basis. Suppose a bill with n days to maturity is currently priced at P percent of its face value, that is, at P per \$100 face value. That bill has a discount rate d defined as

$$d = \frac{360}{n} \frac{100 - P}{100} \tag{1.1}$$

The difference between face value and market price is the discount on the bill. It represents the dollar gain accruing to an investor who holds the bill to maturity. The discount rate is the annualized percentage discount from face value, assuming simple interest (no compounding) over a 360-day year. Market quotations for bills are conventionally made in terms of discount rates rather than dollar prices or percent of face value. The implied price can be computed readily by solving for P in Equation (1.1). Exhibit 1.1 shows an example of Treasury bill price and discount rate calculations.

The Bond Equivalent Yield The discount rate is a poor measure of the rate of return on a Treasury bill, because it is based on face value rather than on the

cost of the bill to the investor, and because it assumes a short (360-day) year. (It is important only because it is so widely used for quotation purposes.)

A more reasonable approximation of the return on a bill held to maturity is the bond equivalent yield. Letting P and n be as above, the bond equivalent yield i is defined as

$$i = \frac{365}{n} \frac{100 - P}{P} \tag{1.2}$$

The yield is the gain per dollar invested (rather than per dollar of face value), assuming simple interest over a 365-day year. The bond equivalent yield corrects for the two most glaring defects of the discount rate as a measure of the rate of return on a Treasury bill: the assumption of a short year and the use of face value as the base on which the gain is calculated. The bond equivalent yield on a Treasury bill is always greater than the discount rate on that bill, because the price of the bill is less than face value. Exhibit 1.1 shows how the bond equivalent yield is computed for a Treasury bill.

The Continuously Compounded Yield The bond equivalent yield on a Treasury bill does not correct the assumption of simple interest used to compute the discount rate. Comparing the yields on two bills with different maturities can, consequently, be misleading. Since there is no obvious choice for the compounding period on a bill, most analysts assume continuous compounding when they want standardized yields for comparative purposes. For given values of P and n, the continuously compounded yield r is defined as

$$r = \frac{365}{n} \ln \left[ \frac{100}{P} \right] \tag{1.3}$$

where  $\ln [x]$  is the natural logarithm of x. Readers unfamiliar with continuous compounding may wish to refer to the appendix to this chapter. Solving for P in Equation (1.3) gives

$$P = 100 \exp\left[-r\frac{n}{365}\right] \tag{1.4}$$

where exp [x] is the base e of natural logarithms raised to the power x. Equation (1.4) shows that the price P is the present value of \$100 to be received n days in the future, discounted at the continuously compounded annual rate r.

 $^{1}$  When a Treasury bill has more than 182 days to run to maturity, the bond equivalent yield i is implicitly defined by the equation

$$100 = P(1 + \frac{1}{2}i) \left( 1 + \frac{1}{2}i \frac{n - 182.5}{182.5} \right)$$

<sup>&</sup>lt;sup>2</sup> The appendix to this chapter shows how rates of return can be computed for investment intervals of arbitrary length using continuously compounded yields.

Exhibit 1.1 shows the computation of continuously compounded yields for several different Treasury bill maturities.

#### **New Issues**

Treasury bills come to market through periodic public auctions open to all investors. The auctions are held by the twelve district Federal Reserve banks acting as fiscal agents for the United States Treasury. New supplies of 91- and

**EXHIBIT 1.1**YIELDS ON TREASURY BILLS ON TUESDAY, MAY 16, 1978

Maturity date	Days to maturity (n)	Quoted discount rate (d), %	Price ( <i>P</i> )	Bond equivalent yield (i), %	Continuously compounded yield (r), %
	2	6.13	\$99.966	6.22	6.21
June 15	30	5.99	99.501	6.10	6.09
July 13	58	6.00	99.033	6.14	6.11
August 17	93	6.41	98.344	6.61	6.55
September 14	121	6.59	97.785	6.83	6.76
October 12	149	6.82	97.177	7.12	7.01

Computations for bill maturing August 17:

To compute the price, solve for P in the equation for the discount rate [Equation (1.1)]: d = (360/n) [(100 - P)/100].

$$P = 100(1 - nd/360)$$

$$= 100(1 - 93(.0641)/360)$$

$$= 100(1 - .01656)$$

$$= 98.344$$

To compute the bond equivalent yield, use Equation (1.2):

$$i = \frac{365}{n} \frac{100 - P}{P}$$
$$= \frac{365}{93} \frac{100 - 98.344}{98.344}$$
$$= .0661, \text{ or } 6.61\%$$

To compute the continuously compounded yield, use Equation (1.3):

$$r = \frac{365}{n} \ln \left[ \frac{100}{P} \right]$$

$$= \frac{365}{93} \ln \left[ \frac{100}{98.344} \right]$$

$$= \frac{365}{93} \ln \left[ 1.01684 \right]$$

$$= \frac{365}{93} \text{ (.01670)}$$

$$= .0655, \text{ or } 6.55\%$$