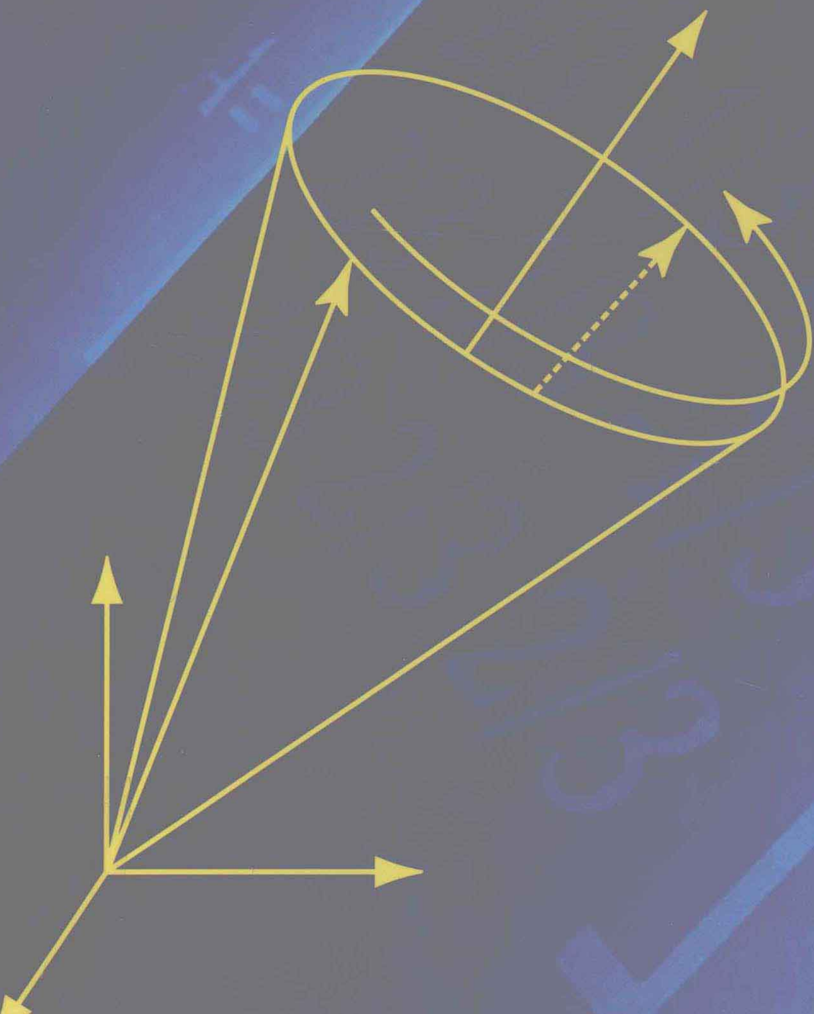
JOHN S. BAY

Fundamentals of

LINEAR STATE SPACE SYSTEMS



Fundamentals of Linear State Space Systems

John S. Bay

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Preface

This book wrote itself over the period of August 1990 to December 1997. It is a result of my teaching the graduate course *Linear System Theory* at Virginia Polytechnic Institute and State University. This is a first course in linear systems taught in the Bradley Department of Electrical and Computer Engineering.

Target Audience

The book is intended to be a comprehensive treatment of the use of linear state space system theory in engineering problems. It is targeted at seniors and first-year graduate students, although much of the material will be accessible to students with only an understanding of basic signals and systems principles. It is intended to gather into a single volume the linear algebra, vector space, and state space theories now used in many engineering texts, but which are often covered in separate courses and separate departments. The book will have appeal to students in all engineering departments.

Whereas many texts introduce state space theory, it is often presented as a supplement to frequency-domain material, such as after classical methods in control systems or after transfer functions in signals and systems texts. Such texts often forsake the mathematical basics necessary for true understanding of state space modeling and analysis. Rather than use frequency-domain analysis as a prelude to state space, this text uses the more natural and meaningful foundation of vector spaces and linear algebra. Thus, state space analysis can be understood from the mathematical foundations of its own domain, rather than as a counterpart to frequency-domain methods. This text would be ideal in a course dedicated to time-domain analysis (both continuous and discrete). It would also be an appropriate text for a school that treats state variable analysis as a stand-alone course, independent of a student's interest or preparation in control systems. It is written in such a way that it can be *read*; it is not merely a collection of linear algebraic facts arranged in an optimal manner.

Content and Organization

The text is organized into two parts. Part 1 begins with a review of linear algebra and vector spaces, both from a geometric viewpoint. This is done in a manner that complements the material presented in a student's mathematics courses, which sometimes leave the student confused by the distinction between linear algebra and matrix theory. It is assumed that students know some matrix theory (determinants, inverses, gaussian elimination, etc.), but not necessarily linear algebra on any abstract level. Furthermore, it exploits the engineering student's training in spatial relationships, facilitating intuitive understanding. By addressing the engineering student, we can focus on the practical matters, geometry, applications, and implementation issues of linear systems, thus maintaining a student's engineering context throughout. This mathematical introduction is rigorous enough to stand on its own, but not so encumbered by proofs that engineering relevance is sacrificed. While graduate students with sufficient mathematical background might skip it, even a student with a good understanding of vector spaces might benefit from the geometric perspective offered.

As part of the discussion of the mathematical preliminaries, linear algebraic systems are treated. Topics such as subspaces, orthogonal projections, basis changes, inner products, and linear transformations are critical to true understanding of the state space, so it is important that they be covered in some detail. Again, these methods are used to study the geometry of physical systems sometimes neglected in engineering texts. A student without knowledge of such issues would otherwise miss the underlying meaning of such common concepts of eigenvalues and eigenvectors, simultaneous equations, Fourier analysis, and similarity transformations.

Only after these algebraic topics are covered are linear differential methods introduced in Part 2. It is then that we cover the topics that are often given in controls texts as the linear system "basics." The latter part of the book contains control system applications and principles. For all of these latter chapters of the book, a familiarity with s-domain and ω-domain analysis is useful, but a deep understanding of classical control or signal processing is not required.

Both continuous-time and discrete-time systems are discussed throughout, although z-domain material is minimized. Because certain developments in state space systems are more easily understood in one domain or the other, this parallel presentation gives us the flexibility to introduce examples from either domain at our convenience. For example, controllability tests are particularly easy to derive in discrete-time, so that is where they should be first introduced.

It is inescapable that computer-aided engineering (CAE) is an integral component of linear system usage. There are now dozens of books dedicated to the use of MATLAB® for linear system and control system design. Recognizing the importance of the computer but wishing to avoid a book tied too closely to computer simulations, we will make use of margin notes wherever a MATLAB command might do the numerical work of the accompanying discussion, denoted by a superscript M on the applicable term, e.g., $rank^{M}$. For example, we use the margin to indicate the availability of certain MATLAB commands, functions, and toolboxes, but we do not assume that MATLAB programming is a required component of the course, nor does the book instruct on its usage. The margin notes refer the reader to Appendix B, which contains summaries for the commands most relevant to the topic at hand. In addition, the end-of-chapter exercises include some computer-based problems, but these problems will not necessarily be tailored to MATLAB.

Most of the material contained in this book can be covered in a single three semester-hour course. If the course is indeed an undergraduate or first-year graduate course, then Chapter 11 might not be covered in that time span. It is recommended that even students with extensive mathematical preparation not omit the early chapters, because the geometric perspective established in these chapters is maintained in the latter chapters. Furthermore, the applications-oriented examples denoted by italics in the table of contents are concentrated in Part 1. In most of the examples thereafter, it will usually be assumed that the physical application has been modeled in state space form. However, if little mathematical review is necessary, then Chapter 11, *Introduction to Optimal Control and Estimation*, can be used to tie together the contents of the previous ten chapters, and make the course more of a *control* than a *systems* course.

John S. Bay
Blacksburg, Virginia
1998

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For my wife, Judy,
And to my mother, Mildred,
And to the memory of my father, John

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Listings in *italics* denote applications-oriented examples.