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DYNAMICS

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DYNAMICS

Also by J. L. Meriam

STATICS

PREFACE

To the Student

The challenge and responsibility of modern engineering practice demand a high level of creative activity which, in turn, requires the support of strong analytical capability. The subject of engineering mechanics, which includes statics and dynamics, constitutes one of the cornerstones of analytical capability, and all engineers should have a basic background in this field of study.

Today's student of engineering becomes tomorrow's practicing engineer who must, through the exercise of his creative imagination and his professional knowledge, successfully combine theory and practice in the development of new structures, machines, devices, and processes which provide benefit to man. This process of modern creative design depends on the ability to visualize new configurations in terms of real materials and processes and the physical laws which govern them. Maximum progress to support the development of this design capability will be made when engineering theory is learned within the context of engineering reality so that the significance of theory can be perceived as it is being studied. This book is written with the foregoing view in mind, and it is hoped that the student will find interest and stimulation in the many problems which are taken from a wide variety of contemporary engineering situations to provide realistic and significant applications of the theory.

The purpose of the study of mechanics is to predict through calculation the behavior of engineering components and systems involving force and motion. Successful prediction in engineering design requires the careful formulation of problems with the aid of a dual thought process of physical understanding and mathematical reasoning. The process of formulating a problem is one of constructing a mathematical model which incorporates appropriate physical assumptions and mathematical approximations and approaches the actual situation with sufficient accuracy for the purpose at hand. Indeed, this process of matching the symbolic model to its physical prototype is, undoubtedly, one of the most valuable experiences of engineering study, and the problems which are included are intended to provide a comprehensive opportunity to develop this ability.

Success in analysis depends to a surprisingly large degree on a well-disciplined method of attack from hypothesis to conclusion where a straight path of rigorous application of principles has been followed. The student is urged to develop ability to represent his work in a clear, logical, and neat manner. The basic training in mechanics is a most excellent place for early development of this disciplined approach which is so necessary in most engineering work which follows.

More material is contained in this text than is covered in the usual first course in engineering mechanics, so that the book will also serve as an introduction to more advanced topics in mechanics and as a future reference for basic principles. The author extends his encouragement to all students of mechanics and hopes that this book will provide substantial assistance to them in their studies.

J. L. Meriam

January 1966

PREFACE

To the Instructor

In recent times the strong trend to increase the analytical capability in engineering has resulted in increased emphasis on the mathematical generalities in mechanics. When adequate emphasis on physical understanding and engineering application is preserved, then the trend is of great benefit in extending capabilities for the analytical description of difficult problems. On the other hand, when primary attention is focused on the mathematical framework of mechanics with secondary attention to physical reality and engineering usage, then the trend is of questionable benefit. Instruction in engineering mechanics has as its basic purpose the development of capacity to predict the effects of force and motion as an aid in carrying out the creative design process of engineering. Therefore the primary focus should be on the engineering significance of physical quantities with the mathematical structure acting in a supporting role. When this basic purpose is kept in mind, a proper balance between theory and application can be realized.

In this same connection there is often the temptation for the instructor of mechanics who has reached a high level of theoretical ability to forget the frame of reference of his students and present the subject with an overemphasis on generalization. There is considerable danger in this approach for the first basic course, since students lack the background necessary to cope with excessive early generality and they are also deprived of experiencing some of the historical and natural development of the subject.

A further consideration of philosophy is the strong need to provide an environment of challenging engineering reality as a means of developing the motives for learning mechanics. The importance of a solid background of analytical capability can be established in no better way than by creating a genuine interest and a compelling engineering need for the effective use of theory.

Dynamics is an engineering text and is written with these views in mind. Effort has been made to present the theory rigorously, concisely, and with a generality commensurate with the background of basic calculus and introductory differential equations assumed of the reader. For two-dimensional analysis the scalar-geometric method is used generally as the simplest and most direct description. For three-dimensional problems vector notation is employed as the most direct and appropriate description. Matrix and tensor methods are used where it becomes necessary to make transformations from one coordinate system to a rotated coordinate system in three dimensions. The exclusive use of scalar notation, of vector notation, or of tensor index notation is rejected in favor of the choice of the mathematical tool which is most appropriate for the situation at hand. In the author's view it is far more important in the basic course in mechanics to preserve and strengthen dependence on geometrical visualization and physical understanding than it is to emphasize the extensive or exclusive use of a tensor notation that reduces geometry essentially to a notational manipulation. The creative ideas that find greatest use in those branches of engineering which are supported by mechanics are born and developed more through the visualization of geometrical configurations than through the manipulation of notation in analysis.

Dynamics is intended for use by engineering students in their second year and beyond. The treatment is arranged under the three main headings of particles, rigid bodies, and nonrigid systems, each with its appropriate coverage of kinematics and kinetics. This arrangement is particularly convenient, since it permits a logical choice of topics to match a variety of course outlines. If primary emphasis on particle dynamics is desired, rigid-body motion is easily minimized, or vice versa. It is also possible to follow an outline based on a two-dimensional treatment with coverage of three-dimensional motion reserved for later study. Included are introductions to more advanced topics such as the space motion of rigid bodies, the general equations for variable mass, wave motion, Lagrange's equations, and Hamilton's principle. These topics will serve as a basis for further study and future reference.

The wide response accorded the two editions of *Mechanics, Parts I* and *II,* written previously by the author has supported the desirability of including in the present book a large number of interesting and practical problems drawn from a broad range of engineering applications. Numerous examples have been taken from the subject of space mechanics and other contemporary developments. The problems are arranged approximately in order of increasing difficulty. Some problems are much longer than others, so that care will be needed in making assignments. Both publisher and author have given special attention to the provision of clear and detailed illustrations in an effort to establish a strong sense of engineering reality. This effort is consistent with the author's firm belief that experience in the formulation of problems which incorporate a high degree of reality, including a choice of the approach for their solution, is perhaps the most important aspect of the study of engineering mechanics. With this approach, theory takes on a significance it can not possibly have when the student encounters primarily idealized and, hence preformulated, problems.

The author wishes to give special recognition to Dr. A. L. Hale of Bell Telephone Laboratories for his detailed review of the manuscript proofs and for his numerous helpful suggestions. Dr. Hale rendered similar assistance to the author during the publication of his previous books on mechanics, and it is a genuine pleasure to have his continued interest and valuable contributions toward the completion of *Dynamics*. The author also wishes to acknowledge the encouragement, patience, and assistance of his wife during the preparation of the manuscript.

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PRINCIPLES OF DYNAMICS

1. **Introduction.** Dynamics is that branch of mechanics which deals with the motion of bodies under the action of forces. The study of dynamics in engineering usually follows the study of statics which deals with the action of forces on bodies at rest. Dynamics has two distinct parts—*kinematics,* which is the study of motion without reference to the forces which cause motion, and *kinetics,* which relates the action of forces on bodies to their resulting motions. The student of engineering will find that a thorough comprehension of dynamics will provide him with one of his most useful and powerful tools for analysis in engineering.

Historically, dynamics is a relatively recent subject compared with statics. The beginning of a rational understanding of dynamics is credited to Galileo (1564–1642) who made careful observations concerning bodies in free fall, motion on an inclined plane, and motion of the pendulum. He was largely responsible for bringing a scientific approach to the investigation of physical problems. Galileo was continually under severe criticism for refusing to accept the established beliefs of his day, such as the philosophies of Aristotle which held, for example, that heavy bodies fall more rapidly than light bodies. The lack of accurate means for the measurement of time was a severe handicap to Galileo, and further significant developments in dynamics awaited the invention of the pendulum clock by Huygens in 1657. Newton (1642–1727), guided by Galileo's work, was able to make an accurate formulation of the laws of motion and, hence, to place dynamics on a sound basis. Newton's famous work was published in the first edition of his *Principia,** which is generally recognized as one of the greatest of all recorded contributions to knowledge. In addition to stating the laws governing the motion of a particle, Newton was the first to formulate correctly the law of universal gravitation. Although his mathematical description was accurate, he felt that the concept of remote transmission of gravitational force without a supporting medium was an absurd

* The original formulations of Sir Isaac Newton may be found in the translation of his *Principia* (1687), revised by F. Cajori, University of California Press, 1934.

notion. Following Newton's time, important contributions to mechanics were made by Euler, D'Alembert, Lagrange, Laplace, Poinsot, Coriolis, Einstein, and others.

In terms of engineering application, dynamics is an even more recent science. Only since machines and structures have operated with high speeds and appreciable accelerations has it been necessary to make calculations based on the principles of dynamics rather than on the principles of statics. The rapid technological developments of the present day require increasing application of the principles of mechanics, particularly dynamics. These principles are basic to the analysis and design of moving structures, to fixed structures subject to shock loads, to high-speed computer mechanisms, to automatic control systems, to rockets, missiles, and spacecraft, to ground and air transportation vehicles, to electron ballistics of electrical devices, and to machinery of all types such as turbines, pumps, reciprocating engines, hoists, machine tools, etc. The student whose interests lead him into one or more of these and many other activities will find a constant need for applying his basic knowledge of dynamics.

2. Basic Concepts. Certain concepts and definitions are basic to the study of dynamics, and they should be thoroughly understood at the outset.

Space is the geometric region in which events take place. In this book the word space will be used to refer to a three-dimensional region. It is not uncommon, however, to refer to motion along a straight line or in a plane as occurring in one- or two-dimensional space, respectively. The concept of n-dimensional space is an abstract device for describing dependence on n independent quantities.

Reference Frame. Position in space is determined relative to some geometric reference system by means of linear and angular measurements. The basic frame of reference for the laws of Newtonian mechanics is the *primary inertial system* or *astronomical frame of reference* which is an imaginary set of rectangular axes assumed to have no translation or rotation in space. Measurements show that the laws of Newtonian mechanics are valid for this reference system as long as any velocities involved are negligible compared with the speed of light.* Measurements made with respect to this reference are said to be *absolute,* and this reference system is considered to be "fixed" in space. A reference frame attached to the surface of the earth has a somewhat complicated motion in the primary system, and a correction to the basic equations of mechanics must be applied for measurements made relative to the earth's reference frame. In the calculation of rocket and space flight trajectories, for example, the absolute motion of the earth becomes an important parameter. For most engineer-

* For velocities of the same order as the speed of light, 186,000 mi./sec., the theory of relativity must be applied. See Art. 16 for a brief discussion of this theory and a numerical example of its effect.

ing problems of machines and structures which remain on the earth's surface, the corrections are extremely small and may be neglected. For these problems the laws of mechanics may be applied directly for measurements made relative to the earth, and in a practical sense such measurements will be referred to as *absolute*.

Time is a measure of the succession of events and is considered an absolute quantity in Newtonian mechanics. The unit of time is the second, which is a convenient fraction of the period of the earth's rotation.

Force is the action of one body on another. A force tends to move a body in the direction of its action upon it.

Matter is substance which occupies space. A *body* is matter bounded by a closed surface.

Inertia is the property of matter causing a resistance to change in motion.

Mass is the quantitative measure of inertia. Mass is also a property of every body which is always accompanied by mutual attraction to other bodies.

Particle. A body of negligible dimensions is called a particle. When the dimensions of a body are irrelevant to the description of its motion, the body may be treated as a particle. At other times a particle may be chosen as a differential element of a body.

Rigid body. A body which has no relative deformation between its parts is known as a rigid body. This is an ideal hypothesis since all real bodies change shape to a certain extent when subjected to forces. When such changes in shape are negligible compared with the changes of position of the body as a whole, the assumption of rigidity is permissible.

Scalar. A quantity with which a magnitude only is associated is known as a scalar. Examples of scalars are time, volume, density, speed, energy, and mass.

Vector. A quantity with which a direction as well as a magnitude is associated is a vector. Examples of vectors are displacement, velocity, acceleration, force, moment, and momentum.

In *Dynamics* boldface type is used for vectors and lightface type is used for scalars. Thus $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$ represents the vector sum of two vectors whereas $S = S_1 + S_2$ represents the scalar sum of two scalars. The magnitude of a vector $\mathbf{V}$ is written as V, and its direction may be indicated by a unit vector, say $\mathbf{n}$, which has a magnitude of unity and the direction of $\mathbf{V}$. Thus $\mathbf{V} = V\mathbf{n}$. Vectors appearing in diagrams are labeled either with boldface or lightface symbols. When the magnitude only of the vector is indicated or specified on the diagram, a lightface symbol is used. Scalar products of vector quantities are indicated by the dot, $\mathbf{V}_1 \cdot \mathbf{V}_2$, and vector products by the cross, $\mathbf{V}_1 \times \mathbf{V}_2$. It is essential that the student make a clear and consistent distinction between vectors and scalars in his written work. If this distinction is not preserved in his notation, serious difficulty can arise. It is recommended that a distinguishing mark for each vector quantity, such as an underline $\underline{V}$, be used in all handwritten work to take the place of the boldface type in print.

It is assumed that the reader is familiar with the physical properties of force vectors and that he has also been introduced to the mathematics of vectors. Reference may be made to Appendix B for a condensed summary of vector algebra and calculus.

Dynamics involves the frequent use of time derivatives of both vectors and scalars. As a notational shorthand a dot over a quantity will frequently be used to indicate a derivative with respect to time. Thus $\dot{x}$ means dx/dt and $\ddot{x}$ stands for d^2x/dt^2.

3. Newton's Laws. Sir Isaac Newton was the first to state correctly the basic laws governing the motion of a particle and to demonstrate their validity. Slightly reworded, these laws are as follows:

Law I. A particle remains at rest or continues to move in a straight line with a uniform velocity if there is no unbalanced force acting on it.

Law II. The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.*

Law III. The forces of action and reaction between contacting bodies are equal in magnitude, opposite in direction, and collinear.

The correctness of these laws has been verified by innumerable accurate physical measurements. The first two laws hold for measurements made in an absolute frame of reference but are subject to slight correction when the motion is measured relative to a reference system having acceleration, such as the earth's surface.

Newton's second law forms the basis for most of the analysis in mechanics. As applied to a particle of mass m it may be stated as

$$\mathbf{F} = m\mathbf{a}, \tag{1}$$

where $\mathbf{F}$ is the resultant force acting on the particle and $\mathbf{a}$ is the resulting acceleration. Newton's first law is a consequence of the second since there is no acceleration when the force is zero, and the particle either is at rest or moves with a constant velocity. The first law adds nothing new to the description of motion but is included since it was a part of Newton's classical statements.

* To some it is preferable to interpret Newton's second law as meaning that the resultant force acting on a particle is proportional to the time rate of change of momentum of the particle and that this change is in the direction of the force. Both formulations are equally correct when applied to a particle of constant mass.