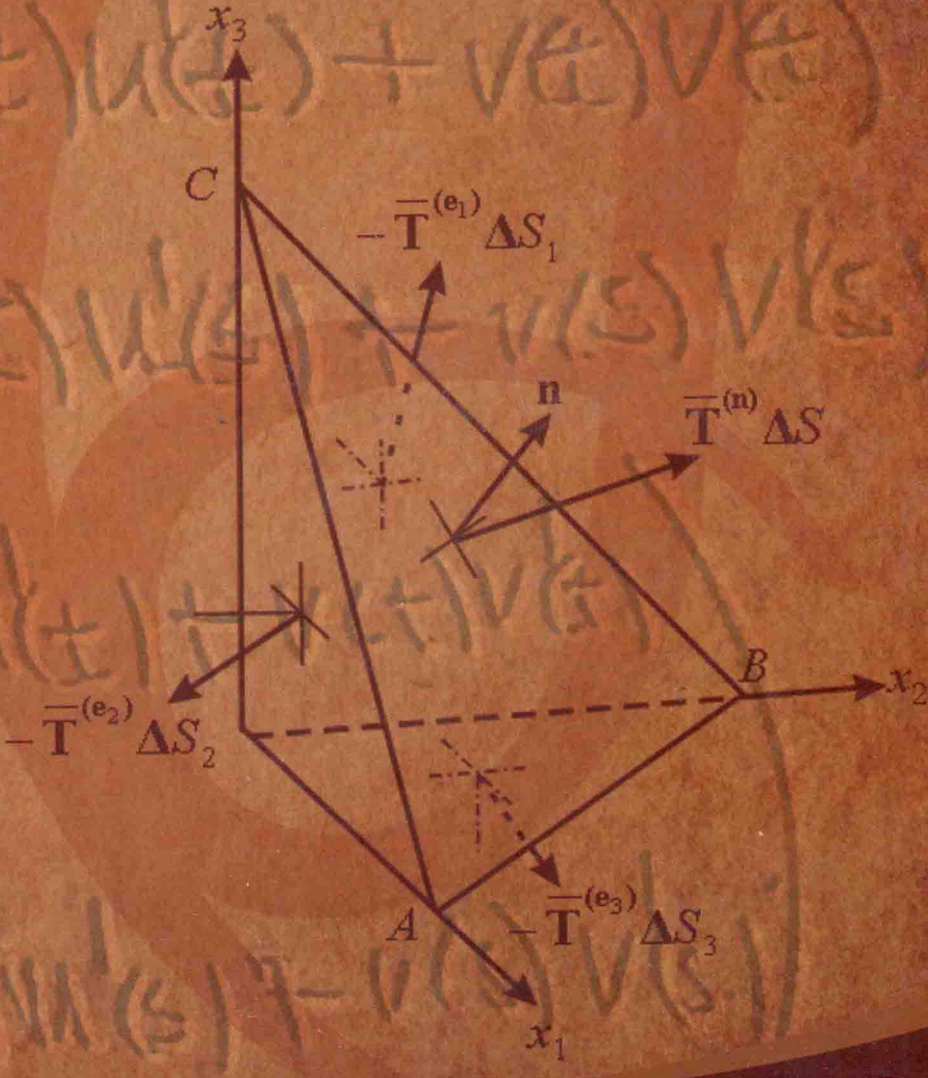
WAVES AND RAYS IN ELASTIC CONTINUA

Michael A. Slawinski



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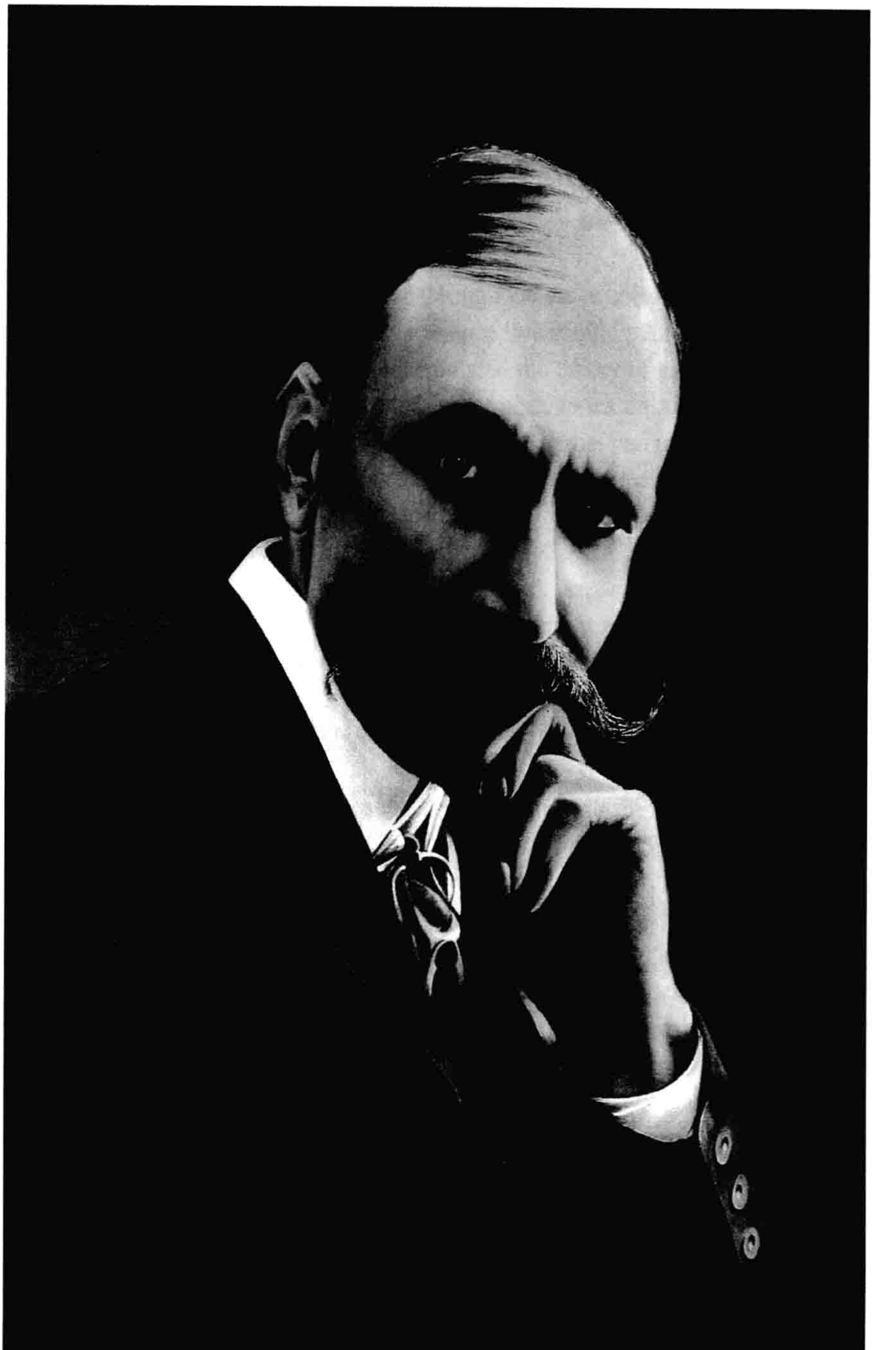

Portrait by Ida Czechowska

Dedication

This book is dedicated to the scientific spirit and accomplishments of Maurycy Pius Rudzki, Chair of Geophysics at the Jagiellonian University, where, in 1895, the first Institute of Geophysics in the world was created.

In order to interpret the wealth of detail contained in the seismographic record of a distant earthquake, we must know the path or trajectory of each ray that leaves the focus or origin in any given direction. An indirect solution of this problem was attempted by several earlier investigators, prominent among whom were Rudzki and Benndorf.

James B. Macelwane (1936) Introduction to theoretical seismology: Geodynamics

Seismological studies appear to have stimulated Rudzki to make the first quantitative calculations on elastic waves.

Michael J.P. Musgrave (1970) Crystal acoustics: Introduction to the study of elastic waves and vibrations in crystals

In the first decade of the [twentieth] century M.P. Rudzki in Cracow began to investigate the consequences of anisotropy in the earth for seismic waves. As far as I can make out, he was the first to determine the wave surface for elastic waves in an anisotropic solid.

Klaus Helbig (1994) Foundations of anisotropy for exploration seismics

Preface

> Il ne suffit pas d'observer, il faut se servir de ses observations, et pour cela il faut généraliser. [. . .] Le savant doit ordonner; on fait la science avec des faits comme une maison avec des pierres; mais une accumulation de faits n'est pas plus une science qu'un tas de pierres n'est une maison.[1]

Henri Poincaré (1902) La science et l'hypothèse

Theoretical formulations of applied seismology are substantiated by observable phenomena. Reciprocally, our perception and understanding of these phenomena necessitate rigorous descriptions of physical behaviours. As stated by Bunge in his book on "Philosophy of Science, Vol. I: From problem to theory",

> A nice illustration of the intertwining of empirical and theoretical events in the actual practice of science is offered by seismology, the study of elastic disturbances of Terra. [...] In conclusion, in order to "read" a seismogram so that it may become a set of data regarding an event (e.g.,

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[1] It is not enough to observe. One must use these observations, and for this purpose one must generalize. [...] The scientist must organize [knowledge]; science is composed of facts as a house is composed of bricks; but an accumulation of facts is no more a science than a pile of bricks is a house.

To emphasize this statement of Poincaré, let us also consider the following quotation.

> As the biggest library if it is in disorder is not as useful as a small but well-arranged one, so you may accumulate a vast amount of knowledge but it will be of far less value to you than a much smaller amount if you have not thought it over for yourself; because only through ordering what you know by comparing every truth with every other truth can you take complete possession of your knowledge and get it into your power.

Arthur Schopenhauer (1851) Parerga and Paralipomena, Volume 2

> an earthquake) or an evidence relevant to a theory (e.g., about the inner structure of our planet), the seismologist employs elasticity theory and all the theories that may enter the design and interpretation of the seismograph.

The present book emphasizes the interdependence of mathematical formulation and physical meaning in the description of seismic phenomena. Herein, we use aspects of continuum mechanics, wave theory and ray theory to explain phenomena resulting from the propagation of seismic waves.

The book is divided into three main parts: *Elastic continua*, *Waves and rays* and *Variational formulation of rays*. There is also a fourth part, which consists of *Appendices*. In *Part 1*, we use continuum mechanics to describe the material through which seismic waves propagate, and to formulate a system of equations to study the behaviour of such a material. In *Part 2*, we use these equations to identify the types of body waves propagating in elastic continua as well as to express their velocities and displacements in terms of the properties of these continua. To solve the equations of motion in anisotropic inhomogeneous continua, we use the high-frequency approximation and, hence, establish the concept of a ray. In *Part 3*, we show that, in elastic continua, a ray is tantamount to a trajectory along which a seismic signal propagates in accordance with the variational principle of stationary traveltime. Consequently, many seismic problems in elastic continua can be conveniently formulated and solved using the calculus of variations. In *Part 4*, we describe two mathematical concepts that are used in the book; namely, homogeneity of a function and Legendre's transformation. This part also contains a *List of symbols*.

The book contains an *Index* that focuses on technical terms. The purpose of this index is to contribute to the coherence of the book and to facilitate its use as a study manual and a reference text. Numerous terms are grouped to indicate the relations among their meanings and nomenclatures. Some references to selected pages are marked in bold font. These pages contain a defining statement of a given term.

This book is intended for senior undergraduate and graduate students as well as scientists interested in quantitative seismology. We assume that the reader is familiar with linear algebra, differential and integral calculus, vector calculus, tensor analysis, as well as ordinary and partial differential equations. The chapters of this book are intended to be studied in sequence. In that manner, the entire book can be used as a manual for a single course. If the variational formulation of ray theory is not to be included in such a course, the entire *Part 3* can be omitted.

Each part begins with an *Introduction*, which situates the topics discussed therein in the overall context of the book as well as in a broader scientific context. Each chapter begins with *Preliminary remarks*, which state the motivation for the specific concepts discussed therein, outline the structure of the chapter and provide links to other chapters in the book. Each chapter ends with *Closing remarks*, which specify the limitations of the concepts discussed and direct the reader to related chapters. Each chapter is followed by *Exercises* and their solutions. While some exercises extend the topics covered, others are referred to in the main text. Reciprocally, the footnotes attached to these latter exercises refer the reader to the sections in the main text, where a given exercise is mentioned. Often, the exercises referred to in the main text supply steps that are omitted from the exposition in the text. Also, throughout the book, footnotes refer the reader to specific sources included in the *Bibliography*.

"Seismic waves and rays in elastic media" strives to respect the scientific spirit of Rudzki, described in the following statement[2] of Marian Smoluchowski, Rudzki's colleague and friend.

> Tematyka geofizyczna musiała nęcić Rudzkiego, tak wielkiego, fantastycznego miłośnika przyrody, z drugiej zaś strony ta właśnie nauka odpowiadała najwybitniejszej właściwości umysłu Rudzkiego, jego dążeniu do matematycznej ścisłości w rozumowaniu.[3]

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[2] Smoluchowski, M., (1916) Maurycy Rudzki jako geofizyk / Maurycy Rudzki as a geophysicist: Kosmos, **41**, 105 – 119

[3] The subject of geophysics must have attracted Rudzki, a great lover of nature. Also, this very science accommodated the most outstanding quality of Rudzki's mind, his striving for mathematical rigour in reasoning.

Changes from First Edition

> Mathematics gives to science the power of abstraction and generalization, and a symbolism that says what it has to say with the greatest possible clarity and economy.
>
> *John Lighton Synge and Byron A. Griffith (1949)*
> *Principles of mechanics*

The conviction expressed by the above quote has inspired both the first and second editions of this book. Significant additions and modifications have been included in the second edition. These changes came about as a result of teaching from this text, readers' enquiries, and research into concepts discussed in the first edition. Summary of these changes and the reason for their inclusion are given below.

Part 1:

Chapter 1: To familiarize the reader with the fundamentals that underlie the seismic theory, we introduce in Section 1.2 the three rudimentary concepts of continuum mechanics, namely, material body, manifold of experience and system of forces.

Chapter 2: To deepen the reader's understanding of the arguments used to formulate the balance principles of continuum mechanics, we include in Sections 2.4 and 2.7 the particle-mechanics motivation of the balance of linear momentum and the balance of angular momentum, respectively. To make the reader aware of possible extensions within continuum mechanics, we discuss in Section 2.7 the distinction between the strong and weak forms of Newton's third law of motion. As a results, we show that the symmetry of the stress tensor is a consequence of the strong form, and not a fundamental physical law.

Chapter 3: To deepen the reader's understanding of the fundamentals that underlie the formulation of seismic theory, we introduce in Section 3.1 the three principles of constitutive equations, namely, determinism, local action and objectivity. To provide the background that allows us to emphasize the meaning of elasticity, we introduce in Section 3.4 constitutive equations of anelastic continua.

Chapter 5: Following a demonstration that only eight classes of the elasticity tensor exist,[4] all these classes are included in this edition. The addition consists of Sections 5.8 and 5.11, where we study trigonal and cubic continua.

Part 2

Chapter 6: Since this chapter deals with the wave equation, which plays an important role in seismology, we wish to deepen the reader's understanding of different aspects of this equation. In Section 6.4, we discuss well-posedness of the wave equation with its initial conditions. To provide further insight into the properties of the wave equation, we compare it in Section 6.6 to the evolution equation, which is ill-posed. Another significant addition consists of the formulation and study of solutions of the wave equation in two and three spatial dimensions, as presented in Section 6.5. Therein, we discuss also the range of influence and the domain of dependence of these solutions. To familiarize the reader with the concepts of reflection and transmission in the context of the wave equation, we study the solutions of one-dimensional scattering, which is discussed in Section 6.7. To familiarize the reader with the several-century-long debate concerning the applicabilty of the wave equation, which is a differential equation, in order to study nondifferentiable solutions, we present the theory of distributions and its application to the wave equation. The crux of this addition is Section 6.8. Furthermore, in Section 6.9, we elaborate on the concept of the reduced wave equation and Fourier's transport of the wave equation, which is used in the following chapter to discuss trial solutions of the equations of motion.

Chapter 7: Since this chapter deals with the equations of motion whose trial solutions necessitate asymptotic methods, we wish to familiarize the reader with the concept of the asymptotic series in the context of ray theory,

[4] Readers interested in the existence of the eight symmetry classes might refer to Bóna, A., Bucataru, I., and Slawinski, M.A., (2005) Characterization of elasticity-tensor symmetries using SU(2). Journal of Elasticity **75**(3), 267 – 289, and to Bóna, A., Bucataru, I., and Slawinski, M.A., (2004) Material symmetries of elasticity tensor. The Quarterly Journal of Mechanics and Applied Mathematics **57**(4), 583 – 598.

which is commonly referred to as the asymptotic ray theory. This addition is contained in Sections 7.2.3 and 7.2.4.

Chapter 8: Since Hamilton's ray equations are the basis of ray theory, we wish to deepen the reader's understanding of their meaning, as well as the meaning of their solutions. To do so, we present an analytic solution of Hamilton's ray equations, which allows us to gain both mathematical and physical insights into Hamilton's equations. Furthermore, an equivalent solution is presented using Lagrange's ray equations. Hence, this analytic solution allows us to relate the Hamiltonian and Lagrangian formulations of ray theory — two distinct approaches. The crux of this addition consists of Section 8.5.

Chapter 9: The convexity and detachment of the innermost phase-slowness surface are commonly invoked in seismology. In this edition, we show particular cases for which this surface is not detached, and we comment on the fact that detachment is not necessary to prove the convexity of the innermost surface.

Chapter 10: In this edition, we use Newton's third law of motion — rather than the second one used in the first edition — to derive in Section 10.2.1, the dynamic boundary conditions on the interface between two media. Since we are dealing with two media acting on one another, the third law lends itself more naturally than the second one for which we had to treat the interface as a layer with a vanishing thickness.

Part 3

Chapter 14: In this edition, the traveltime expression for a signal in a continuum exhibiting an elliptical velocity dependence with direction and a linear dependence with depth is valid for the entire ray: both the downgoing and upgoing segments. In the first edition, the expression was valid for a downgoing ray only.

In this edition, the chapter on Lagrange's ray equations is found at the end of *Part 2,* rather than immediately after the chapter on Hamilton's ray equations. This placement of the chapter emphasizes the physical motivation; namely, we can study the entire ray theory in the context of Hamilton's ray equations. Lagrange's ray equations, on the other hand, open a new path of study to which we devote *Part 3.*

As a result of teaching from the first edition, the second edition acquired more exercises and figures. There is also a *List of Figures* included.

With the above additions, the title of the book has been modified to emphasize the generality of the approach and the importance of the concept of continuum. Thus, "Seismic waves and rays in elastic media" became "Waves and rays in elastic continua".

Acknowledgments

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Editorial revisions of the text with Cathy Beveridge, from the original sketch to the final drafts for both editions, enhanced the structure and coherence of this book.

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