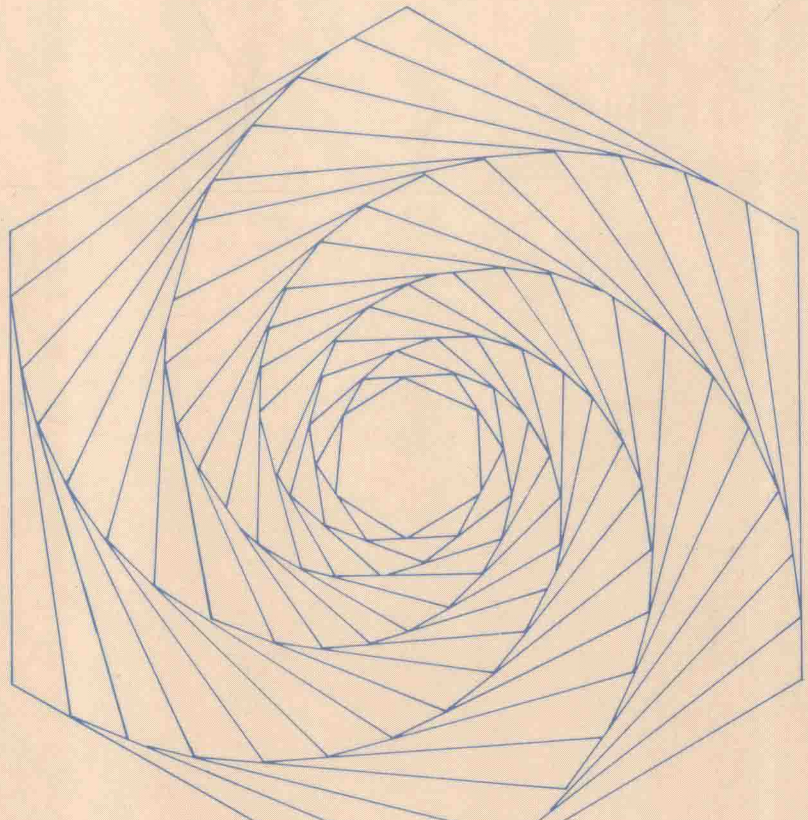
STUDY GUIDE TO ACCOMPANY

KOLMAN • DENLINGER

APPLIED CALCULUS



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Preface

This Study Guide is designed to be used in conjunction with *Applied Calculus* by Bernard Kolman and Charles G. Denlinger. It contains solutions to nearly one-half the odd-numbered exercises in that textbook. The techniques used in the solutions here are identical to those used in the text; thus, this Study Guide reinforces the material presented. However, since the textbook exercises are not retyped herein, a copy of the textbook should be available whenever this guide is used.

CONTENTS

1 Functions

Key Ideas for Review

* A function is a rule or formula that determines the unique value of one variable (the dependent variable) once the value of another variable (the independent variable) has been specified.

* A single equation is not the only way to define a function. Sometimes a function is defined by a table, chart, or by several equations, in a piecewise definition.

* The domain of a function consists of the set of all real numbers at which the function is defined and yields a real number.

* The graph of the function f is the graph of the equation $y = f(x)$.

* The vertical line test: if any vertical line cuts a curve at more than one point, then the curve is not the graph of any function of x.

* $(f \circ g)(x) = f(g(x))$.

* In business and economics problems, the cost, revenue, and profit functions occur frequently; moreover, $P(x) = R(x) - C(x)$. The "break-even" point occurs where $R(x) = C(x)$.

* The slope of a nonvertical line is given by

$$m = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$

  where $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ are any two distinct points on the line.

* A vertical line has no slope.

* If the slope m is positive, then y increases as x increases (the line *rises* from left to right); if m is negative, then y decreases as x increases (the line *falls* from left to right).

* The slope-intercept form of a line is $y = mx + b$.

* If two nonvertical lines have the same slope, then they are parallel. Conversely, if two nonvertical lines are parallel, then they have the same slope.

* The point-slope form of a line that passes through the point $P(x_1, y_1)$ and has slope m is $y - y_1 = m(x - x_1)$.

* The slope of a line parallel to the x-axis is zero.

* The equation of a vertical line through (a,b) is $x = a$. The equation of a horizontal line through (a,b) is $y = b$.

* The equation of a straight line can be written as $Ax + By = C$, where A and B are not both zero. Conversely, the graph of the linear equation $Ax + By = C$ (A and B not both zero) is a straight line.

* Two lines are parallel or identical or intersect at only one point.

* When a principal P is invested for t years at interest rate r, the interest earned is $I = Prt$ and the value of the investment after t years is

$$S = P+I = P(1 + rt).$$

* If a merchant marks up an item that costs him C by the rate r, then the selling price is

$$S = C + rC = C(1 + r).$$

  Similarly, if an item with list price L is discounted at the rate r, then the selling price is

$$S = L - rL = L(1 - r).$$

* If an item with net cost C is depreciated linearly over n years, then the annual depreciation is $D = c/n$. After t years the total accumulated depreciation is Dt and the book value of the item is $V(t) = V(0) - Dt$.

* Given a set of n data points (x_1, y_1), (x_2, y_2), . . . (x_n, y_n), the line of best fit obtained by the method of least squares can be found by solving the system (16) of Section 1.4 for the slope m and the y-intercept b.

* The graph of a second degree function $f(x) = ax^2 + bx + c$ is a parabola, opening upward if $a > 0$ and downward if $a < 0$. The vertex has x-coordinate $x = -b/2a$. The axis of symmetry is the vertical line through the vertex.

* The parabola $y = ax^2 + bx + c$ has either a highest point (if $a < 0$) or a lowest point (if $a > 0$). This is important when we are looking for the largest or smallest value of a quadratic function.

* The point of intersection of the graphs of two equations can be found algebraically by solving the two equations simultaneously.

Exercise Set 1.1, (Page 8)

3. Given the function $F(x) = \dfrac{x^2 + 1}{3x - 1}$

(a) To find F(1) we replace x by 1. Thus

$$F(1) = \frac{(1)^2 + 1}{3(1) - 1} = \frac{2}{2} = 1$$

(b) To find F(-2) we replace x by -2. Thus

$$F(-2) = \frac{(-2)^2 + 1}{3(-2) - 1} = \frac{5}{-7} = \frac{-5}{7}$$

(c) To find F(4), we replace x by 4. Thus

$$F(4) = \frac{(4)^2 + 1}{3(4) - 1} = \frac{17}{11}$$

(d) To find F(0), we replace x by 0. Thus

$$F(0) = \frac{(0)^2 + 1}{3(0) - 1} = \frac{1}{-1} = -1$$

(e) To find F(a), we replace x by a. Thus

$$F(a) = \frac{(a)^2 + 1}{3(a) - 1} = \frac{a^2 + 1}{3a - 1}$$

(f) To find F(a - 2), we replace x by a - 2. Thus

$$F(a - 2)) = \frac{(a - 2)^2 + 1}{3(a - 2)-1} = \frac{a^2 - 4a + 4 + 1}{3a - 6 - 1} = \frac{a^2 - 4a + 5}{3a - 7}$$

(g)To find F(-x), we replace x by -x Thus

$$F(-x) = \frac{(-x)^2 + 1}{3(-x) - 1} = \frac{x^2 + 1}{-3x - 1}$$

(h)To find $F(x^2)$, we replace x by x^2 Thus

$$F(x^2) = \frac{(x^2)^2 + 1}{3(x^2) - 1} = \frac{x^4 + 1}{3x^2 - 1}$$

(i)

$$\frac{1}{F(x)} = \frac{1}{\frac{x^2 + 1}{3x - 1}} = \frac{3x-1}{x^2 + 1}$$

5. Given the formula relating Fahrenheit temperature F to Celsius temperature C

$$F = \frac{9}{5}C + 32$$

(a) To write C as a function of F, we solve the formula for C.

$$F - 32 = \frac{9}{5}C$$

or

$$C = \frac{5}{9}(F - 32)$$

(b) To find the Celsius equivalents of Fahrenheit temperatures, we substitute in the formula of part (a)

(i) If F = 4°, then $C = \frac{5}{9}(4 - 32) = \frac{5}{9}(-28) = -15.6°$

(ii) If F = 0°, then $C = \frac{5}{9}(0 - 32) = -17.8°$

(iii) If F = -10°, then $C = \frac{5}{9}(-10 - 32) = \frac{5}{9}(-42) = -23.3°$

(iv) If F = 32°, then $C = \frac{5}{9}(32 - 32) = 0°$

(v) If F = 98.6°, then $C = \frac{5}{9}(98.6 - 32) = \frac{5}{9}(66.6) = 37°$

(vi) If F = 212°, then $C = \frac{5}{9}(212 - 32) = \frac{5}{9}(180) = 100°$

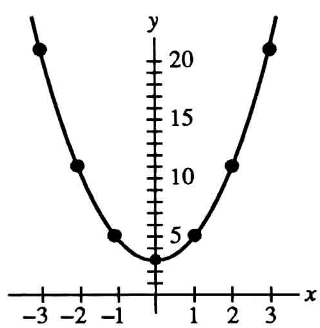
9. Since a negative number has no real square root, we must have $x - 1 \geq 0$ for x to be in the domain of $h(x) = \sqrt{x - 1}$. Thus, the domain of h consists of all real numbers which are greater than or equal to 1.

11. We have $f(x) = (x - 2)/(x + 1)$. Since division by zero is undefined, f(-1) is undefined. Thus, the domain of f consists of all real numbers except -1.

19. The graph of $f(x) = 2x^2 + 3$. is the graph of $y = 2x^2 + 3$. We choose values of x arbitrarily and calculate the corresponding values of y. The results are

$y = 2x^2 + 3$

x	y
0	3
±1	5
±2	11
±3	21

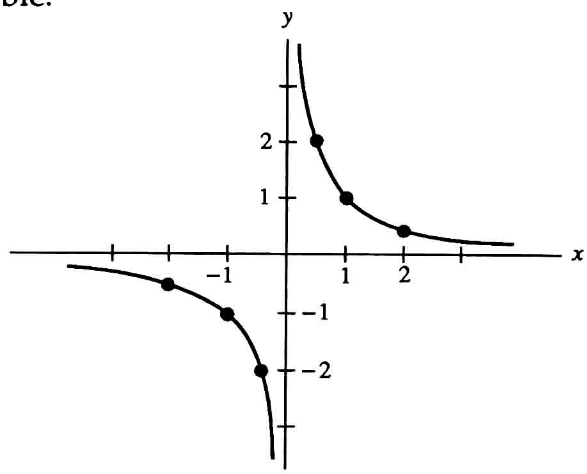


These points are plotted to obtain the graph.

23. The graph of $f(x) = \frac{1}{x}$ is the graph of $y = \frac{1}{x}$. We choose values of x arbitrarily and calculate the corresponding values for y. We note that x=0 is not in the domain of this function, so we must exclude this from our table.

$y = \frac{1}{x}$

x	y
-2	-1/2
-1	-1
-1/2	-2
1/2	2
1	1
2	1/2



These points are plotted to obtain the graph.

<u>*Exercise Set 1.2, (Page 19)*</u>

1. The graph of $f(x) = |x| + 1$ is the graph of $y = |x| + 1$, which may also be written as

$$y = \begin{cases} x + 1 & \text{if } x \geq 0 \\ -x + 1 & \text{if } x \leq 0 \end{cases}$$

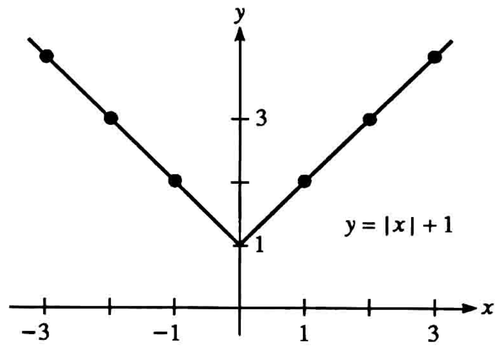
We sketch the graph in two stages. If $x \geq 0$, we have

x	0	1	2	3
f(x) = x+1	1	2	3	4

which is a linear graph in the first quadrant. If $x < 0$, we have

x	-1	-2	-3
f(x) = -x+1	2	3	4

which is a linear graph in the second quadrant. We sketch the two stages together as



9. The graph of

$$y=f(x) = \begin{cases} x + 2 & \text{if } x < 0 \\ x - 2 & \text{if } x \geq 0 \end{cases}$$

is obtained in two stages.

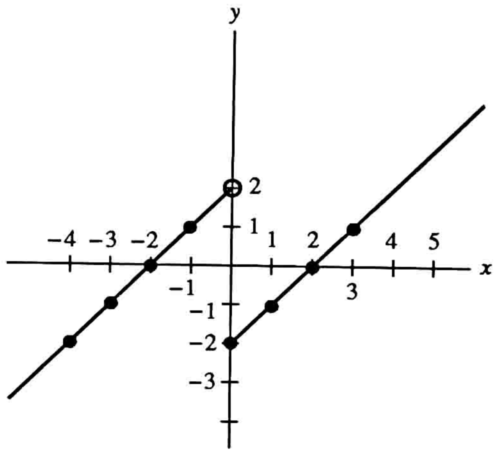
If x<0, we have

x	-4	-3	-2	-1
y=x+2	-2	-1	0	1

and if x≥0, we have

x	0	1	2	3
y=x-2	-2	-1	0	1

Plotting these points gives the graph



13. Given $f(x) = \frac{3}{x - 1}$ and $g(x) = x^2 + 2$:

(a) To find $(f \circ g)(2) = f(g(2))$, we first find $g(2)$.

$g(2) = (2)^2 + 2 = 6$

Thus $f(g(2)) = f(6) = \frac{3}{6 - 1} = \frac{3}{5}$

(b) To find $(g \circ f)(2) = g(f(2))$, we first find $f(2)$.

$f(2) = \frac{3}{2 - 1} = \frac{3}{1} = 3$

Thus $g(f(2)) = g(3) = (3)^2 + 2 = 11$

(c) To find $(f \circ g)(x) = f(g(x))$, we substitute $g(x)$ for every occurrence of x in the rule for f. Thus

$$f(g(x)) = \frac{3}{(x^2 + 2) - 1} = \frac{3}{x^2 + 1}$$

(d) To find $(g \circ f)(x) = g(f(x))$, we substitute $f(x)$ for every occurrence of x in the rule for g. Thus

$$g(f(x)) = \left(\frac{3}{x-1}\right)^2 + 2 = \frac{9}{(x-1)^2} + 2 = \frac{9 + 2(x-1)^2}{x^2 - 2x + 1} = \frac{2x^2 - 4x + 11}{x^2 - 2x + 1}$$

(e) To find $(f \circ f)(1) = f(f(1))$, we first find f(1).
Since x = 1 is not in the domain of f, f(1) is undefined. Thus $(f \circ f)(1)$ is undefined.

(f) To find $(g \circ g)(2) = g(g(2))$, we first find g(2).

$$g(2) = (2)^2 + 2 = 6$$

Thus $g(g(2)) = g(6) = (6)^2 + 2 = 38$.

19. Given $h(x) = \left(\frac{3x-5}{x+4}\right)^{1/3}$, we want to write this function as a composite of two simpler functions. We see that $h(x) = u^{1/3}$ where $u = \frac{3x-5}{x+4}$. Thus we let $f(u) = u^{1/3}$ and

$g(x) = \frac{3x-5}{x+4}$. Then $h(x) = f\left(\frac{3x-5}{x+4}\right) = f(g(x)) = (f \circ g)(x)$.

25. We are given a fixed cost F(x) = \$2438 and a variable cost $V(x) = 4x^2 - 2x$.

(a) The cost function C(x) = F(x) + V(x). Thus $C(x) = 2438 + 4x^2 - 2x$.

(b) If x = 50 units then the cost will be $C(50) = 2438 + 4(50)^2 - 2(50) =$ \$12,338.

29. If x dollars are invested at 7% compounded annually, the interest earned in one year will be .07x dollars. The amount A will be

A = x (original amount) + .07x (interest earned) = (1.07)x

Exercise Set 1.3 , (Page 32)

5. To find the slope of a line, we put the line into slope-intercept form y = mx + b. The coefficient of x will be the slope.

(a) $y = 3x + 2$. This line is in slope-intercept form and $m = 3$.

(b) $y = 3$. Since $y = 0 \cdot x + 3$, the slope $m = 0$.

(c) $x = \frac{2}{3}y + 2$

$$x - 2 = \frac{2}{3}y \quad \text{or} \quad y = \frac{3}{2}x - 3$$

The slope $m = \frac{3}{2}$

7. When m is positive, the line rises from left to right; when m is negative, the line falls from left to right.

(a) $y = 2x+3$	$m = 2 > 0$	line rises from left to right
(b) $y = \frac{-3}{2}x+5$	$m = \frac{-3}{2} < 0$	line falls from left to right
(c) $y = \frac{4}{3}x - 3$	$m = \frac{4}{3} > 0$	line rises from left to right
(d) $y = \frac{-2}{5}x - 3$	$m = \frac{-2}{5} < 0$	line falls from left to right

15. We want to find the point-slope form for the line. This form is $y - y_1 = m(x - x_1)$.

(a) The given points are (-1,2) and (3,5)

$$\text{The slope is } m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{5 - 2}{3 - (-1)} = \frac{3}{4}$$

We use this slope and either point in the form. Taking the point (-1,2), we have $y - 2 = \frac{3}{4}(x + 1)$

(b) The given points are (-3,-4) and (0,0)

$$\text{The slope is } m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{-4 - 0}{-3 - 0} = \frac{4}{3}$$

Using the point (0,0) in the point-slope form yields

$$y - 0 = \frac{4}{3}(x - 0) \qquad \text{or} \qquad y = \frac{4}{3}x$$

17. We first solve x + 2y = 3 for y obtaining 2y = -x + 3, or equivalently, y = -x/2 + 3/2. Hence, the slope of any line parallel to the given line is -1/2. Since the desired line passes through (1,-2), we can use the point-slope equation with m = -1/2, $x_1 = 1$, and $y_1 = -2$ so that

$$y + 2 = \frac{-1}{2}(x - 1)$$

Multiplying both sides by -2 yields

$$-2y - 4 = x - 1$$

or

$$0 = 2y + x + 3$$

23. (a)
$$\begin{aligned} x + y &= 6 \\ 2x + 3y &= 15 \end{aligned}$$

Multiplying the top equation by -3 and adding to the bottom equation gives -x = -3 or x = 3. Substituting x = 3 into the top equation, we have y = 3. Thus the only solution of the system is x = 3, y = 3, and the lines intersect at the point (3,3).

(b)
$$\begin{aligned} x - 2y &= 7 \\ 3x - 6y &= 14 \end{aligned}$$

The system has no solution. There is no pair of real numbers x and y such that x - 2y = 7, then 3x - 6y = 14, for if x - 2y = 7, then 3x - 6y = 3(x - 2y) = 3(7) = 21. In this case the lines are parallel.

(c)
$$\begin{aligned} x + 3y &= 1 \\ -2x - 6y &= -2 \end{aligned}$$

Since the second equation is -2 times the first, there are infinitely many solutions of the system and the lines are identical.

(d) $x - 3y = -5$
$2x + 3y = -1$

Adding the two equations gives $3x = -6$ or $x = -2$. Substituting $x = -2$ into the top equation gives $-3y = -3$ or $y = 1$. Thus the only solution of the system is $x = -2$, $y = 1$, and the lines intersect at the point $(-2, 1)$.

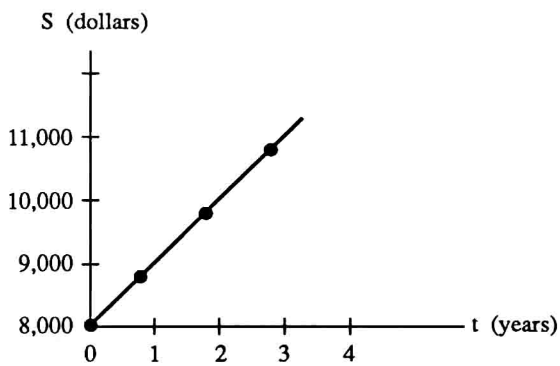
Exercise Set 1.4, (Page 46)

1. (a) The principal P is \$8000 and the rate of interest is 12% so $r = .12$. The simple interest formula is:

$$S = P + Prt = 8000 + 8000\,(.12)\,t$$

The amount owed $S = 8000 + 960t$

(b)



t	S = 8000 + 960
0	8,000
1	8,960
2	9,920
3	10,880
4	11,840

(c) After 6 years, $t = 6$, and the amount owed is

$$S = 8000 + (960)(6) = \$13{,}760$$

(d) After 9 months, $t = \frac{3}{4}$, and the amount owed is

$$S = 8000 + (960)\left(\frac{3}{4}\right) = \$8{,}720$$

11. The original value of the car is $C = \$8000$. The dollar value of the car after t years of ownership is $V = C(1 - rt) = 8000(1 - rt)$. When $t = 2$ the car's value is \$4800, thus