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Electron Radial Wave Functions and Nuclear Beta-decay

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PREFACE

As there are already a number of excellent books on the subject of beta decay, all published in the late sixties, or early seventies, one could ask the question: why another book of this kind?

Well, these last years have, after all, brought some basic changes in the way of looking at the whole complex of beta decay, and, at the same time, weak interactions as a whole. As far as the beta decay treatment is concerned, problems are tackled nowadays with the so-called elementary particle treatment, i.e. one now uses methods which first emerged in elementary particle physics for low energy beta decay as well. The background for this change in philosophy, which is quite contrary to former practice (of considering the decaying nucleons inside the nucleus as independent), is to be seen in the fact that today there is a great tendency to achieve model-independent statements from experiments; 'model-independent' in this case means independent of details of the underlying nuclear structure. In conflict with this model independency, where observables are expressed in terms of form factors, is the Coulomb final state interaction between the charged electron and the charged nucleus. Because of this, a lot of model dependencies crop up which could otherwise be ignored. These dependencies arise because the number of form factors becomes too great. For this reason it is particularly important to pay attention to the Coulomb interaction.

Of course, there is another reason. The precision of experiments in the last decade has increased so much for various observables that even small effects have to be considered nowadays, whereas in the past these could be omitted. Sometimes it is particularly the Coulomb interaction which is either directly responsible for such small effects or which makes the interpretation of observables, which are difficult to measure, more complicated. Thus, the interdependence of Coulomb interaction and beta decay is one of the main aspects of the book.

The present book is certainly not an elementary introduction to the subject of nuclear beta decay, and the prospective reader is expected to have already some knowledge of the basic physical phenomena and their elementary theoretical description. Further prerequisites are some knowledge of the techniques of the quantum-mechanical theory of angular momenta, of nuclear physics and of the physical aspects of relativistic quantum mechanics. From the topic of special functions, the gamma function and spherical Bessel functions are assumed to be well-known subjects, but lack of familiarity with them is not expected to be too prohibitive since all properties and equations for these quantities are explicitly stated where they are needed. Similarly, familiarity with linear ordinary differential equations and complex integration would be helpful but is not necessary.

The composite title *Electron radial wave functions and nuclear beta decay* has been chosen so as to indicate that the electron radial wave functions, in fact, play a major role for our treatise on nuclear beta decay, and that in this respect the present book is significantly different from the well-known earlier books on beta decay. Also, the composite title is to indicate that the book contains a more detailed and further-reaching account of electron radial wave functions than would necessarily be needed in the context of nuclear beta decay. In particular, Chapter 3 is hoped also to be of interest to readers who do not work on beta decay but who deal with electron radial wave functions in another context.

In fact, Chapters 3 and 4 can be read independently of the other parts of the book. Chapter 4, of course, uses many results from Chapter 3, but we expect that the user who is mainly interested in Chapter 4 can read it independently of Chapter 3 if he is willing to accept the statements based on and the results quoted from Chapter 3, and in any case for a more detailed understanding he would not need the whole of Chapter 3. Similarly, the chapters on beta decay can be read independently of the chapters on electron wave functions if the reader is willing to accept the results quoted from Chapter 4. In order to support such independent reading we have sometimes repeated important formulae which are needed from other chapters (or even other sections, if far away, of the same chapter).

What the book contains may best be seen from the detailed table of contents, so only a few additional remarks are needed. Chapters 12 and 13 merely report, without any kind of derivation, some results on gauge theories and radiative corrections, topics which to some extent have been included for the convenience of the reader although they are essentially beyond the scope of our treatise. For in order to treat these phenomena in detail, we would need quite different techniques, such as those based on Feynman graphs, which otherwise have not been applied at all in this book. Other topics which are not dealt with are double beta decay, inner bremsstrahlung, atomic effects accompanying beta decay, and gross theories.

Although this is a theoretical book, occasionally the formulae are also compared with corresponding experiments. It should be pointed out, however, that the book does not contain a compilation of all existing experiments. The authors have just selected suitable ones when opportune.

To choose an appropriate notation is a particularly cumbersome task for a book of this scope. We would have liked to have used many more different symbols than are available. Many of our symbols therefore have a local meaning for one or a few sections only, and are used in other sections for different quantities. The coefficients of various different power series, for instance, are all denoted by a_n. For the global variables, on the other hand, which have the same meaning throughout the whole book, we sometimes use a simplified notation, omitting subscripts or indexes which are kept fixed. The momentum of the electron p_e, for instance, is simply denoted by p in Chapter 4 where almost exclusively electrons are considered, or the scattering phase shift Δ_κ is simply denoted by Δ as long as the angular momentum quantum number κ need not be specified. As for the more mathematical parts of the book we have decided not to use the modern notation, like $x \in]0, 1[$, for instance, in place of $0 < x < 1$, in view of the fact that many of the workers on beta decay belong, like ourselves, to an older generation.

Although our list of references is not short, we want to point out that we did not intend to give a complete bibliography, but have included when appropriate some of those references which are of immediate relevance to our text in giving support to our statements or in providing additional information beyond our treatment. In addition some of the references have been included which treat a subject for the first time or otherwise seem to be interesting for the historical development.

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March, 1981

H. B.
W. B.

CONTENTS

1
INTRODUCTION

FOR A LONG time the strong, electromagnetic, weak, and gravitational interactions were believed to be the four basic interactions. Generally, their processes were treated separately. A unified theory which was able to describe them all did not exist. As far as weak interaction theory is concerned, to which our interest is directed in the following, its development is strongly connected with the understanding of beta-decay. Many of the basic ideas in weak interactions were originally discovered from theoretical and experimental investigations of nuclear beta-decay. The theoretical game started when Fermi (1934) first suggested his famous theory of beta-decay. In his attempt only the vector interaction was considered. At that early stage it was also Fermi who recognized that beta-decay is strongly influenced by the Coulomb interaction between the beta-particle and the charged nucleus. The function which takes care of the essential effects in that context, and later on was usually called the 'Fermi function', was already explicit in his article. It should also be mentioned that even the existence of forbidden beta-transitions was globally discussed in that paper. Shortly after Fermi's first formulation of beta-decay theory the formalism was completed by showing that the Hamiltonian must be a linear combination of the five possible relativistic invariants, i.e. scalar, vector, tensor, axial vector, and pseudoscalar interaction (Bethe and Bacher 1936, Fierz 1937). Corresponding selection rules for beta-disintegrations were additionally discussed (Gamow and Teller 1936). Electron capture was also soon included (Yukawa and Sakata 1935, 1936, 1937; Bethe and Bacher 1936; Möller 1937). Thus, the form of the beta-decay or weak interaction theory, which is today usually called the phenomenological one, had finally been worked out by about 1937.

This form has been remarkably stable over the last forty years. With some minor changes it remains up to now a fundamental corner-stone for a theoretical description of nuclear beta-decay. It has also been applied with success to many other weak processes. The question, however, which, or which combination, of the five possible interactions (scalar, vector, tensor, axial vector, pseudoscalar) really contributes could only be decided experimentally. This point was not definitively cleared up until twenty years later. In the meantime, the theory of forbidden transitions was elaborated in detail for beta-decay (Konopinski and Uhlenbeck 1941) and for electron capture (Marshak 1942). In that context for the first time the necessary beta-decay Coulomb functions were additionally considered in more detail because of the greater sensitivity of the relevant observables to such details. A summary of the status of knowledge at that time was given by Konopinski (1943).

With the end of the forties, beta-radioactive sources of high specific activities became available. A very active era of experimental beta-spectroscopy began. One after another, more and more puzzles in the theory were experimentally solved (Konopinski and Langer 1953). A further modification of the theory became necessary, however, when parity violation in beta-decay was discovered by Wu and co-workers in 1956 (Wu *et al.* 1957). Their experiment was initiated by Lee and Yang who claimed that no experimental proof of the hypothesis of parity conservation for beta-decay existed at that time (Lee and Yang 1956). Two years later, by the end of 1958, the final form of the V–A interaction was then conclusively established, mainly by electron neutrino angular correlation experiments and by the determination of the electron asymmetry of polarized neutrons. After the concept of the universal Fermi interaction had been proposed in 1948 to 1949 the near equality of the coupling constants G_β in beta-decay and of the Fermi constant g_μ in μ-decay led to the introduction of the conserved vector current (CVC) hypothesis (Gershtein and Zeldovich 1956; Feynmann and Gell-Mann 1958). Experimental evidence for the CVC theory was then obtained up to 1964 (Wu 1964). A second symmetry concept which makes use of the behaviour of the weak interacting currents under a G-parity transformation was suggested by Weinberg (1958). In this framework one distinguishes between first and second class currents, both types being distinguished by their G-parity transformation properties. The fifties and the sixties can be considered as the most fruitful epoch of beta-spectroscopy. Thus a number of excellent reviews (Konopinski 1959; Weidenmüller 1961; Wu *et al.* 1965; Blin-Stoyle 1966) and textbooks (Wu and Moszkowski 1966; Konopinski 1966; Schopper 1966; Morita 1973; Blin-Stoyle 1973) about beta-decay appeared in or shortly after that period, and all aspects were fully elucidated. Electron capture especially had been considered in detail (Brysk and Rose 1958; Bouchez and Depommier 1960; Berenyi 1968; Bambynek *et al.* 1977). The progress of experimental techniques in beta-decay increased the accuracy of the experimental data which, on the other hand, required more accurate theoretical calculations for their interpretation. Thus, greater attention was also given to a more exact treatment of the necessary Coulomb functions (Bühring 1963, 1965; Behrens and Bühring 1971). Also, a number of comprehensive tables for these beta-decay Coulomb functions (Fermi function, etc.) were published (Dzhelepov and Zyrianova 1956; Bhalla and Rose 1960, 1962; Behrens and Jänecke 1969; Dzhelepov and Zyrianova 1972; Bambynek *et al.* 1977).

In the period before 1964 the approximation of considering the nucleons inside the nucleus to be independent (impulse approximation) was introduced at the beginning of every theoretical treatment of beta-decay. All formulae obtained up till then were therefore expressed in terms of nuclear matrix elements. But the results derived in this way are, of course, burdened by approximations and not of general validity. In view of this situation the theory was reformulated by expressing all observables model-independently in terms of form factors, a method which was well-known before from elementary particle physics (Stech and Schülke 1964; Kim and Primakoff 1965). This elementary particle theory (EPT) formalism, which, so to speak, forgets that the nucleus consists of nucleons, became more and more used (see Schopper 1966). The reason was that this approach offered excellent possibilities for establishing model-independent constraints. In that context a particular version of this formalism introduced by Holstein (Holstein *et al.* 1972; Holstein 1974*a*) should be mentioned especially since it was this special formulation which was applied to various analyses of allowed transitions.

Now we come to the last ten years. As far as beta-decay and the weak interaction are concerned the activities within the seventies can, *cum grano salis*, be summarized under the heading 'symmetries'. In the field of beta-decay where many original ideas were intimately connected with the name of Wilkinson, one was interested in a possible failure of the CVC hypothesis, in the renormalization of the axial vector coupling constant in complex nuclei, in the validity of time reversal invariance, and in the possible existence of second class currents. For that purpose, experiments of the highest precision have been carried out in order to determine *ft*-values of the 0^+–0^+ superallowed transitions (for a review see Wilkinson 1978; Wilkinson *et al.* 1978) and in order to search for small deviations from the usual behaviour of allowed transitions (see, for example, Wu 1977; Wilkinson 1978; Calaprice 1978; Behrens *et al.* 1978; Telegdi 1979). Naturally, the highest precision was now also required for the theoretical interpretation of these results. Coulomb effects, of which the main part can be taken care of via the electron radial wave functions, had to be treated more and more accurately. Even very small terms which could be omitted in earlier times without any problems are of importance today. No dramatic changes, however, emerged in the end. Up to now, CVC and time reversal invariance are believed to be valid and second class currents to be absent, as was assumed before. But it should be noted that a certain increase in precision with respect to every aspect should be achieved in order to come to definite conclusions.

From a universal point of view, on the other hand, dramatic changes have occurred in the last decade. The so-called phenomenological theory of weak interactions which was originally developed by Fermi, as mentioned before, is in fact not renormalizable. That means, in a usual perturbation expansion, every order beyond the first diverges but these infinities can not be made to disappear as in quantum electromagnetics. In fact, no basic theory is normally expressed by the name phenomenological. That problem was not solved until the so-called gauge theories, first proposed by Weinberg (1967) and Salam (1968). These gauge theories predicted that the weak interaction density contains neutral currents in addition to charged ones. Indeed, these neutral currents have been found (Hasert *et al.* 1973). This discovery can be considered as the breakthrough for gauge theories, but this type of theory is able to bring about even more. Actually, they overcame the other old problem of how to treat the four fundamental interactions separately and to consider them to be basically different ones. As a first step, both weak and electromagnetic interactions could be described in an unified way on the basis of a spontaneously broken gauge symmetry (Weinberg 1980). Next, a synthesis of weak, electromagnetic, and strong interactions, which is usually called the grand unified theory, was established by assuming all these interactions to be gauge ones (Glashow 1980). But beyond that, the great vision of an ultimate unification of all forces could be seen on the horizon for the first time (Salam 1980). Although the topic of gauge theories is beyond the scope of this book, we would not omit at least a mention of these fundamentally important developments.

2
DIRAC EQUATION

2.1. Dirac equation, units, and conventions

SEVERAL different conventions are in use for writing the Dirac equation and related quantities. It is therefore the main purpose of this chapter to establish the conventions and definitions to be used in this book, rather than to given an introduction to Dirac theory.

The Dirac equation for a free particle of mass m may be written in Hamiltonian form as

$$\{-(\boldsymbol{\alpha}\boldsymbol{p})-\beta m\}\psi(\mathbf{r},t)=\mathcal{W}\psi(\mathbf{r},t) \tag{2.1}$$

where $\boldsymbol{p}$ is the vector differential operator

$$\boldsymbol{p}=-\mathrm{i}\,\mathrm{grad} \tag{2.2}$$

with respect to the space co-ordinates $\mathbf{r}=(x,y,z)$ and $\mathcal{W}$ the differential operator

$$\mathcal{W}=\mathrm{i}\,\partial/\partial t \tag{2.3}$$

with respect to time. Natural units are being used such that

$$\hbar=1,$$
$$c=1,$$
$$m_e=1,$$

where $2\pi\hbar=h$ is Planck's constant, c the velocity of light, and m_e the rest mass of the electron. The term $(\boldsymbol{\alpha}\boldsymbol{p})$ denotes a scalar product with

$$\boldsymbol{\alpha}=(\alpha_1,\alpha_2,\alpha_3)$$

a vector the components of which are four by four hermitian matrices. Similarly, β is a hermitian four by four matrix. Equation (2.1) therefore represents a system of four coupled linear partial differential equations. The Dirac matrices $\alpha_1,\alpha_2,\alpha_3,\beta$ satisfy

$$\alpha_1^2=\alpha_2^2=\alpha_3^2=\beta^2=1, \tag{2.4}$$

$$\alpha_k\beta+\beta\alpha_k=0, \tag{2.5}$$

$$\alpha_k\alpha_l+\alpha_l\alpha_k=2\delta_{kl}1 \tag{2.6}$$

with

$$\delta_{kl}=\begin{cases}1 & \text{if } k=l\\ 0 & \text{if } k\neq l\end{cases} \tag{2.7}$$

$$k,l=1,2,3.$$

Here **0** and **1** are the four by four zero and unit matrix, respectively. While eqns (2.4)–(2.6) describe the essential properties of the Dirac matrices, they do not fix them uniquely. By convention we may choose the following explicit representation:

$$\beta = \begin{Bmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{Bmatrix}, \tag{2.8}$$

$$\alpha_1 = \begin{Bmatrix} & & & 1 \\ & & 1 & \\ & 1 & & \\ 1 & & & \end{Bmatrix}, \tag{2.9a}$$

$$\alpha_2 = \begin{Bmatrix} & & & -i \\ & & i & \\ & -i & & \\ i & & & \end{Bmatrix}, \tag{2.9b}$$

$$\alpha_3 = \begin{Bmatrix} & & 1 & \\ & & & -1 \\ 1 & & & \\ & -1 & & \end{Bmatrix}, \tag{2.9c}$$

where the elements not filled in are zero. These matrices may be represented conveniently in partitioned form

$$\beta = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & -\mathbf{1} \end{pmatrix}, \tag{2.10}$$

$$\alpha_1 = \begin{pmatrix} \mathbf{0} & \sigma_1^{\mathrm{P}} \\ \sigma_1^{\mathrm{P}} & \mathbf{0} \end{pmatrix}, \tag{2.11a}$$

$$\alpha_2 = \begin{pmatrix} \mathbf{0} & \sigma_2^{\mathrm{P}} \\ \sigma_2^{\mathrm{P}} & \mathbf{0} \end{pmatrix}, \tag{2.11b}$$

$$\alpha_3 = \begin{pmatrix} \mathbf{0} & \sigma_3^{\mathrm{P}} \\ \sigma_3^{\mathrm{P}} & \mathbf{0} \end{pmatrix}, \tag{2.11c}$$

in terms of the two by two submatrices

$$\mathbf{0} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \tag{2.12a}$$

$$\mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{2.12b}$$