The Finite Element Method in Heat Transfer and Fluid Dynamics

Third Edition

J. N. Reddy
D. K. Gartling

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To our wives

Aruna and *Laura*

Preface to the Third Edition

Computational fluid mechanics (CFD) and computational heat transfer (CHT) continue to evolve at a significant pace and have become an ever increasing presence in standard engineering design and analysis practice. The seemingly endless increase in computing power in both single processor and parallel environments has allowed realistic problems of significant complexity and fidelity to be routinely solved and utilized in technological advances. Commercial software has made rapid progress in providing a broad spectrum of analysis capabilities to a variety of industries. Though software is increasingly robust, accurate CFD and CHT simulations still require a knowledgeable user, with a background in both mechanics and numerical methods. The present edition of this book remains focused on providing the information required by an individual who is interested in good numerical methods for the study and understanding of fluid mechanics and heat transfer phenomena.

This book remains practical in scope and content with an emphasis on computational procedures that we have found effective on a wide spectrum of applications. Little, if any, material has been deleted from the second edition. New material has been added primarily in the first five chapters and reflects the research thrusts over the last eight years. Chapter 1 contains the general description of the boundary value problems of interest and has been expanded with a section on mathematical preliminaries and a section on low-speed compressible flows. Chapter 2 continues with the introduction of the finite element method and is essentially unchanged. The thermal conduction and radiation problem is discussed in great detail in Chapter 3, and has a new discussion of mode superposition methods and a more detailed account of radiation solution methods. The isothermal, viscous flow problem is the topic of Chapter 4. The section on stabilized methods has been expanded with more discussion of variational multiscale methods (VMM), and a new section on least-squares finite element models (LSFEM) has been added. Chapter 5 extends the finite element method to non-isothermal flows and now includes a section on the formulation of low-speed, compressible flows. Non-Newtonian flow problems, both inelastic and viscoelastic, are included in Chapter 6, which is largely unchanged. Chapter 7 remains focused on the formulations and algorithms for multidisciplinary problems involving fluid mechanics, heat transfer, solid mechanics, and electromagnetics; the last chapter is on parallel computing, including a general discussion of the parallel architecture and the implementation of finite element models. Organization of the text, equation numbering, references, and symbols retain the same style as used in the previous editions. References have been added as needed without the removal of older citations. We believe it is important to retain the historical record.

The authors thank the publisher for the opportunity to prepare this third edition. The first author thanks colleagues Drs. Vinu Unnikrishnan and Ginu Unnikrishnan for their help with the scanning of figures from the first two editions and preparing the subject index, and Feifei Cheng for proofreading of the manuscript. The second author again thanks his numerous present and former colleagues at Sandia National Laboratories who continue to present challenging engineering applications in applied and computational mechanics. Specific acknowledgments must go to Drs. Mike Glass, Rick Givler, Charles Hickox, Roy Hogan, Mario Martinez and Phil Sackinger for collaboration and assistance in much of the algorithm development and demonstration simulations cited in this work. Portions of this book are taken from research work performed at Sandia National Laboratories under Contract No. DE–AC04–94AL85000 awarded by the U.S. Department of Energy and are used with permission. The authors dedicate this book to their wives, Aruna and Laura, who have graciously tolerated the authors' preoccupation with the writing of the book.

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Preface to the Second Edition

In the six years since the first edition of this book appeared some significant changes have occurred in the area of computational mechanics in general, and in computational fluid mechanics and heat transfer in particular. Foremost among these changes has been the extraordinary increase in performance in desktop computing platforms and the arrival in significant numbers of parallel computers. This widespread availability of capable computing hardware has predictably lead to the increased demand for computer simulation of products and processes during the engineering design and manufacturing process. Our original thesis that the finite element method was very well suited to general purpose and commercial software continues to hold true, as numerous programs are now available for the simulation of all types of applied mechanics problems. The range of applications of finite element analysis in fluid mechanics and heat transfer has become quite remarkable with complex, realistic simulations being carried out on a routine basis. The combination of hardware performance and reliable finite element algorithms has made these advances possible. Another significant change in computational mechanics is the increase in multidisciplinary (multiphysics) problems and their solution via finite element methods. Again, the increase in hardware performance has contributed to these types of computationally intensive problems. However, the inroads made by the finite element method in all areas of mechanics have also had a positive influence on coupled analysis. The commonality of finite element formulation, approximation and solution among the various boundary value problems in mechanics eases considerably the contemplation of multiphysics solutions and software. A final change in the numerical simulation arena comes from the implementation side of the finite element method. The demand for software capability and reliability has increased in step with the hardware performance. The use of parallel computers has added to the complexity of the implementation. All of these attributes lead to the conclusion that finite element implementation, if done well, will require some significant knowledge from areas in computer science. The time of general purpose codes being developed and maintained by one or two individuals is past and multi-talented teams now provide the most modern software.

Our focus for the present edition of this book remains the same — the education of the individual who is interested in good numerical methods for the study of fluid mechanics and heat transfer phenomena. The text remains practical in scope and content with an emphasis on computational procedures that we have found effective on a wide spectrum of applications. Little, if any, material has been deleted from the first edition. New material has been added in almost all chapters along with some rearrangement of topics to improve overall clarity and maintain the step-wise addition of increasingly complex material. Chapter 1 contains the general description of the boundary value problems of interest and has been augmented with a section on chemically reactive systems and additional discussion of change of phase. Chapter 2 continues with the introduction of the finite element method and is essentially unchanged. The thermal conduction and radiation problem is discussed in great detail in Chapter 3, and it has new sections covering specialized finite elements and advanced topics in thermal analysis. The advanced topics section includes descriptions of difficult boundary conditions, such as multipoint constraints, contact and bulk nodes, material motion and kinematics, and methods for chemically reactive solids. The isothermal, viscous flow problem is the topic of Chapter 4. New sections in this chapter cover stabilized finite element methods and a general discussion of methods for free surface problems; the section on turbulence modeling has also been moved to this chapter. Chapter 5 extends the finite element method to non-isothermal flows and is largely unchanged. Non-Newtonian flow problems, both inelastic and viscoelastic, are now included in a revised and updated Chapter 6. A completely new Chapter 7 is focused on formulations and algorithms for multidisciplinary problems involving fluid mechanics, heat transfer, solid mechanics, and electromagnetics. This chapter outlines many of the possible types of coupling, describes the finite element equations for each mechanics area, and presents a number of realistic numerical examples. The last chapter on advanced topics is now devoted exclusively to a discussion of parallel computing including some general discussion of the parallel architecture and sections on parallel implementation of finite element models. Organization of the text, equation numbering, references, and symbols retain the same style as used in the first edition.

The second author thanks his numerous present and former colleagues at Sandia National Laboratories who continue to provide a wealth of challenging problems in applied and computational mechanics. Specific acknowledgments must go to Drs. Mike Glass, Rick Givler, Charles Hickox, Roy Hogan and Phil Sackinger for collaboration and assistance in much of the algorithm development and demonstration simulations cited in this work. Portions of this book are adapted from work performed at Sandia National Laboratories under Contract No. DE–AC04–94AL85000 awarded by the U.S. Department of Energy and are used with permission. The authors dedicate this book to their wives, Aruna and Laura, who have graciously tolerated the authors' preoccupation with the writing of the book.

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Preface to the First Edition

The numerical simulation of fluid mechanics and heat transfer problems has become a routine part of engineering practice as well as a focus for fundamental and applied research. Though there are still various topical areas where our physical understanding and/or ineffective numerical algorithms limit the investigation, a large number of complex phenomena can now be confidently studied via numerical simulation. Though finite difference methods have and will continue to play a major role in computational fluid dynamics (CFD) and heat transfer, finite element techniques have spurred the explosive development of "general purpose" methods and the growth of commercial software. The inherent strengths of the finite element method such as unstructured meshes, element-by-element formulation and processing, and the simplicity and rigor of boundary condition application are being coupled with modern developments in automatic mesh generation, adaptive meshing, and improved solution techniques to produce accurate and reliable simulation packages that are widely accessible. Improvements in computer hardware and system software (e.g., powerful workstations and window environments) have contributed significantly to streamlining the numerical simulation process. The finite element method in fluid mechanics and heat transfer has rapidly caught up with the well-established solid mechanics community in simulation capabilities.

As in any rapidly developing field, the education of the non-expert user community is of primary importance. The present text is an attempt to fill a need for those interested in using the finite element method in the study of fluid mechanics and heat transfer. It is a pragmatic book that views numerical computation as a means to an end—we do not dwell on theory or proof. Other fundamental and theoretical textbooks that cover these aspects are available or anticipated. The emphasis here is on presenting a useful methodology for a limited but significant class of problems dealing with heat conduction, incompressible viscous flows, and convection heat transfer.

The text has been developed out of our experience and course notes used in teaching graduate courses and continuing education courses to a wide spectrum of students. To gain the most from the book the student should have a reasonable background in fluid mechanics and heat transfer as would normally be found in most mechanical, aerospace, chemical or engineering mechanics curriculums. An introductory knowledge of finite element techniques would be very helpful but not essential; some familiarity with basic numerical analysis, linear algebra, and numerical integration would also be of assistance.

Our approach to the finite element method for fluid mechanics and heat transfer has been designed as a series of incremental steps of increasing complexity. In Chapter 1, the continuum boundary value problems that form the central focus of the book are described in some detail. We have tried to be as general as possible in describing the varied physical phenomena that may be encountered within the limits of non-isothermal, incompressible, viscous flows. Chapter 2 introduces the finite element method by application to a simplified, two-dimensional heat conduction problem. All of the necessary machinery for constructing weak forms of a partial differential equation and building a finite element model are introduced here and demonstrated by application. Chapter 3 recaps parts of Chapter 2 and extends the finite element method to three dimensions, time dependence, and practical applications in conduction heat transfer. Isothermal viscous fluid mechanics formulations are described in Chapter 4 along with a significant section on the solution of nonlinear equations developed from the flow problem. Chapter 5 extends the viscous flow problem to consider convective heat transfer formulations and applications. Inelastic non-Newtonian flows and free surface problems are covered in Chapter 6. The complex topic of viscoelastic flow simulation is surveyed in Chapter 7. The last chapter concludes the text with a survey of several advanced topics, including turbulence modeling. The coverage of each topic is sufficient to allow the reader to understand the basic methodology, use existing simulation software with confidence, and allow development of some simpler, special purpose computer codes. Example problems ranging from simple benchmarks to practical engineering solutions are included with each topical area. Adequate references to the relevant literature have also been included for those desiring a more encyclopedic coverage of a specific topic.

The text is organized into major sections within each chapter. Equations are numbered consecutively within each major section. Within a section, reference to an equation is by its sequential number; references to equations outside the current section have a full section, equation number citation. Vectors, tensors, and matrices are denoted by boldface letters. The vectors of interpolation (shape) functions in this book are denoted by Greek symbols ($\mathbf{\Psi}, \mathbf{\Theta}, \mathbf{\Phi}$). Bibliographic information for literature cited in the text is numbered sequentially within each chapter and collected at the end of the chapter.

The first author would like to thank M. S. Ravisankar for his help with Chapter 8 topics and Praveen Gramma for reading the manuscript. The second author would like to thank his numerous present and former colleagues at Sandia National Laboratories who have provided a seemingly endless stream of challenging problems and taught him much about the practice of computational mechanics. Specific acknowledgments must go to Drs. Charles Hickox, Rick Givler, Roy Hogan, Phil Sackinger, Randy Schunk, Rekha Rao, and Steve Rottler for their suggestions and comments on early versions of the text. Portions of this book are adapted from work performed at Sandia National Laboratories under Contract No. DE–AC04–76DP00789 awarded by the U.S. Department of Energy and are used with permission.

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About the Authors

J. N. Reddy earned a Ph.D. in Engineering Mechanics from the University of Alabama in Huntsville, worked as a Postdoctoral Fellow at the University of Texas at Austin, was Research Scientist for Lockheed Missiles and Space Company during l974–75, and taught at the University of Oklahoma from 1975 to 1980 and Virginia Polytechnic Institute & State University from 1980 to 1992. Currently, he is a Distinguished Professor and the inaugural holder of the Oscar S. Wyatt Endowed Chair at Texas A&M University, College Station. Dr. Reddy has published over 400 journal papers and 16 textbooks on theoretical formulations and numerical simulations of problems in solid and structural mechanics, computational fluid dynamics, numerical heat transfer, computational biology, geology and geophysics, mechanics of nanosystems, and applied mathematics.

Dr. Reddy is the recipient of numerous honors and awards, including the 1998 Nathan M. Newmark Medal from the American Society of Civil Engineers, the 2003 Computational Solid Mechanics award from the U.S. Association of Computational Mechanics, the 2004 Distinguished Research Award from the American Society for Composites, and an honorary degree (Honoris Causa) from the Technical University of Lisbon, Portugal (2009). Dr. Reddy is a fellow of the American Academy of Mechanics, the American Institute of Aeronautics and Astronautics, the American Society of Civil Engineers, the American Society of Mechanical Engineers, the American Society for Composites, International Association of Computational Mechanics, U.S. Association of Computational Mechanics, the Aeronautical Society of India, and the Institution of Structural Engineers, U.K. Dr. Reddy serves on the editorial boards of two dozen journals, and as the Editor-in-Chief of *Applied Mechanics Reviews*, *Mechanics of Advanced Materials and Structures*, *International Journal of Computational Methods in Engineering Science and Mechanics*, and *International Journal of Structural Stability and Dynamics.*

As a result of his extensive publications of archival journal papers and books, Dr. Reddy is recognized by ISI Highly Cited Researchers with over 10,000 citations and an H-index of over 40 to his credit. A more complete resume with links to journal papers can be found at http://authors.isihighlycited.com/ or http://www.tamu.edu/acml.

David K. Gartling is a Senior Scientist in the Engineering Sciences Center at Sandia National Laboratories, Albuquerque, New Mexico. He earned his B.S. and M.S. in Aerospace Engineering at the University of Texas at Austin and completed the diploma course at the von Kármán Institute for Fluid Dynamics in Brussels, Belgium. After completion of his Ph.D. in Aerospace Engineering at the University of Texas at Austin, he joined the technical staff at Sandia National Laboratories. Dr. Gartling was a Visiting Associate Professor in the Mechanical Engineering Department at the University of Sydney, Australia, under a Fulbright Fellowship, and later he was a Supervisor in the Fluid and Thermal Sciences Department at Sandia National Laboratories. Dr. Gartling has published numerous papers dealing with finite element model development and finite element analysis of heat transfer and fluid dynamics problems of practical importance. He is the recipient of the 2001 Computational Fluid Dynamics Award from the U.S. Association of Computational Mechanics and is a fellow of the American Society of Mechanical Engineers. Dr. Gartling is presently a member of several professional societies, serves on the editorial boards of several journals, and is the Co-Editor of *International Journal for Numerical Methods in Fluids.*

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