Mathematics and Control in Smart Structures

Volume 3667

PROCEEDINGS OF SPIE

SPIE—The International Society for Optical Engineering

Smart Structures and Materials 1999

Mathematics and Control in Smart Structures

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Chair/Editor

1-4 March 1999
Newport Beach, California

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Published by
SPIE—The International Society for Optical Engineering

Volume 3667

SPIE is an international technical society dedicated to advancing engineering and scientific applications of optical, photonic, imaging, electronic, and optoelectronic technologies.

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Please use the following format to cite material from this book:
Author(s), "Title of paper," in *Smart Structures and Materials 1999: Mathematics and Control in Smart Structures,* Vasundara V. Varadan, Editor, Proceedings of SPIE Vol. 3667, page numbers (1999).

ISSN 0277-786X
ISBN 0-8194-3141-9

Published by
SPIE—The International Society for Optical Engineering
P.O. Box 10, Bellingham, Washington 98227-0010 USA
Telephone 360/676-3290 (Pacific Time) • Fax 360/647-1445

Printed in the United States of America.

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SESSION 1

Finite Element Modeling and Optimization

Computer-aided optimization of smart structures

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ABSTRACT

In this paper, the finite-element/boundary-element program CAPA[1] is presented, which has been developed by the authors during the last decade. With this software environment we are able to model quite different types of transducers which mostly ask for the numerical solution of a multifield problem, such as coupled electric-mechanical fields or magnetic-mechanical fields. Practical applications in the area of smart structures will demonstrate the applicability of the developed software.

1. INTRODUCTION

Short product lifetime cycles, fast time to market and cost reduction as well as an increasing technical complexity are only some of the challenges developers of electromechanical transducers are faced with. Since the fabrication of prototypes and experimental based design is a lengthy and costly process, the need for appropriate numerical simulation tools arises. These transducers, however, are based on the mutual interaction of different physical fields:

- Coupling *Electric Field - Mechanical Field* [2]
  This coupling is either based on the electrostrictive or piezoelectric effect or, results from the force on an electric charge in an electric field (electrostatic force).
- Coupling *Magnetic Field - Mechanical Field* [3]
  This coupling is twofold. We first have the electromotive force (emf) which describes the generation of an electric field (electric voltage resp. current) when a conductor is moved in a magnetic field and, second, the electromagnetic force.
- Coupling *Mechanical Field - Acoustic Field*
  Very often an electromechanical transducer is surrounded by a fluid or a gaseous medium in which an acoustic wave is launched (actuator) or is impinging from an outside source towards the receiving transducer.

Figure 1 shows the basic elements and interfaces of our software system CAPA.[1] In this environment we are able

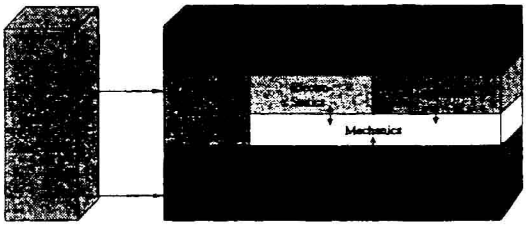

Figure 1. Software System CAPA

to precisely model nearly any type of piezoelectric, electrostatic or electromagnetic transducer including an eventual coupling to a surrounding medium, either a gas, a fluid or a solid.
The software even allows the modeling of major nonlinear effects arising in electromechanical transducers, such as dependence of the elastic moduli on deformation, the nonlinear magnetization curves of ferroelectric materials or the geometric nonlinearities due to large displacements.

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Part of the SPIE Conference on Mathematics and Control in Smart Structures • Newport Beach, California • March 1999
SPIE Vol. 3667 • 0277-786X/99/$10.00

2. PIEZOELECTRIC TRANSDUCERS

2.1. Basic Equations of Piezoelectricity

The development of piezoelectric finite elements is based on three equations, namely the material equations, Newton's law and the potential equation. The material equations, which are the basis of linear piezoelectricity, take into account the piezoelectric effect both in the description of the mechanical stresses as well as the dielectric displacement. According to[5] , these equations may be written as

$$\vec{T} = \mathbf{c}^E \vec{S} - \mathbf{e}^T \vec{E} \tag{1}$$

$$\vec{D} = \mathbf{e}\vec{S} + \boldsymbol{\varepsilon}^S \vec{E}. \tag{2}$$

Herein, $\vec{T}$ denotes the mechanical stress, $\mathbf{c}^E$ the mechanical material tensor at constant electric field $\vec{E}$, $\mathbf{e}$ the piezoelectric coupling tensor, $\boldsymbol{\varepsilon}^S$ the dielectric tensor at constant strain $\vec{S}$ and $\vec{D}$ the electric displacement.

Newton's law, which describes the elastic behaviour of a finite, deformable body can be expressed by the partial. differential equation

$$\left(\frac{\partial T_1}{\partial x} + \frac{\partial T_4}{\partial y} + \frac{\partial T_6}{\partial z}\right)\vec{e_1} + + \left(\frac{\partial T_2}{\partial y} + \frac{\partial T_4}{\partial x} + \frac{\partial T_5}{\partial z}\right)\vec{e_2}$$
$$+ \left(\frac{\partial T_3}{\partial z} + \frac{\partial T_5}{\partial y} + \frac{\partial T_6}{\partial x}\right)\vec{e_3} = \rho\frac{\partial^2 \vec{u}}{\partial t^2}. \tag{3}$$

Herein, ρ denotes the density of the material and $\vec{e_1}, \vec{e_2}$ and $\vec{e_3}$ are unit vectors in cartesian coordinates for the x-, y- and z-direction, respectively.
Using the relation between the electric field $\vec{E}$ and the scalar electric potential ϕ according to

$$\vec{E} = -\nabla\phi, \tag{4}$$

the constituitive equation for the electric field results in

$$\nabla \cdot \varepsilon\nabla\phi = q\ , \tag{5}$$

where q represents the free electric volume charge.

2.2. Fluid-Solid Coupling

The propagation of acoustic waves in non-viscous media is completely described by the scalar velocity potential ψ. The particle velocity $\vec{v}$ and the velocity potential ψ are related by

$$\vec{v} = -\nabla\psi, \tag{6}$$

and the governing equation for the acoustic field is the linear wave equation

$$\nabla \cdot \nabla\ \psi = \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2}, \tag{7}$$

where c is the sound velocity. In the case of a solid-fluid interface the normal component of the surface velocity v_n of the solid must meet the normal component of the particle velocity $\partial\psi/\partial n$

$$v_n = \vec{n} \cdot \left(\frac{\partial \vec{d}}{\partial t}\right) = -\vec{n} \cdot \nabla\psi = -\frac{\partial\psi}{\partial n}. \tag{8}$$

2.3. Finite Element Formulation

Combining (1)-(8) we get a full description of the dynamic behavior of a piezoelectric body immeresd in an acoustic fluid. The application of a finite element discretization scheme to these equation ends up with a linear system of equations, which can be summarized as follows[2,4]

$$\begin{pmatrix} M_{uu} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & M_{\psi\psi} \end{pmatrix} \begin{pmatrix} \left\{\frac{\partial^2 u}{\partial t^2}\right\} \\ \left\{\frac{\partial^2 \psi}{\partial t^2}\right\} \\ \left\{\frac{\partial^2 \psi}{\partial t^2}\right\} \end{pmatrix} + \begin{pmatrix} C_{uu} & 0 & C_{u\psi} \\ 0 & 0 & 0 \\ C_{u\psi}^T & 0 & C_I \end{pmatrix} \begin{pmatrix} \left\{\frac{\partial u}{\partial t}\right\} \\ \left\{\frac{\partial \phi}{\partial t}\right\} \\ \left\{\frac{\partial \psi}{\partial t}\right\} \end{pmatrix} \tag{9}$$

$$+ \begin{pmatrix} K_{uu} & K_{u\phi} & 0 \\ K_{u\phi}^T & K_{\phi\phi} & 0 \\ 0 & 0 & K_{\psi\psi} + K_I \end{pmatrix} \begin{pmatrix} \{u\} \\ \{\phi\} \\ \{\psi\} \end{pmatrix} = \begin{pmatrix} \{F\} \\ \{Q\} \\ 0 \end{pmatrix} . \tag{10}$$

Herein $\mathbf{K}_{uu}$, $\mathbf{C}_{uu}$ and $\mathbf{M}_{uu}$ denote the mechanical stiffness, damping and mass matrix, $\mathbf{K}_{\psi\psi}$ and $\mathbf{M}_{\psi\psi}$ the acoustic stiffness and mass matrix, $\mathbf{K}_{\phi\phi}$ and $\mathbf{K}_{u\phi}$ the dielectric stiffness- and the piezoelectric coupling matrix, $\mathbf{C}_{u\psi}$ the solid-fluid coupling matrix, $\mathbf{K}_I$ and $\mathbf{C}_I$ the infinite acoustic stiffness and damping matrix, $\{F\}$ and $\{Q\}$ the external mechanical forces and electric charges, $\{u\}$ the nodal vector of displacement, $\{\psi\}$ the nodal vector of acoustic velocity potential and and $\{\phi\}$ the nodal vector of scalar electric potential.

Starting with the semidiscrete formulation, time stepping procedures based on the Newmark or Hilber-Hughes-Taylor method[6] may be applied. Furthermore, standard algorithms for eigenvalue calculations, like the subspace and Lanczos procedures, as well as harmonic excitations are easily extended.

3. ELECTROSTATIC TRANSDUCERS

3.1. Basic Equations

In the case of an electrostatic-mechanical-transducer the coupling between the electric and the mechanical field is caused by the electrostatic force between the electrodes. This force is calculated based on the electrostatic force tensor $\mathbf{T}_E$, where $\vec{E} = (E_x,\ E_y,\ E_z)$ denotes the electric field

$$\mathbf{T}_E = \begin{bmatrix} \varepsilon E_x^2 - \frac{1}{2}\varepsilon|E|^2 & \varepsilon E_x E_y & \varepsilon E_x E_z \\ \varepsilon E_y E_x & \varepsilon E_y^2 - \frac{1}{2}\varepsilon|E|^2 & \varepsilon E_y E_z \\ \varepsilon E_z E_x & \varepsilon E_z E_y & \varepsilon E_z^2 - \frac{1}{2}\varepsilon|E|^2 \end{bmatrix} . \tag{11}$$

The electrostatic force $\vec{F}_E$ is given by

$$\vec{F}_E = \iint_A \mathbf{T}_E \vec{n}\, dS\,, \tag{12}$$

where $\vec{n}$ is the normal vector on the surface A. The electrostatic force leads to a deformation of the electrodes, which is described by (3) and, therefore, introduces a geometric nonlinearity in (5).

3.2. FEM-BEM-Formulation

The boundary element discretization of (5) yields to the following BE-matrix equation

$$\mathbf{H}_\phi\{\phi\} + \mathbf{G}_\phi\{E_n\} = 0 \tag{13}$$

with the two boundary element matrices $\mathbf{H}_\phi$ and $\mathbf{G}_\phi$, the nodal vector $\{\phi\}$ of the scalar electric potential and the nodal vector $\{E_n\}$ of the normal component of the electric field.

As described in section 2.3 the application of a FE-formulation to (3) leads to the wellknown matrix equation for the mechanical quantities

$$\mathbf{M}\{\ddot{u}\} + \mathbf{C}\{\dot{u}\} + \mathbf{K}\{u\} - \{F(\phi, E_n)\} = \{0\} \tag{14}$$