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Chaotic Transitions in Deterministic and Stochastic Dynamical Systems

Applications of Melnikov Processes in Engineering, Physics, and Neuroscience

Emil Simiu

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Et ce monde rendait une étrange musique,
    Comme l'eau courante et le vent,
Ou le grain qu'un vanneur d'un mouvement rhytmique
    Agite et tourne dans son van.

—Charles Baudelaire, *Une charogne* (Les Fleurs du Mal)

Preface

A wide variety of natural phenomena can be modeled as planar dynamical systems with motions that may exhibit deterministically or stochastically induced transitions, that is, escapes from and captures into distinct regions in which the systems' motions can evolve. For example, the rolling motion of a vessel may escape from a safe region and be captured into a region wherein capsizing occurs. Mathematically, the transitions entail the crossing of barriers between potential wells.

The Melnikov method provides necessary conditions for the occurrence of such transitions, as well as useful insights into how the transitions are affected by the system and excitation characteristics. It provides a unified treatment of deterministic systems, systems with various types of stochastic excitation—additive, state dependent, white, colored, Gaussian, continuously distributed non-Gaussian, or dichotomous—and systems with combinations of deterministic and stochastic excitation. It has wide-ranging application in science and engineering: in this book alone examples of applications range from physics to mechanical engineering, naval architecture, oceanography, nonlinear control, stochastic resonance, and neurophysiology. It is an effective tool for modeling complex dynamical systems. It allows a chaotic dynamics interpretation of stochastically induced transitions. Last but not least, it is elegant and transparent.

This book is designed to introduce the Melnikov method to readers interested primarily in applications. Prerequisites for the development of stochastic Melnikov theory are (1) deterministic Melnikov theory and (2) basic elements of the theory of stochastic processes. Therefore, following an introduction (Chapter 1), Melnikov theory is presented in a deterministic context, first as a tool applicable to motions with transitions in planar systems, without reference to chaotic dynamics (Chapter 2), then in relation to the chaotic nature of such motions (Chapter 3). These chapters are designed for readers with no previous exposure to dynamical systems theory. Chapter 4 presents requisite elements of the theory of stochastic processes. Based on material presented in Chapters 2, 3, and 4, Chapter 5 extends Melnikov theory to motions with transitions in stochastic planar systems. Chapters 2 through 5 form Part 1 of the book and are devoted to fundamentals. Part 2 consists of Chapters 6 through 12 and is devoted to applications.

The material of Chapters 2 to 4 allows readers not familiar with the theory of nonlinear dynamical systems and/or the theory of stochastic processes to acquire the background needed for the applications without having to resort to a mass of specialized texts less focused with respect to the material specifically needed in this book and more elaborate mathematically. The fundamental material presented in Chapter 3 can be omitted on a first reading as it is not used in all applications. It is needed, however, for understanding the chaotic behavior of the systems being studied, in particular systems exhibiting stochastic resonance.

One of the themes emerging from this book is the hitherto virtually unexplored relationship between chaotic and stochastic dynamics. To our knowledge this contains the first published material that deals with this relationship and explains it within the framework of Melnikov theory.

In Part 2 of the book each of the chapters is concerned with a particular type of application and can be read independently of the others. Material covered in the Appendixes may be omitted on a first reading with no significant prejudice to the reader's ability to apply the method's basic results.

The book is to a large extent self-contained, and numerous references are provided for basic material and additional details. Prerequisites for the book are the equivalent of a first-year graduate course in applied mathematics, including systems of linear differential equations. Laborious mathematical derivations were kept to a minimum, with a few exceptions: the derivation of the expression for the Melnikov function (Chapter 2 and Appendix A1), and the material on the Smale horseshoe map and the shift map (Chapter 3), which convincingly reveals the essence and beauty of chaotic dynamics.

I am indebted to Dr. John W. Lyons and Dr. Richard N. Wright of the National Institute of Standards and Technology for their steadfast support of my initial efforts in the field of chaotic dynamics, and to Professor Stephen Wiggins of the California Institute of Technology for his encouragement and helpful advice during the initial phases of my research. Support by the Ocean Engineering Division of the Office of Naval Research (Dr. Steven Ramberg, Dr. Michael Shlesinger, and Dr. Tom Swean) is also acknowledged with thanks. Dr. Michael R. Frey and Dr. Marek Franaszek contributed many original and stimulating ideas to the research covered in this book. Their contributions are gratefully acknowledged, as are those of Dr. Graham R. Cook, Dr. Charles Hagwood, and Dr. Yudaya Sivathanu. I am also grateful to Dr. Kevin J. Coakley, Professor Mircea Grigoriu, Dr. Howland A. Fowler, Dr. Agnessa Kovaleva, Dr. David Sterling, and Professor Timothy M. Whalen, and to the Princeton University Press reviewers, for many valuable comments and criticisms. It is a pleasure to acknowledge the professionalism and helpfulness of Mr. Trevor Lipscombe, Mr. David Ireland, and Ms. Vickie Kearn (editors), Ms. Anne Reifsnyder (production editor), and Ms. Jennifer Slater, of Princeton University Press.

I dedicate this book to Devra.

Credits

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Contents

Chapter One

Introduction

This work has two main objectives: (1) to present the Melnikov method as a unified theoretical framework for the study of transitions and chaos in a wide class of deterministic and stochastic nonlinear planar systems, and (2) to demonstrate the method's usefulness in applications, particularly for stochastic systems. Our interest in the Melnikov method is motivated by its capability to provide criteria and information on the occurrence of transitions and chaotic behavior in a wide variety of systems in engineering, physics, and the life sciences.

To illustrate the type of problem to which the Melnikov method is applicable we consider a celebrated experiment on a system known as the magnetoelastic beam. The experiment demonstrates the remarkable type of dynamic behavior called *deterministic chaos* (Moon and Holmes, 1979). The system consists of (a) a rigid frame fixed onto a shaking table that may undergo periodic horizontal motions, (b) a beam with a vertical undeformed axis, an upper end fixed onto the frame, and a free lower end, and (c) two identical magnets equidistant from the undeformed position of the beam (Fig. 1.1). The beam experiences nonlinear displacement-dependent forces induced by the magnets, linear restoring forces due to its elasticity, dissipative forces due to its internal friction, the viscosity of the surrounding air, and magnetic damping, and periodic excitation forces due to the horizontal motion of the shaking table. Neither the system properties nor the forces acting on the beam vary randomly with time: the system is fully deterministic.

In the absence of excitation, and depending upon the initial conditions, the beam settles on one of two possible stable equilibria, that is, with the beam's tip closer to the right magnet or closer to the left magnet. The beam also has an unstable equilibrium position—its vertical undeformed axis.

If the excitation is periodic, three distinct types of steady-state dynamic behavior can occur:

1. For sufficiently small excitations, depending again upon the initial conditions, the beam moves periodically about one of its two stable equilibria. The periodic motion is confined to a half-plane bounded by the beam's unstable equilibrium position (Fig. 1.2(a)); in this type of motion there can be no escape from that half-plane.

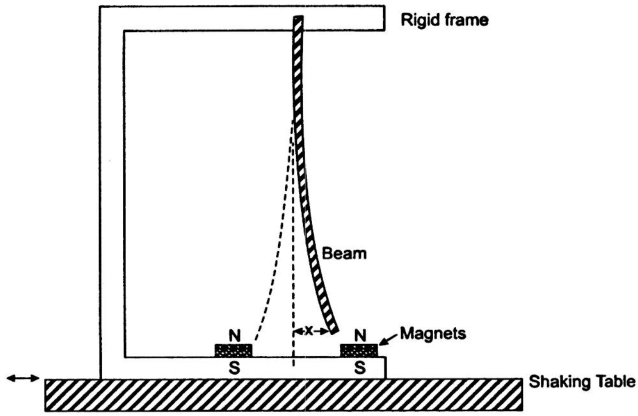

Figure 1.1. The magnetoelastic beam (after Moon and Holmes, 1979).

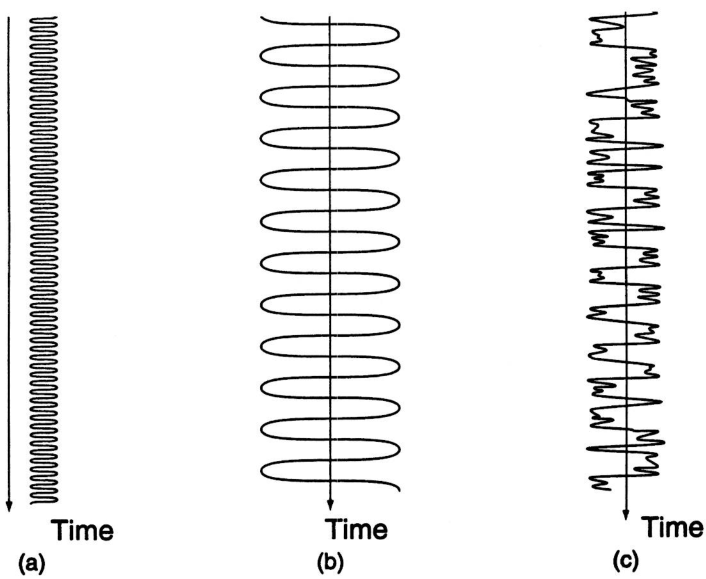

Figure 1.2. Types of steady-state dynamic behavior that may be observed in the periodically excited magnetoelastic beam: (a) periodic motion confined to one of the half-planes bounded by the beam's unstable equilibrium position; (b) periodic motion visiting both half-planes; (c) irregular motion with transitions.

2. For sufficiently large excitations the motion is periodic about—and crosses periodically—the unstable equilibrium position (Fig. 1.2(b)).
3. For intermediate excitation amplitudes, and for restricted sets of initial conditions and excitation frequencies, the steady-state motion is *irregular*, even though the system is fully deterministic; hence the term deterministic chaos. The motion evolves about one of the three equilibria, then it undergoes successive *transitions*, that is, it changes successively to motion about one of the other two equilibria (Fig. 1.2(c)). Transitions in such irregular, deterministic motion are referred to as *chaotic*. A transition away from motion in a half-plane bounded by the beam's unstable equilibrium position is called an *escape*. A transition to motion occurring within such a half-plane is called a *capture*. A succession of escapes and captures is referred to as *hopping*.

The system just described may be modeled as a *dynamical system*—a system that evolves in time in accordance with a specified mathematical expression. In this book we are primarily concerned with dynamical systems capable of exhibiting all three types of behavior illustrated in Fig. 1.2. One basic feature of such systems is that they are *multistable*, meaning that their unforced counterparts have at least two stable equilibria (the term applied to the case of two stable equilibria is *bistable*). In the particular case of mechanical systems, the dynamic behavior is modeled by nonlinear differential equations expressing a relationship among terms that represent

- inertial forces
- dissipative forces
- potential forces, that is, forces derived from a potential function and dependent solely upon displacements; for Fig. 1.1 these forces are due to the magnets and the elasticity of the beam
- excitation forces dependent explicitly on time

Similar terms occur in equations modeling other types of dynamical system, for example, electrical, thermal, or chemical systems.

For a large number of systems arising in engineering or physics safe operation requires that steady-state motions occur within a restricted region, called a *safe region* (in Fig. 1.2(a), the displacement coordinates in the restricted regions are bounded by the vertical line that coincides with the axis of the undeformed beam); transitions to motions visiting another region are undesirable. However, for some systems (e.g., systems that enhance heat transfer, and neurological systems whose activity entails escapes or, in neurological terminology, firings) the occurrence of such transitions is a functional requirement.

Although we will also examine systems with a slowly varying third variable, our main focus will be on planar systems, that is, continuous systems

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