MANORANJAN SINGH

Theory of Fuzzy Structures and Applications

Fuzzifications of Mathematical Notions

LAP LAMBERT Academic Publishing

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ACKNOWLEDGEMENT

MAN HAS FAITH in the divine whom he addresses reverentially as Almighty. It is this divine power that regulates life. It directs us to live and perform assigned duties and carry out responsibilities with sincerity. I being a devotee of "**Maa Mangla**", beg to offer this flower: my **D.Sc.** dissertation "**THEORY OF FUZZY STRUCTURES AND APPLICATIONS**" (**Fuzzification of Mathematical Notions**) having thirteen petals at the feet of my deity. I believe that her blessing has been strength to me.

I have always been thinking of crowning my academic achievement with a **D.Sc. degree**. My constant endeavour and teaching experiences of over two decades encouraged me a lot and I shared it with my respected teacher **Dr. Krishna Acharya**, retired University Professor and Head, Magadh University, Bodh-Gaya, who very kindly agreed to be my supervisor. I have more than a quarter of century of association with him as a student as well as a colleague. All my endeavour would have been an exercise in futility had it not been for his able guidance, advice, encouragement and timely help. I am deeply indebted to him from the core of my heart.

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My son **Mr. Aniket Ranjan**, a student of Computer Science Engineering has taken keen interest in typing of this manuscript. My daughter **Miss Ananta Ranjan** has also helped in the typing of this manuscript. I wish to express my love to both of you. God bless you. I gracefully acknowledge their help and caring attitude.

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THIS WORK IS DEDICATED
TO
THE LOVING MEMORY
OF
MY LATE PARENTS
LALAN PRASAD SINGH
AND
KAMALA DEVI
AN EXEMPLARY
PERSONALITY
WHOSE
LOVE, BLESSINGS
AND
INSPIRATION
MADE ME WHAT I AM TO-
DAY.

PREFACE

BY ACKNOWLEDGING THE POWER of abstract approach, the twentieth century has been a witness to the innovative ideas of unified mathematics. For this unification fuzzy mathematics plays an important role. Modern mathematics is characterized by its emphasis on the systematic investigation of the process of fuzzification of abstract mathematical structures. Many such fuzzifications are incorporated in this research project.

There are many uncertainties in the real world. Fuzzy theory treats a kind of uncertainty called fuzziness, where it displays that the boundary of "**yes**" or "**no**" is ambiguous. In other words, it is open-ended and it includes the subordinates/subjunctives or the recognition of individuals. Fuzzy mathematics is essential and is applicable to every branch of human knowledge from consumer products like refrigerators or washing machine to big communication networks like trains or subways. Recently, it has been used as a tool combining new theories (called soft computing) such as genetic algorithms or neural network to get knowledge from real data. Fuzzy mathematics offer greater richness in application than the classical mathematics. It provides ample motivation to review various concepts/results of abstract algebra, linear algebra and topology in the broader frame work of the fuzzy setting.

Today fuzzy set theory (FST) provides language for all disciplines in contemporary mathematics in much the same way as mathematics has been an essential language for many sciences.

This synthesis is an exposition of some basic ideas of fuzzy algebra, fuzzy linear algebra and fuzzy topology with application of fuzzy sets to some areas other than mathematics. The key idea in this post doctoral dissertation revolves around the concept of fuzzy subgroupoids, fuzzy subgroups, fuzzy normal subgroups, fuzzy rings, fuzzy ideals, fuzzy fields, fuzzy linear spaces, fuzzy modules, fuzzy topological spaces, fixed point in fuzzy metric space, fuzzy normed linear space and fuzzy topological vector space with some applications of fuzzy logic to some other branches of human knowledge, namely sports and medical science. These topics have been included because of their fundamental importance and collaborative relevance.

For convenience, this **D.Sc.** dissertation has been organized in three sections. The first section consists of eight chapters dealing with the fuzzification of "algebraic structures". The second section comprises four chapters and it deals with the application of fuzzy sets in the realm of topology and topological-algebraic structure. The third section synthesizes the applicational aspects of the fuzzy set theory in two diverse areas of knowledge – sports and medical science. Some chapters have been split into two parts which discuss individual topics with some degree of thoroughness forming the basic organizational units of the work. This work is an effort to promote further studies and research in this discipline of fuzzification. The main objectives are as follows:-

(i) To develop techniques for the fuzzification of various notions of algebra.

(ii) To study non-fuzzy algebraic objects as well as its fuzzy algebraic objects.

(iii) To focus on the fuzzification of topological structures and topological-algebraic structures.

(iv) To study some peculiarities of the fuzzy setting.

(v) To describe an activity carried out by applying FST and fuzzy logic in other areas of knowledge.

It is much more illuminating to solve one problem by two or more different methods than to solve any number of problems by the same method. With these things in mind an effort has been made to develop a uniform and systematic theory for fuzzy algebraic structures, fuzzy topological structures and extending the pattern of classical algebra and topology.

The details of the thirteen chapters, in all are as follows :

The **introductory chapter** sketches in brief the ***Cantor's*** intuitive theory of sets and the causes behind the shift to fuzzy set theory (FST). It also deals with the word "fuzzification" and lays out the framework of the study presented in the subsequent chapters.

The **second chapter** offers the smallest fuzzy algebraic structure called "fuzzy subgroupoids". The process of fuzzification of algebraic structure was initiated by ***Rosenfeld*** in 1971, who gave the notion of fuzzy subgroupoids. Here the notion of fuzzy subgroupoids proposed by ***Rosenfeld*** has been examined and analyzed. Then the concept of fuzzy subgroupoid has been redefined. This redefined concept is found more appealing and more general than that of ***Rosenfeld***. The notion of product (*) of fuzzy subgroupoids has also been introduced and some new results have been derived there upon.

Chapter III discusses the improvement of the theory of fuzzy subgroupoids. For this purpose the universe of discourse has been changed from groupoid to group. The primary purpose of this chapter is to give a description of these improvements. Here the concept of fuzzy subgroups introduced by ***Rosenfeld*** has been investigated and analyzed under different conditions. Finally, the redefined version of fuzzy subgroups has been proposed and some new fascinating results have been derived. It has been established that the product (*) of two fuzzy subgroups of a group G is again a fuzzy subgroup of G.

In the **fourth chapter** "level subsets" and "level subgroups" of a fuzzy subgroup have been studied. The notion of level subset was first introduced by ***Zadeh*** and that of level subgroups by ***P.S.Das*** in 1981. Here modified versions of "level subsets" and "level subgroups" have been presented and results of ***Das*** and others have been substantiated in terms of the new definitions of level subset and level subgroups and fuzzy subgroups proposed in second chapter.

Chapter V concentrates on the fuzzification of the theory of normal subgroups. It has been observed that fuzzy normal subgroups (FNSG) are a generalization of normal subgroups. FNSG were also studied by ***Mukherjee & Bhattacharya*** and also by ***M.Akgul***. But the present study proposes the definition of FNSG differently. Several analogs of classical normal subgroups for fuzzy normal subgroups have also been derived. In this chapter some results on fuzzy cosets, fuzzy quotient group, and fuzzy version of the famous ***Lagrange's Theorem*** have also been achieved. Again, the notion of product (*) of two FNSG has been introduced and some new results have been found.

In the **VIth chapter** an attempt has been made to study the methods of fuzzification of the classical algebraic structures such as rings, ideals, sub- rings and fields. It has been dealt with in two parts. The first part covers the fuzzy rings and fuzzy ideals and the second part covers the fuzzy field. The concepts of fuzzy rings, fuzzy sub-rings, fuzzy ideals and fuzzy fields introduced by ***Liu, Dixit, Kumar*** and ***Ajmel; Nanda*** and ***Biswas*** respectively have been analyzed and investigated. It has been observed that they are more general than their classical counterparts. Then, redefined versions of these algebraic structures have also been presented. It would be proper to mention here that these definitions are more appealing than the definitions of ***Liu, Dixit, Kumar and Ajmel; Nanda*** and ***Biswas***. Besides, the notion of product (*) of two fuzzy rings, two fuzzy ideals and two fuzzy fields have been introduced and it has been established that these structures are preserved under the product (*).

The **seventh chapter** is devoted to the study of the fuzzifications of linear spaces which was proposed by ***Nanda*** in 1986 and redefined by ***Biswas*** in 1989. It has been presented in two parts. The first part details the notion of a geometrical set in a linear space and derives some results for geometrical sets in a linear space, which are analogous to the results of ***Chandra*** & ***Rehman***. And the second part analyzes the concept of fuzzy linear spaces (FLS) and observes that it is more general than its classical counter part. Here, the redefined notion of FLS has been proposed and it has been shown that the proposed definition is more general and appealing than that of ***Nanda*** and ***Biswas***. The notion of product (*) of two fuzzy linear spaces has been proposed and it has been observed that the product is again a FLS under the new definition.

Chapter VIII is dedicated to the fuzzification of module theory. ***Negoita*** and ***Ralescu*** did this coveted work by applying fuzzy sets in the realm of module theory and proposed the notion of fuzzy modules. It is more general than its classical counterpart. Here, once again we have proposed the redefined version of fuzzy modules and it has been shown that it is more appealing than those of ***Negoita*** and ***Ralescu***. Finally, the notion of product (*) of fuzzy modules has been introduced, which is again a fuzzy module as per the new definition.

Chapter IX is concerned with the study of fuzzy topological spaces which was initiated by ***C.L.Chang***. Here the concepts such as compactness, separation axioms and continuous maps for fuzzy topological spaces have been dealt with and some good results have been discovered.

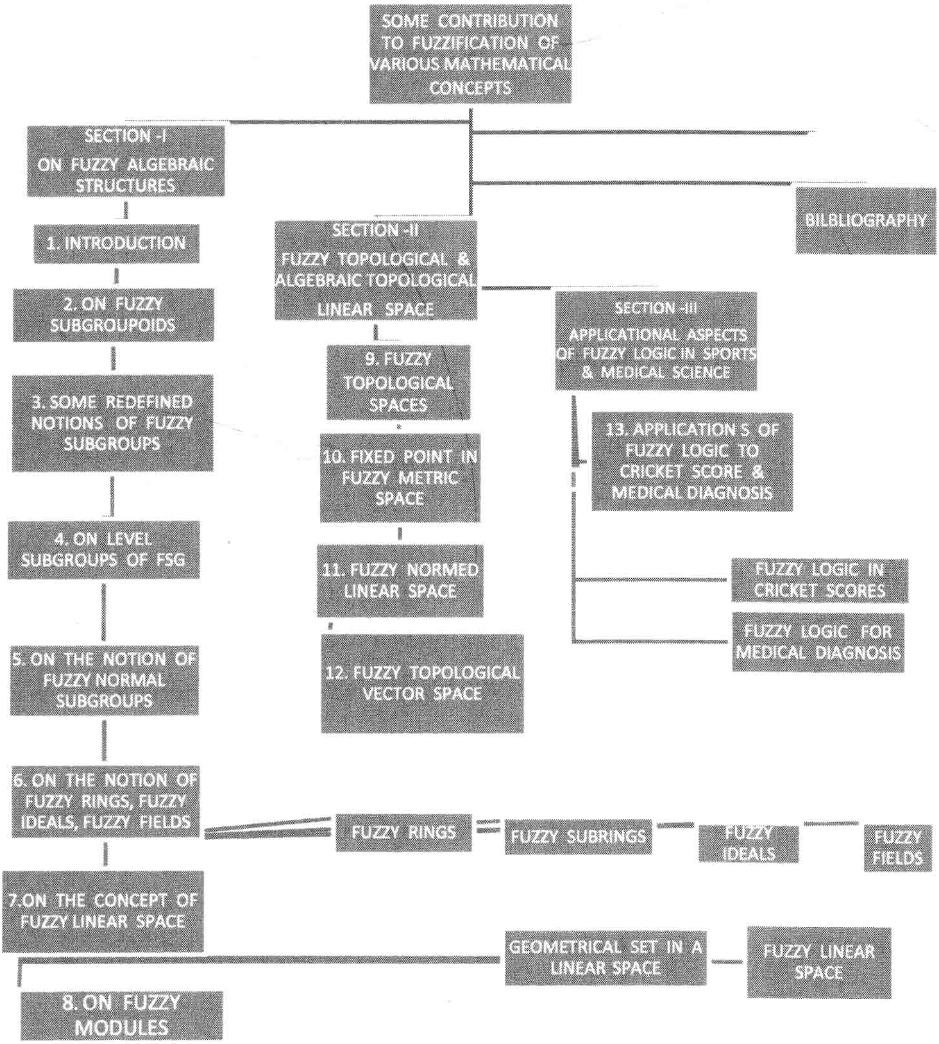
Chapter X is connected with the notion of fixed point in fuzzy metric space and complete fuzzy metric space. The study was set forth by ***O.Kalevo*** and ***S.Seikkla*** in 1984. Here the existence of unique common fixed point for four self maps satisfying certain properties called expansive condition has been examined.

Chapter XI aims at providing an introduction to fuzzy normed linear space which was proposed by ***C.Felbin*** in 1992. In this chapter, we have redefined the notion of fuzzy normed linear space in such a way that it appears to be an extension of the definition of a classical normed linear space. Following this definition of fuzzy normed linear space, we have shown that vector addition and scalar multiplication are continuous functions.

Chapter XII discusses some basic investigations about fuzzy topological vector spaces, propounded by ***Katsara*** in 1981. Here the properties of fuzzy semi-normed spaces have been studied. The bornological fuzzy topological vector spaces have also been studied and some results have been derived.

In the **concluding chapter**, the entire research has been directed to the application of fuzzy sets and fuzzy logic in two diverse fields, other than mathematics namely, sports and medical science. Firstly, fuzzy logic has been employed in cricket scores visualizing the result of the match. In the second case, fuzzy sets and fuzzy logic have been applied to a very important area of medical science such as the diagnostic problem related to the detection of blood sugar in human beings.

Thus, the mathematical theories of fuzzy algebraic structures, fuzzy topological structures and fuzzy topological algebraic structures have been crafted and advanced for further innovative application in different areas of knowledge and investigation such as sports and medical science. This is to demonstrate in no uncertain terms how a result of classical mathematics can be transformed in terms of fuzzy mathematics.



CHAPTER DEPENDENCY DIAGRAM

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