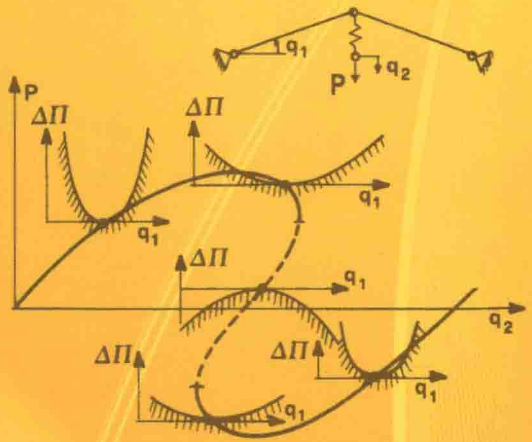
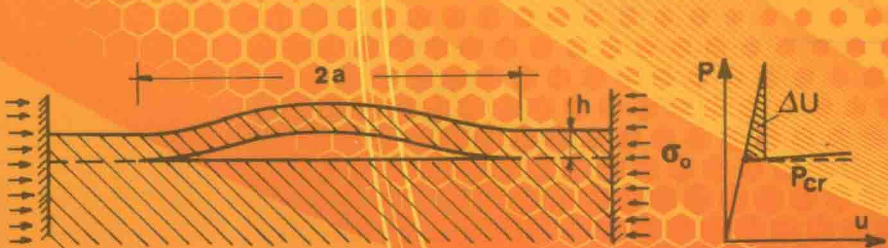
ZDENĚK P BAŽANT • LUIGI CEDOLIN



STABILITY OF STRUCTURES

Elastic, Inelastic, Fracture and Damage Theories



World Scientific

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ZDENĚK P BAŽANT
Northwestern University, USA

LUIGI CEDOLIN
Politecnico di Milano, Italy

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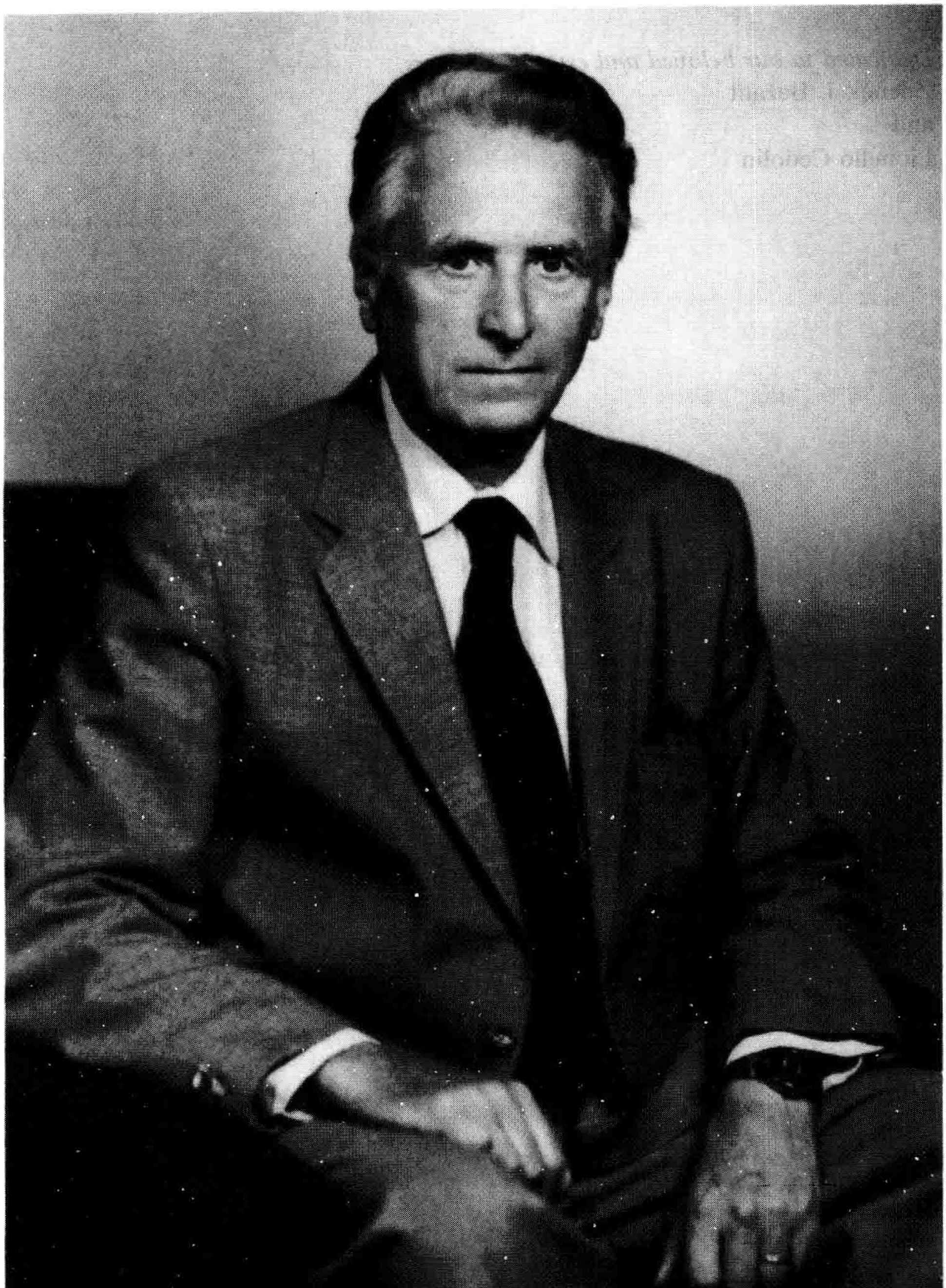
STABILITY OF STRUCTURES

Elastic, Inelastic, Fracture and Damage Theories

Dedicated to our beloved and esteemed fathers
Zdeněk J. Bažant
and
Lionello Cedolin



Dr. ZDENĚK J. BAŽANT, one of the two persons to whom this book is dedicated, was born on June 11, 1908, in Nové Město in Moravia, Czechoslovakia. He was Professor of Foundation Engineering at the Czech Technical University at Prague during 1947–1975. Prior to that he had been the chief design engineer of Lanna Co. in Prague (1932–1947). Since 1975, he has been Professor Emeritus and Consultant to Geoindustria Co. in Prague. He has been a fellow of ASCE without interruption since 1947. He has authored a number of books and numerous research articles.

Preface

It is our hope that this book will serve both as a textbook for graduate courses on stability of structures and a reference volume for engineers and scientists. We assume the student has a background in mathematics and mechanics only at the level of the B.S. degree in civil or mechanical engineering, though in the last four chapters we assume a more advanced background. We cover subjects relevant to civil, structural, mechanical, aerospace, and nuclear engineering, as well as materials science, although in the first half of the book we place somewhat more emphasis on the civil engineering applications than on others. We include many original derivations as well as some new research results not yet published in periodicals.

Our desire is to achieve understanding rather than just knowledge. We try to proceed in each problem from special to general, from simple to complex, treating each subject as concisely as we can and at the lowest possible level of mathematical apparatus we know, but not so low as to sacrifice efficiency of presentation. We include a large number (almost 700) of exercise problems. Solving many of them is, in our experience, essential for the student to master the subject.

In some curricula, the teaching of stability is fragmented into courses on structural mechanics, design of steel structures, design of concrete structures, structural dynamics, plates and shells, finite elements, plasticity, viscoelasticity, and continuum mechanics. Stability theory, however, stands at the heart of structural and continuum mechanics. Whoever understands it understands mechanics. The methods of stability analysis in various applications are similar, resting on the same principles. A fundamental understanding of these principles, which is not easy to acquire, is likely to be sacrificed when stability is taught by bits, in various courses. Therefore, in our opinion, it is preferable to teach stability in a single course, which should represent the core of the mechanics program in civil, mechanical, and aerospace engineering.

Existing textbooks of structural stability, except for touching on elastoplastic columns, deal almost exclusively with elastic stability. The modern stability problems of fracture and damage, as well as the thermodynamic principles of stability of irreversible systems, have not been covered in textbooks. Even the catastrophe theory, as general is it purports to be, has been limited to systems that possess a potential, which implies elastic behavior. Reflecting recent research results, we depart from tradition, devoting about half of the book to nonelastic stability.

Various kinds of graduate courses can be fashioned from this book. The first-year quarter-length course for structural engineering students may, for example, consist of Sections 1.2–1.7, 2.1–2.4, 2.8, 3.1, 3.2, 3.5, 3.6, 4.2–4.6, 5.1–5.4, 6.1–6.3, 7.1–7.3, 7.5, 7.8, 8.1, 8.3, and 8.4, although about one-third of these sections can be covered in one quarter only partly. A semester-length course can cover them fully and may be expanded by Sections 1.8, 1.9, 2.7, 3.3, 4.5, 4.6, 5.5, 7.4, 7.8, 8.2, and 8.6. The first-year course for mechanical and aerospace engineers may, for example, be composed of Sections 1.1–1.5, 1.7, 1.9, 2.1–2.3, 3.1–3.7, 4.2–4.6, 5.1–5.4, 6.1–6.3, 7.1–7.3, 7.5, 7.8, 8.1–8.3, and 9.1–9.3, again with some sections covered only partly. A second-year sequel for structural engineering students, dealing with inelastic structural stability, can, for example, consist of Sections 8.1–8.6, 9.1–9.6, 10.1–10.4, 13.2–13.4, and 13.6, preceded as necessary by a review of some highlights from the first course. Another possible second-year sequel, suitable for students in theoretical and applied mechanics, is a course on material modeling and stability, which can be set up from Sections 11.1–11.7, 10.1–10.6, 13.1–13.4, 13.8–13.10, and 12.1–12.5 supplemented by a detailed explanation of a few of the constitutive models mentioned in Section 13.11. A course on Stability of Thin-Wall Structures (including plates and shells) can consist of a review of Sections 1.1–1.8 and detailed presentation of Chapters 6 and 7. A course on Inelastic Columns can be based on a review of Sections 1.1–1.8 and detailed presentation of Chapters 8 and 9. A course on Stability of Multidimensional Structures can be based on a review of Sections 1.1–1.9 and detailed presentation of Chapters 7 and 11. A course on Energy Approach to Structural Stability can be based on a review of Sections 1.1–1.8 and detailed presentation of Chapters 4, 5, and 10. A course on Buckling of Frames can be based on Chapters 1, 2, and 3. Chapter 3, along with Section 8.6, can serve as the basis for a large part of a course on Dynamic Stability.

The present book grew out of lecture notes for a course on stability of structures that Professor Bažant has been teaching at Northwestern University every year since 1969. An initial version of these notes was completed during Bažant's Guggenheim fellowship in 1978, spent partly at Stanford and Caltech. Most of the final version of the book was written during Professor Cedolin's visiting appointment at Northwestern between 1986 and 1988, when he enriched the text with his experience from teaching a course on structural analysis at Politecnico di Milano. Most of the last six chapters are based on Bažant's lecture notes for second-year graduate courses on inelastic structural stability, on material modeling principles, and on fracture of concrete, rock, and ceramics. Various drafts of the last chapters were finalized in connection with Bažant's stay as NATO Senior Guest Scientist at the Ecole Normale Supérieure, Cachan, France, and various sections of the book were initially presented by Bažant during specialized intensive courses and guest seminars at the Royal Institute of Technology (Cement och Betonginstitutet, CBI), Stockholm; Ecole des Ponts et Chaussées, Paris; Politecnico di Milano; University of Cape Town; University of Adelaide; University of Tokyo; and Swiss Federal Institute of Technology. Thanks go to Northwestern University and the Politecnico di Milano for providing environments conducive to scholarly pursuits. Professor Bažant had the good fortune to receive financial support from the U.S. National Science Foundation and the Air Force Office of Scientific Research, through grants to Northwestern University; this funding supported research on which the last six chapters are partly based. Professor Bažant wishes to express his thanks to his father, Zdeněk J. Bažant, Professor Emeritus of Foundation Engineering at the

Czech Technical University (ČVUT) in Prague and to his grandfather, Zdeněk Bažant, late Professor of Structural Mechanics at ČVUT, for having introduced him to certain stability problems of structural and geotechnical engineering.

We are indebted for many detailed and very useful comments to Leone Corradi and Giulio Maier, and for further useful comments to several colleagues who read parts of the text: Professors J. P. Cordebois, S. Dei Poli, Eduardo Dvorkin, Theodore V. Galambos, Richard Kohoutek, Franco Mola, Brian Moran, and Jaime Planas. Finally, we extend our thanks to M. Tabbara, R. Gettu, and M. T. Kazemi, graduate research assistants at Northwestern University, for checking some parts of the manuscript and giving various useful comments, to Vera Fisher for her expert typing of the manuscript, and to Giuseppe Martinelli for his impeccable drawings.

Evanston, Ill.
October, 1989

Z. P. B. and L. C.

Introduction

One of the principal objectives of theoretical research in any department of knowledge is to find the point of view from which the subject appears in its greatest simplicity.

—J. Willard Gibbs
(acceptance letter of Rumford Medal, 1881)

Failures of many engineering structures fall into one of two simple categories: (1) material failure and (2) structural instability. The first type of failure, treated in introductory courses on the strength of materials and structural mechanics, can usually be adequately predicted by analyzing the structure on the basis of equilibrium conditions or equations of motion that are written for the initial, undeformed configuration of the structure. By contrast, the prediction of failures due to structural instability requires equations of equilibrium or motion to be formulated on the basis of the deformed configuration of the structure. Since the deformed configuration is not known in advance but depends on the deflections to be solved, the problem is in principle nonlinear, although frequently it can be linearized in order to facilitate analysis.

Structural failures caused by failure of the material are governed, in the simplest approach, by the value of the material strength or yield limit, which is independent of structural geometry and size. By contrast, the load at which a structure becomes unstable can be, in the simplest approach, regarded as independent of the material strength or yield limit; it depends on structural geometry and size, especially slenderness, and is governed primarily by the stiffness of the material, characterized, for example, by the elastic modulus. Failures of elastic structures due to structural instability have their primary cause in geometric effects: the geometry of deformation introduces nonlinearities that amplify the stresses calculated on the basis of the initial undeformed configuration of the structure.

The stability of elastic structures is a classical problem which forms the primary content of most existing textbooks. We will devote about half the present treatise to this topic (Part I, Chapters 1–7).

We begin our study of structural stability with the analysis of buckling of elastic columns and frames, a bread-and-butter problem for structural engineers. Although this is a classical research field, we cover in some detail various recent advances dealing with the analysis of very large regular frames with many members, which are finding increasing applications in tall buildings as well as certain designs for space structures.

The study of structural stability is often confusing because the definition of structural stability itself is unstable. Various definitions may serve a useful purpose for different problems. However, one definition of stability—the dynamic definition—is fundamental and applicable to all structural stability problems. Dynamic stability analysis is essential for structures subjected to nonconservative loads, such as wind or pulsating forces. Structures loaded in this manner may falsely appear to be stable according to static analysis while in reality they fail through vibrations of ever increasing amplitude or some other accelerated motion. Because of the importance of this problem in modern structural engineering we will include a thorough treatment of the dynamic approach to stability in Chapter 3. We will see that the static approach yields correct critical loads only for conservative structural systems, but even for these it cannot answer the question of stability completely.

The question of stability may be most effectively answered on the basis of the energy criterion of stability, which follows from the dynamic definition if the system is conservative. We will treat the energy methods for discrete and discretized systems in Chapter 4 and those for continuous structures in Chapter 5, in which we will also focus on the approximate energy methods that simplify the stability analysis of continuous structures.

In Chapters 6 and 7 we will apply the equilibrium and energy methods to stability analysis of more complicated thin structures such as thin-wall beams, the analysis of which can still be made one-dimensionally, and of two-dimensional structures such as plates and shells. Because many excellent detailed books deal with these problems, and also because the solution of these problems is tedious, requiring lengthy derivations and mathematical exercises that add little to the basic understanding of the behavior of the structure, we limit the treatment of these complex problems to the basic, prototype situations. At the same time we emphasize special features and approaches, including an explanation of the direct and indirect variational methods, the effect of imperfections, the postcritical behavior, and load capacity. In our computer era, the value of the complicated analytical solutions of shells and other thin-wall structures is diminishing, since the solutions can be obtained by finite elements, the treatment of which is outside the scope of the present treatise.

While the first half of the book (Part I, Chaps. 1–7) represents a fairly classical choice of topics and coverage for a textbook on structural stability, the second half of the book (Part II, Chaps. 8–13), devoted to inelastic and damage theories of structural stability, attempts to synthesize the latest trends in research. Inelastic behavior comprises not only plasticity (or elastoplasticity), treated in Chapters 8 and 10, but also creep (viscoelastic as well as viscoplastic), treated in in Chapter 9, while damage comprises not only strain-softening damage, treated in Chapter 13, but also fracture, which represents the special or limiting case of localized damage, treated in Chapter 12. Whereas the chapters dealing with plasticity and creep present for the most part relatively well-established theories, Chapters 10–13, dealing with thermodynamic concepts and finite strain effects in three dimensions, as well as fracture, damage, and friction, present mostly fresh results of recent researches that might in the future be subject to reinterpretations and updates.

Inelastic behavior tends to destabilize structures and generally blurs the aforementioned distinction between material failures and stability failures. Its effect can be twofold: (1) it can merely reduce the critical load, while instability is still caused by nonlinear geometric effects and cannot occur in their absence—this is typical of plasticity and creep (with no softening or damage); or (2) it can cause instability by itself, even in the absence of nonlinear geometric effects in the structure—this is typical of fracture, strain-softening damage, and friction and currently represents a hot research subject. An example of this behavior is fracture mechanics. In this theory (outlined in Chapter 12), structural failure is treated as a consequence of unstable crack propagation, the instability being caused by the global structural action (in which the cause of instability is the release of energy from the structure into the crack front) rather than the nonlinear geometric effects.

Stability analysis of structures that are not elastic is complicated by the fact that the principle of minimum potential energy, the basic tool for elastic structures, is inapplicable. Stability can, of course, be analyzed dynamically, but that too is complicated, especially for inelastic behavior. However, as we will see in Chapter 10, energy analysis of stability is possible on the basis of the second law of thermodynamics. To aid the reader, we will include in Chapter 10 a thorough discussion of the necessary thermodynamic principles and will then apply them in a number of examples.

Irreversibility, which is the salient characteristic of nonelastic behavior, produces a new phenomenon: the bifurcation of equilibrium path need not be associated with stability loss but can typically occur in a stable manner and at a load that is substantially smaller than the stability limit. This phenomenon, which is not found in elastic structures, will come to light in Chapter 8 (dealing with elastoplastic columns) and will reappear in Chapters 12 and 13 in various problems of damage and fracture. A surprising feature of such bifurcations is that the states on more than one postbifurcation branch of the equilibrium path can be stable, which is impossible for elastic structures or reversible systems in general. To determine the postbifurcation path that will actually be followed by the structure, we will need to introduce in Chapter 10 a new concept of stable path, which, as it turns out, must be distinct from the concept of stable state. We will present a general thermodynamic criterion that makes it possible to identify the stable path.

The stability implications of the time-dependent material behavior, broadly termed creep, also include some characteristic phenomena, which will be explained in Chapter 9. In dealing with imperfect viscoelastic structures under permanent loads, we will have to take into account the asymptotic deflections as the time tends to infinity, and we will see that the long-time (asymptotic) critical load is less than the instantaneous (elastic) critical load. In imperfect viscoelastic structures, the deflections can approach infinity at a finite critical time, and can again do so under a load that is less than the instantaneous critical load. For creep buckling of concrete structures, we will further have to take into account the profound effect of age on creep exhibited by this complex material.

The most important consequence of the instabilities caused by fracture or damage rather than by geometric effects is that they produce size effect, that is, the structure size affects the nominal stress at failure. By contrast, no size effect exists according to the traditional concepts of strength, yield limit, and yield surface in the stress or strain space. Neither does it according to elastic stability theory. The most severe and also the simplest size effect is caused by failures due to propagation of a sharp fracture where the fracture process happens at a point. A less severe size effect, which represents a transition from failures governed by strength or yield criteria to failures governed by instability of sharp fractures, is produced by instability modes consisting either of propagation of a fracture with a large fracture process zone (Chap. 12) or of damage localization (Chap. 13). As a special highlight of the present treatise, these modern problems are treated in detail in the last two chapters.

The practical design of metallic or concrete columns and other structures is an important topic in any stability course. In this text, the code specifications and design approaches are dispersed through a number of chapters instead of being presented compactly in one place. This presentation is motivated by an effort to avoid a cookbook style and present each aspect of design only after the pertinent theory has been thoroughly explained, but not later than that. It is for this reason, and also because fundamental understanding of inelastic behavior is important, that the exposition of column design is not completed until Chapters 8 and 9, which also include detailed critical discussions of the current practice.

The guiding principle in the presentation that follows is to advance by induction, from special and simple to general and complex. This is one reason why we choose not to start the book with general differential equations in three dimensions and thermodynamic principles, which would then be reduced to special cases. (The general three-dimensional differential equations governing stability with respect to nonlinear geometric effects do not appear in the book until Chap. 11.) There is also another reason—the three-dimensional analysis of stability is not necessary for slender or thin structures made of structural materials such as steel or concrete, which are relatively stiff. It is only necessary for dealing with incremental deformations of massive inelastic structures or structures made of highly anisotropic or composite materials which can be strained to such a high level that some of the tangential moduli of the material are reduced to values that are of the same order of magnitude as the stresses.

As another interesting phenomenon, which we will see in Chapter 11, various possible choices of the finite-strain tensor lead to different expressions for the critical loads of massive bodies. It turns out that the stability formulations corresponding to different choices of the finite-strain tensor are all equivalent, but for each such formulation the tangential moduli tensor of the material has a different physical meaning and must be determined from experimental data in a different manner. If this is not done, then three-dimensional finite-strain stability analysis makes no sense.

As we live in the new era of computers, stability of almost any given structure could, at least in principle, be analyzed by geometrically nonlinear finite element codes with incremental loading. This could be done in the presence of complex nonlinear behavior of the material as well. Powerful though this approach is, the value of simple analytical solutions that can be worked out by hand must not be underestimated. This book attempts to concentrate on such solutions. It is these solutions that enhance our understanding and also must be used as test cases for the finite element programs.

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