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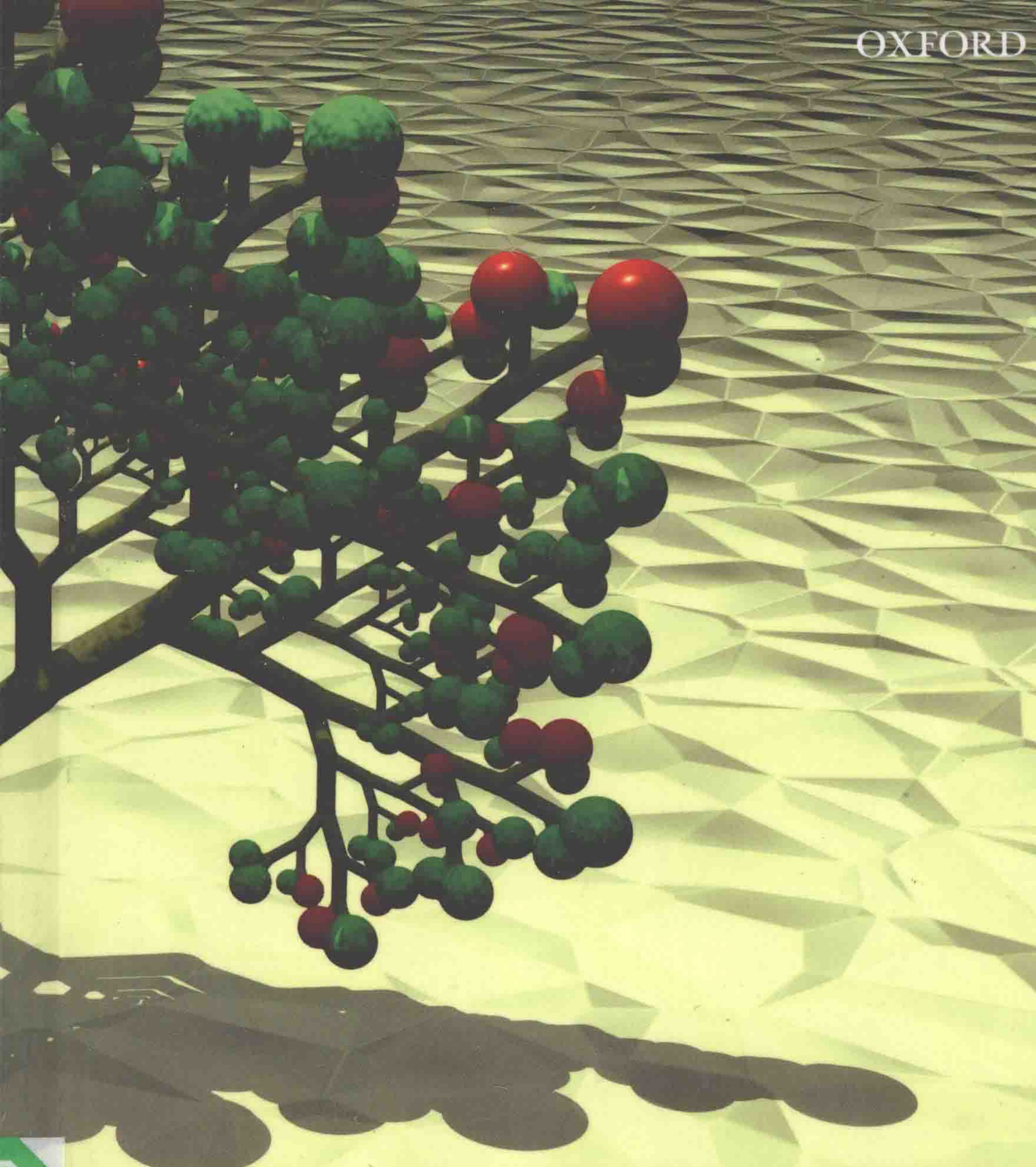

Modeling Reality

HOW COMPUTERS MIRROR LIFE

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Modeling Reality

How Computers Mirror Life

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Modeling Reality

We dedicate this book to Zofia Białynicka-Birula

Preface

This book originated from a series of lectures delivered by the first author at the Warsaw School of Social Psychology and at Warsaw University over the last six years. The purpose of these lectures was to give a very broad overview of various aspects of modeling for a mixed audience, from students of mathematics, computer science, and physics to students of biology and social sciences. Considering the different levels of mathematical literacy among those who attended the lectures, we have relied only on the mathematical concepts known to high school graduates. Therefore, our book can be understood by a wide spectrum of readers—from ambitious high school students to graduate students of all specialities. We were trying to keep the mathematics at the high school level; however, some chapters may require an additional effort since they describe modern advances in computer modeling.

The book was originally published in Polish in 2002 by the publishing house Prószyński and Ska in Warsaw. The English translation is substantially expanded and modified.

The material presented in this book is illustrated with twenty-five computer programs written especially for this purpose. All of the programs have undergone a substantial upgrade and face-lift for the English edition.

Acknowledgements

We warmly thank Vittorio Bertocci for many valuable comments and suggestions regarding both the text of the book and the form of the programs. This work has been partly supported by the Polish Ministry of Scientific Research Grant Quantum Information and Quantum Engineering.

All portraits of the scientists and the chapter opening images were drawn by Maria Białynicka-Birula.

Publisher's Note

This print-on-demand edition of Modelling Reality does not include a CD-ROM. The textual references to the CD-ROM are from a previous print edition. The CD-ROM contents are now available on the following website, http://www.modelingreality.net/getprogs.html

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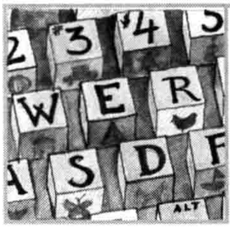

From building blocks to computers

Models and modeling

The aim of science is to describe, explain, and predict the behavior of the world which surrounds us. Reality is, however, much too complex to be described accurately, without any simplification or approximation. That is why, when describing a given phenomenon, we take into account only the elements of reality which we think have a significant influence over the phenomenon we are describing. Let us suppose, for example, that we are describing the motion of an apple falling from a tree. To describe it we would take into account the weight of the apple, the height of the tree, and perhaps the air resistance. On the other hand, we would omit such factors as the taste of the apple, its color, or the configuration of the planets in the sky at that given moment. The planet configuration does in fact influence the motion of the apple, but that influence is so small that we can easily omit it.

Specifying the significant factors is perhaps the most important element of research. Many fields of science, especially astronomy, physics, and chemistry, started developing rapidly only when the scientists learned to recognize the significance of different factors and to omit the insignificant ones. In other fields, such as social science, psychology, or medicine, the significance of different factors has not yet been precisely established.

Conducting research in a given field can be made easier if one first tries to understand the behavior of very simple objects, hoping to thus shed light on rules governing the behavior of the more complex ones. The concepts and methods covered in this book can considerably aid such an approach.

The word **model** appeared in written English for the first time in 1575. Nowadays it has many meanings. In Webster's *New encyclopedic dictionary* we find nine and in *Merriam–Webster dictionary* even thirteen different meanings of the term *model*. In a recent book (Müller and Müller, 2003), the authors give nineteen illuminating examples of different uses of this notion. There are two meanings listed in Webster that are perhaps the closest to our usage of this term. These are: 'a system of assumptions, data, and inferences used to describe mathematically an object or state of affairs' and 'a theoretical projection of a possible or imaginary system'. In this book, by a **model** we shall mean the set of elements of reality considered to be significant for a given phenomenon and the rules governing the behavior of these elements. The choice of significant elements and the definition of the rules is indeed the essence of modeling. We can judge the suitability of this choice by comparing the results obtained from the model with reality.

From cradle days, every one of us comes across many different models. Even the cradle itself is a model. Most children's toys are models of objects from the adult world and games are models of real social situations. Our adult life is full of models as well. A map is a model of a city or a country, a globe is a model of the Earth, and a calendar is a model of a year. When conducting a poll, as a model of society, we often use a small sample of its representatives. A computer is a model of evolution's grandest creation—the human brain. All computers can in turn be modeled by a single object, called the Turing machine, an abstract device described in detail later in this book. Even the human intellect can be modeled, its model being what is called artificial intelligence.

The act of modeling can be split into three phases. The first is the most difficult of all as it does not conform to any precise rules—it is **choosing the model** to describe a given aspect of reality. The second phase is **constructing an algorithm** according to which the model will function. An algorithm is a set of rules, which when applied systematically leads to a solution to the problem. The third phase, easiest, though often arduous and time-consuming, is checking the obtained results against the initial hypothesis and **drawing conclusions**. Naturally, in the end it is the conformity of the results with the reality that we meant to model that serves to measure the model's worth.

Algorithms were known to ancient Greek mathematicians (the Euclides algorithm for finding the greatest common divisor, and the Eratosthenes algorithm for building a sequence of consecutive prime numbers), but the term itself was introduced much later and comes from the Latin form (Algorismi) of the name of a ninth-century mathematician from Baghdad—Muhammad ibn Musa al-Khwarizmi.

What is amazing about modeling is how accurately we can often describe the world, notwithstanding the enormous simplifications we are forced to apply. In this respect, nature is kind to us, even though it could have been altogether different. All phenomena could have been intrinsically intertwined with one another, and without disentangling the entire knot of dependencies we would not be able to draw conclusions regarding the behavior of any single thread. Moreover, it is possible that there exist fields in which no simplified models will yield a suitable description of reality, and in which one must take into account an entire multitude of mutually-dependent factors to obtain a result. One good candidate for such a model-defiant object is our consciousness. In most other areas of research, however, modeling seems to bring success upon success and so far no limits imposed by nature are making themselves apparent. Progress seems to be purely a function of time.

One could come to the conclusion that *every* theory—every description of reality—is a model, since evidently no theory is perfect. What is then the difference between the modeling of a phenomenon and the creation of a theory describing it? The borderline between these two concepts is indeed diffused. Some theories are nothing but simple models, while some models deserve the rank of a fully-fledged theory. In our opinion, however, the difference lies in the fact that, when forming a **theory**, we try to take into account all of the factors we know to have an influence on the studied phenomenon. When building a theory we aim at perfection (even though we rarely achieve it). When constructing a model, however, we *purposely* exclude some of the factors and leave only a few chosen ones for the sake of obtaining a simpler scheme.

The omission of influential factors can be complemented by the use of **probability theory**, a very helpful instrument, to which we devote a big part of this book. It allows us to replace *full knowledge* of the studied phenomenon with what we might call *average knowledge*. In this way we are able to achieve approximate results or the most probable results, which are often the best we can get when full knowledge is practically beyond our reach.

We start by introducing a simple model, which shall serve us as a paradigm of modeling reality throughout this book, be it social, biological, physical, or any other kind of reality. It is the cellular automaton called *The game of life*.

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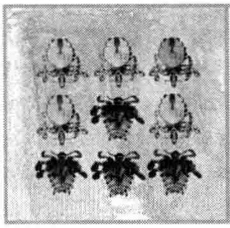

The game of life

A legendary cellular automaton

In the late 1960s John Conway devised a one-person game, which he called *The game of life*. In 1970 Martin Gardner, the author of a mathematical games section in *Scientific American*, popularized Conway's idea. Gardner's two articles caused utter fervor among many computer enthusiasts. Personal computers were not yet around at that time (they appeared only in the mid-1970s) and all experiments with *The game of life* were conducted on giant, expensive mainframes. It has been said that such *Life* games cost American companies several million dollars. The source of such fascination must have been the extreme simplicity of the game's rules combined with the vast variety of resulting 'life-forms' and wealth of interesting problems which arose from it.

John Horton Conway (1937–); English mathematician from Cambridge; known, among others, as coauthor of Conway and Guy (1996).

The game of life represents a very simple model of the birth, evolution, and death of a colony of living organisms—let us call these organisms bacteria. Theoretically, one can play the game by moving around tokens on a *Go* board, although this is a very tedious task. That is, nonetheless, how the game was played in the beginning. To avoid mistakes, two colors of tokens were used. Each move was realized by replacing all of the tokens on the board with tokens of the other color, representing the next generation of bacteria, according to the game's rules described below. Yet it was only the utilization of computers that gave the game its full brilliance.

The rules of the game

Like many models of reality, as well as many board games, *The game of life* takes place in space and time. Space in this game is a two-dimensional lattice divided into square cells (Fig. 2.1). Each cell contains exactly one bacterium—this bacterium can be *dead* or *alive*. A cell containing a living bacterium is marked, while an unmarked cell contains a dead bacterium. The flow of time is also measured discretely, by numbering the consecutive generations of the colony.

The game begins by selecting the cells occupied by the initial generation of living bacteria. The second generation results from the first one as the rules of the game are applied, and so on. In this fashion any number of subsequent generations can be constructed.

Conway experimented for a long time, trying out different rules for the development of the colony. Finally, he came across a set of rules that makes

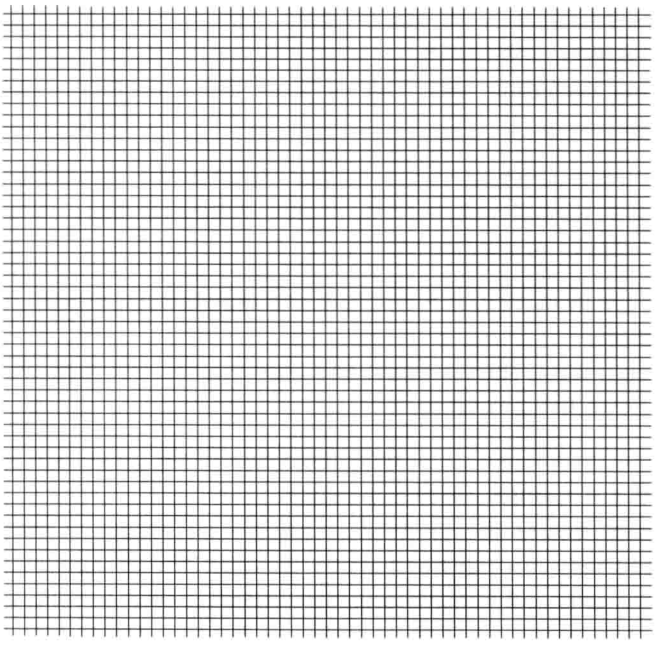

Fig. 2.1 *The game of life* empty board (or computer screen).

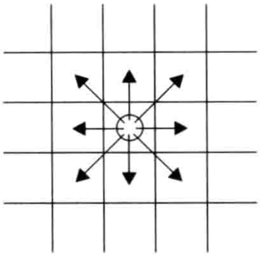

Fig. 2.2 The eight neighbors of a cell.

the evolution of the system both interesting and hard to predict. Conway's rules determine

(i) when a bacterium survives and passes on to the next generation,
(ii) when a new bacterium is born in place of a dead one, and
(iii) when a living bacterium dies.

Which of the events is to take place is determined by the number of living neighbors of a given cell. Each cell neighbors with eight others, as depicted in Fig. 2.2. Thus, the rules are as follows.

- A bacterium with zero or one living neighbors dies of solitude.
- A living bacterium with two or three living neighbors is happy and survives on to the next generation.
- A dead bacterium with exactly three living neighbors is reborn due to optimal living conditions.
- A living bacterium with four or more living neighbors dies of overcrowding.

To make it more straightforward, we can abandon the bacteria concept and refer simply to the cells as being dead or alive. Following this, we can formulate Conway's rules in two sentences.

- If a cell is dead, then it is reborn if and only if it has exactly three living neighbors.
- If a cell is alive, then it dies if and only if it has less than two, or more than three, living neighbors.

We leave the demonstration of the equivalence of the above two sets of rules to the reader as a simple exercise in logic.

With each tick of the clock we conduct a full survey of our cells and, according to the rules explained above, we apply the changes by registering all newborn cells and deregistering the ones that died of solitude or overcrowding. Of course, all of these tasks can be done for us by a computer. Figures 2.3 and 2.4 portray the development of a small colony of bacteria

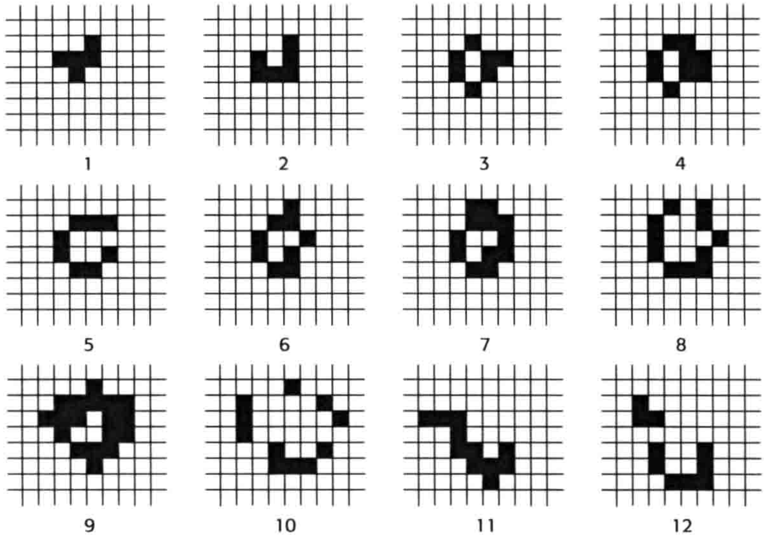

Fig. 2.3 The evolution of a colony composed initially of five living bacteria.

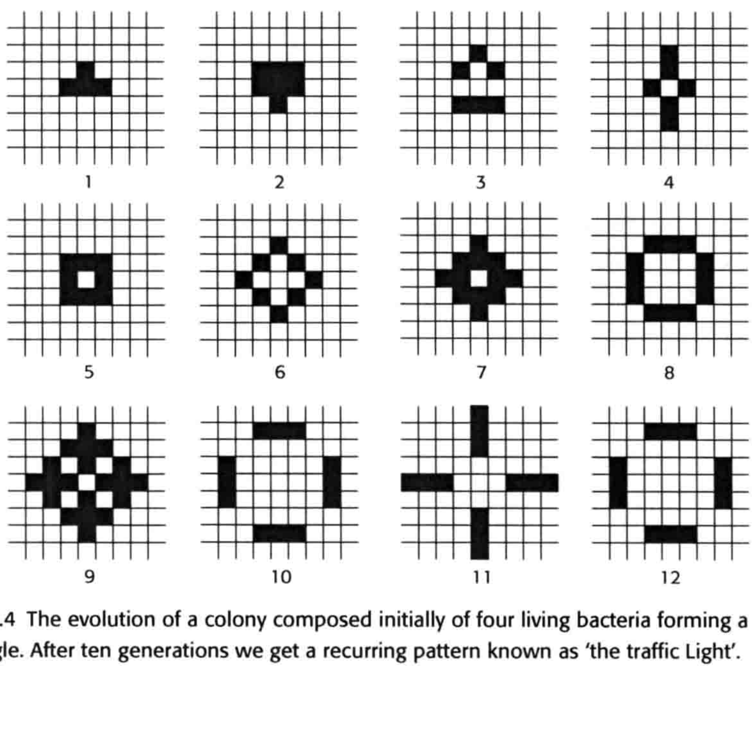

Fig. 2.4 The evolution of a colony composed initially of four living bacteria forming a triangle. After ten generations we get a recurring pattern known as 'the traffic Light'.

throughout twelve generations. We encourage readers to check that each step is constructed from the previous one in accordance with the rules of the game.

The program **Conway** serves to generate a sequence of *Life* generations on a computer. We can use it to track the evolution of a colony constructed by us, or start from one of the pre-built configurations known to yield interesting results. Of the latter, we recommend trying the 'glider factory' configuration—it was devised in answer to one of the problems posed by Conway: is there a configuration whose size (number of living cells) grows without limits?

What we find most amazing when observing the successive generations of bacteria is the giant contrast between the complexity of the development of a colony and the simplicity of both the rules, and the initial configurations. A wealth of forms can therefore be a consequence of very simple rules, and in this respect *The game of life's* case is not an isolated one. More 'scientific' examples can be found in physics or chemistry. Millions, or even billions, of organic chemical compounds possess an immense range of remarkable properties; and yet, they are constructed out of carbon, oxygen, and hydrogen atoms, and a few other elements, in accordance with rules which can be inscribed in the form of one equation—the Schrödinger equation. Finding such simple rules to explain a complex set of phenomena is indeed the quintessence of modeling.

Cellular automata

The game of life is an example of what scientists call a *cellular automaton*. Cellular automata, the theory behind them, and their application are now considered to be a separate field of science. Its beginnings date back to the late 1950s, when Stanislaw Ulam and John von Neumann devised a method for computing the motion of a liquid. The principle of the method was to divide the liquid into cells and determine the motion of each cell on the basis of the behavior of the neighboring ones.

Some consider the Pascal triangle (see Chapter 4) to be the first cellular automaton, with its subsequent rows signifying the successive generations.

Stanislaw Marcin Ulam (1909–1984); a Polish mathematician residing in the United States from 1939. During the Second World War he took part in the Manhattan Project in Los Alamos, where he was the first to present a realistic technique for the construction of the hydrogen bomb.

John von Neumann (1903–1957); an American mathematician of Hungarian origin. Renowned not only as a mathematician, but also as a theoretical physicist and initiator of many fields of applied mathematics. He created game theory (1928), presented the first algorithm for generating random numbers (1946), formulated the functioning principles of a modern computer containing a central processor (CPU) and data storage memory (RAM) (1945), and played a large role in the construction of the first nuclear bomb.

In general, a cellular automaton can be characterized by the following three parameters:

- the structure of the space in which all events take place;
- the definition of objects inhabiting that space;
- a set of rules defining the behavior of these objects.

Due to the huge diversity of conceivable cellular automata, a uniform theory of such structures has not yet been developed. Only in the simplest case of a one-dimensional space and with the introduction of other limiting assumptions did Stephen Wolfram arrive at a complete classification of cellular automata. If we decided to allow each cell of an automaton to have one of only two possible values (like in *The game of life*) and if the evolution of a cell depended only on its own value and on the value of two of its closest neighbors, then we would already have 256 possible automata. This follows from the fact that there are $2^3 = 8$ possible states of three neighboring cells, and for each such state we can independently select one of two possible outcomes—the resulting state of the middle cell. We get $2^8 = 256$ possibilities. The number of possible automata grows dramatically with the increase in the number k of possible cell states and the number s of cells influencing the evolution of a given cell. The general equation is k^{k^s}. If we increase the range of influence to include two other neighbors (now $s = 5$, i.e. the cell itself and four of its neighbors), then we already have $2^{2^5} = 2^{32} = 4\,924\,967\,296$ possible different cellular automata.

Stephen Wolfram (1959–); creator of Mathematica® and founder of Wolfram Research (http://www.wolfram.com). His recently published book (Wolfram, 2002) is a giant tribute to cellular automata, which the author claims underlie all of the universe's phenomena.

It is worth noting that the future of a cellular automaton can always be predicted, as long as its rules do not contain any elements of chance. Usually, however, knowing the state of an automaton does not allow us to guess

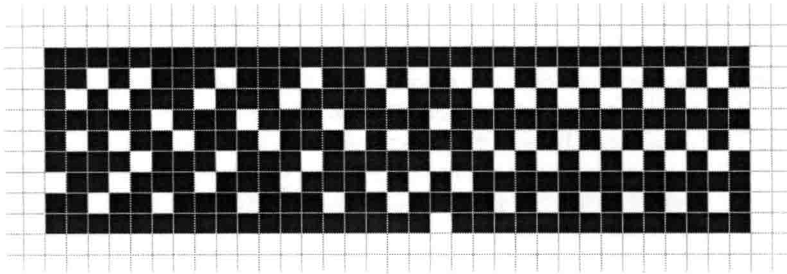

Fig. 2.5 The first 'Garden of Eden'-type configuration, found by Roger Banks—a configuration for which no previous generation exists.

its past. A configuration can have many possible predecessors, or it can be a configuration for which no previous configuration exists. A configuration with no possible predecessors has been given the evocative name of a 'Garden of Eden'. The quest for Gardens of Eden was a popular activity during the beginnings of *The game of life* craze. Roger Banks was the first to find such a configuration (depicted in Fig. 2.5) in 1971, with the use of a computer program searching all possible preceding configurations.

Cellular automata have found many applications in physics, chemistry, and technical sciences. We can use them to model such diverse processes as the motion of loose matter (such as sand piles), the flow of fluid through porous materials, the spread of forest fires, traffic jams, the interaction of elementary particles, and many others. Also, the models of society described in Chapter 14 are very much like cellular automata. Neural networks, which model the human brain and are described in Chapter 13, have a lot in common with cellular automata as well, since each element of such a network functions, to a large degree, independently.