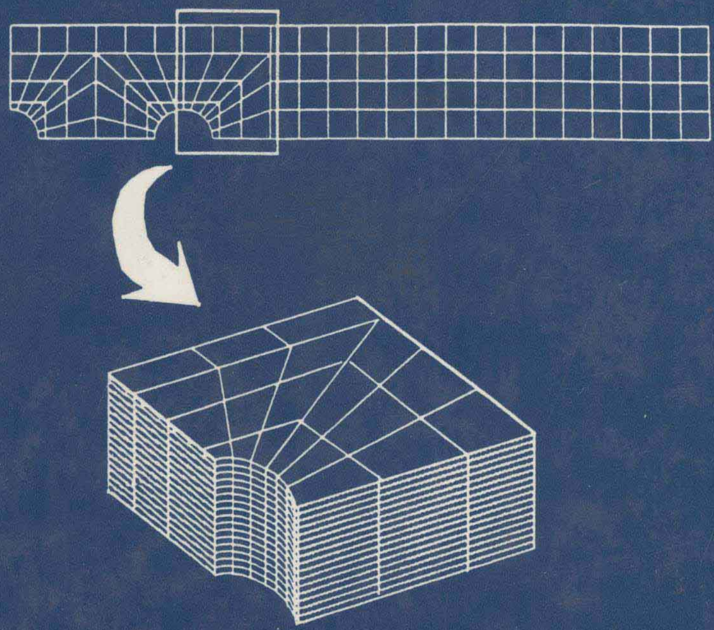
SOLID MECHANICS AND ITS APPLICATIONS

O. O. Ochoa and J. N. Reddy

Finite Element Analysis of Composite Laminates



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Finite Element Analysis of Composite Laminates

by

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Volume 7

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About the Authors

Dr. Ozden Ochoa is a professor of Mechanical Engineering at Texas A&M University. Her research in composites focuses on a broad range of problems in mechanics of composite structures, with applications in aerospace, automotive and construction industries. She authored over sixty publications on these topics. She taught shortcourses on the finite element method and principles of composites. Dr. Ochoa serves on national technical committees of Aerospace, Applied Mechanics, and Petroleum Divisions of American Society of Mechanical Engineers. At Texas A&M University, she serves as a faculty advisor to Engineering Scholars Program, University Honors Program, and Accelerated BS/MS Program. She is a member of the Center for Mechanics of Composites and the Offshore Technology Research Center. Dr. Ochoa is recognized as a 1991 University Honors Teacher/Scholar award recipient.

Dr. Reddy is the first holder of the *Oscar S. Wyatt, Jr. Endowed Chair* in Mechanical Engineering at Texas A&M University. Prior to the current appointment, he was the *Clifton C. Garvin Chaired Professor* of Engineering Science and Mechanics at Virginia Polytechnic Institute and State University. He authored over 150 journal papers and six books on the theory and applications of the finite element method, refined theories of laminated composite plates and shells and their finite element analyses, computational fluid mechanics, numerical heat transfer, and applied mathematics. Dr. Reddy serves on the editorial boards of a dozen journals, including: *Journal of Applied Mechanics*, *International Journal for Numerical Methods in Engineering*, and *Journal of Composites Technology and Research*. He received the *1983 Walter L. Huber Research Prize* from the American Society of Civil Engineers and *1992 Worcester Reed Warner Medal* of the American Society of Mechanical Engineers. He is a *Fellow* of the American Academy of Mechanics, the American Society of Mechanical Engineers, the American Society of Civil Engineers, and the Aeronautical Society of India.

Preface

Composite materials are increasingly used in aerospace, under water, and automotive structures. The application of composite materials to engineering components has spurred a major effort to analyze structural components made from them. Composite materials provide unique advantages over their metallic counterparts, but they also present complex and challenging problems to analysts and designers. To take advantage of the full potential of composite materials, structural analysts and designers must have accurate mathematical models and design methods at their disposal. The most common structural elements are plates and shells. An accurate modelling of stress fields and failures is of paramount importance in the design of such components.

The present monograph has the objective of introducing the mechanics concepts, structural theories, and finite element models of composite laminates. Detailed coverage of the basic mechanics of composite materials, theories of composite plates and shells, and the finite element method are avoided in the interest of providing a general background necessary for engineers to analyze composite structures.

The authors are very grateful to Professor G. M. L. Gladwell of the University of Waterloo for his technical as well as editorial comments and for suggesting improvements. It is a pleasure to acknowledge the help of Mr. Y. S. N. Reddy (Virginia Polytechnic Institute and State University) in reading the manuscript and offering useful suggestions during the preparation of the manuscript. The authors also wish to thank Dr. Nigel Hollingworth (Kluwer) for the encouragement and support in publishing the monograph. The authors are very thankful to Mrs. Vanessa McCoy of the Department of Engineering Science and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, for typing of the manuscript. Without the patience and cooperation of Mrs. McCoy, it would not have been possible to publish this monograph.

O. O. Ochoa and *J. N. Reddy*

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Chapter One

Overview

1.1 Introduction

The phrase 'composite material' refers to a material that is formed by combining two or more materials on a macroscopic scale. Structures made of such materials are called composite structures. Composite materials are fabricated to have better engineering properties than the conventional materials, for example, metals. Some of the properties that can be improved by forming a composite material are: stiffness, strength, weight, corrosion resistance, thermal properties, fatigue life and wear resistance. Most man-made composite materials are made from two materials: a reinforcement material and a parent or matrix material. Composite materials are finding applications in a variety of systems; including aircraft and submarine structures, space structures, automobiles, sports equipment, medical prosthetic devices, and electronic circuit boards. They are most suitable in applications that require high strength-to-weight and stiffness-to-weight ratios. With the increased use of fiber-reinforced composites in structural components, studies involving the behavior of components made of composites are receiving considerable attention. Functional requirements and economic considerations of design are forcing engineers to seek reliable and accurate yet economical methods of determining static and dynamic characteristics of the structural components.

The analytical study and design of composite materials requires knowledge of anisotropic elasticity, structural theories and failure/damage criteria. Unlike isotropic materials, anisotropic materials exhibit complicated mechanical behavior. For example, consider rectangular blocks of isotropic and anisotropic (monoclinic) materials. When an isotropic block is subjected to pure shear stress, it develops only shear strain and no normal strains. Similarly, if an isotropic block is subjected to normal stress, it develops only normal strain and no shear strains. Under identical loads, a block made of an anisotropic material deforms differently from an isotropic block , as shown in Fig. 1.1–1. When a shear stress is applied to an anisotropic body, it develops normal strains in addition to shear strain; a normal stress similarly produces shear strain as well as normal strain.

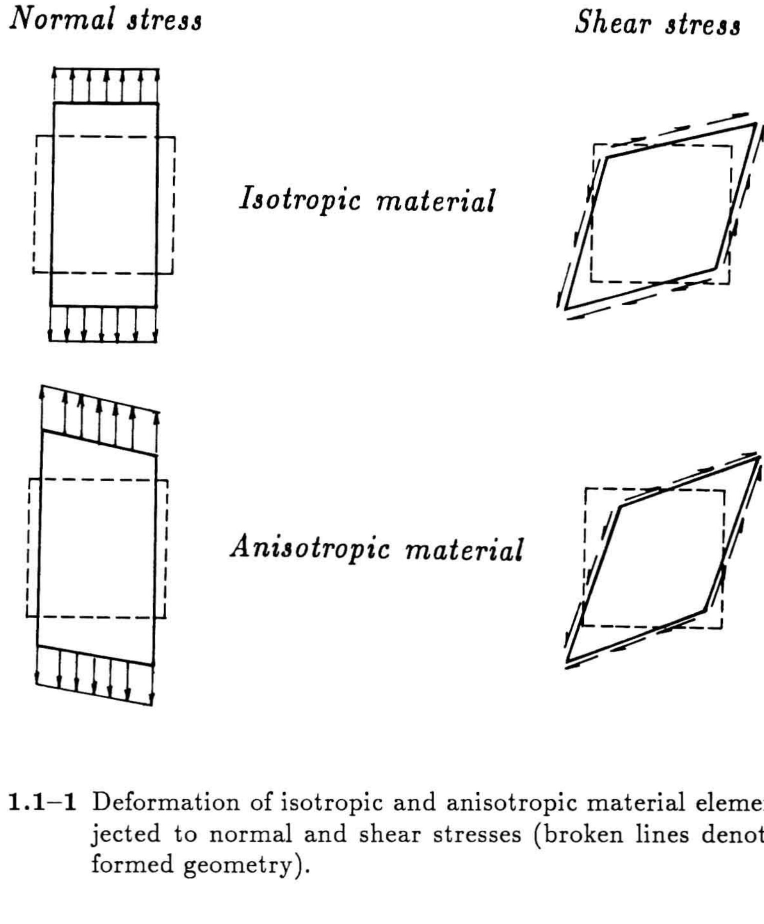

Figure 1.1–1 Deformation of isotropic and anisotropic material elements subjected to normal and shear stresses (broken lines denote undeformed geometry).

Most real world problems involving composite structures do not admit exact solutions, requiring one to find approximate, but representative solutions. The finite element method is an effective approximate method of obtaining numerical solutions to boundary-value, initial-value and eigen-value problems. The method is the most powerful numerical tool available today for predicting the response of composite structures. In the formulation and analysis of any mathematical model of a physical process, we should include all appropriate details consistent with the objective of the study. For example, representation of true loads and boundary conditions can be achieved to any desired closeness in a numerical model. However, the representation of material properties [physical constants] is inevitably an approximation which demands careful physical measurements. Gross approximation of some parameters and accurate representation of others does not lead to realistic modeling of the overall problem. One must accurately represent the kinematics of deformation as well as the material behavior.

1.2 Present study

The objective of this monograph is to present the laminated plate theories and their finite element models to study the deformation, strength and failure of composite structures. Emphasis is placed on engineering aspects, such as the analytical descriptions, effective analysis tools, modeling of physical features, and evaluation of approaches used to formulate and predict the response of composite structures.

We share our experiences in terms of guidelines and recommendations throughout the book, while focussing on fundamental aspects of finite element modeling of composite laminates. One may consult the list of references provided at the end of each chapter to gain further insight into mechanics and materials aspects and for advanced modeling of composites structures (see [1-8] and references therein).

Chapter 2 is devoted to the introduction of the definitions and terminology used in composite materials and structures. Anisotropic constitutive relations and laminate plate theories are also reviewed. Finite element models of laminated composite plates are presented in Chapter 3. Numerical evaluation of element coefficient matrices, postcomputation of strains and stresses, and sample examples of laminated plates in bending and vibration are discussed. Chapter 4 introduces damage and failure criteria in composite laminates. Finally, Chapter 5 is dedicated to case studies involving various aspects and types of composite structures. Joints, cutouts, woven composites, environmental effects, postbuckling response and failure of composite laminates are discussed by considering specific examples.

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Chapter Two
Mechanics of Composite Laminates

2.1 Introduction

Analysis of structures made of composite materials requires a knowledge of anisotropic elasticity, an appropriate structural theory that accounts for desired kinematics, failure criteria to determine if the structure has failed, and a numerical method to solve the boundary-value problem associated with the structure. The study of anisotropic elasticity and structural theories used to analyze composite laminates constitute the topics for this chapter.

The traditional undergraduate engineering education that emphasizes isotropic materials, e.g., metals, is neither adequate nor appropriate for analyzing this new class of engineered material systems. If the component is a heterogeneous anisotropic medium, we must have a proper structural theory to model the stiffness correctly, predict the stress fields, and determine the initiation and growth mechanisms of different failures. Drawing analogies between isotropic and anisotropic media, with the hope of using existing knowledge of metals and making deductions from them, can lead to incorrect results and conclusions.

In the present study, we shall focus on the fundamentals of anisotropic elasticity and a study of the mechanics of composite laminates. We will review basic assumptions, study constitutive relations of composite materials, and derive structural theories of composite laminates.

2.2 Anisotropic elasticity

2.2–1 Definitions

Since we seek a fundamental understanding of the mechanics of composites materials, we will start with the study of their constitutive behavior. First, we identify the differences between homogeneous and heterogeneous material systems.

A material is said to be *homogeneous* if the material properties remain unchanged throughout. In a heterogeneous system, the material properties are a function of position. Next, we distinguish between isotropic and anisotropic materials. A material is said to be *isotropic* if all its material properties at a point are independent of the direction. That is, if one refers to point A in an isotropic medium of Figure 2.2–1 with a coordinate system with its origin at A, the physical properties at A will remain invariant for any arbitrary rotation of axes. An *anisotropic* material is one which exhibits material properties that are directionally dependent, i.e., a given material property can have different values in different directions. We shall discuss various special cases of anisotropic materials in the next section. Note that a material can be isotropic and homogeneous, isotropic and heterogeneous (or inhomogeneous), anisotropic and homogeneous, or anisotropic and heterogeneous.

2.2–2 Constitutive relations

In a three-dimensional Cartesian coordinate system (see Figure 2.2–2), it is customary to describe the state of deformation by six components of strain and stress, namely, three normal and three shear components. A linear relation between the six stresses and six strains is known as the generalized Hooke's law, and it can be expressed as,

$$\sigma_k = C_{kj}\epsilon_j \quad (k = 1, 2, ..., 6) \tag{2.2 - 1}$$

where C_{kj} are known as the *elastic coefficients*. Note that (2.2–1) is an abbreviation of the proper tensor form of Hooke's law, namely (see [1-5]),

$$\sigma_{ij} = c_{ijk\ell}\ \epsilon_{k\ell}$$

When C_{kj} are functions of position the material is heterogeneous, and when they are constant throughout the material it is homogeneous. We note that C_{kj} are entries in the k-th row and j-th column of a 6×6 square matrix; however, C_{kj} are *not* components of a second-order tensor. Also note that the single subscript notation for stress and strain components is based on the convention,

$$\begin{aligned}
&\sigma_1 = \sigma_{11},\ \sigma_2 = \sigma_{22},\ \sigma_3 = \sigma_{33},\ \sigma_4 = \sigma_{23},\ \sigma_5 = \sigma_{13},\ \sigma_6 = \sigma_{12} \\
&\epsilon_1 = \epsilon_{11},\ \epsilon_2 = \epsilon_{22},\ \epsilon_3 = \epsilon_{33},\ \epsilon_4 = 2\epsilon_{23},\ \epsilon_5 = 2\epsilon_{13},\ \epsilon_6 = 2\epsilon_{12}
\end{aligned} \tag{2.2 - 2}$$

Here $(\sigma_1, \sigma_2, \sigma_3)$ denote the normal stresses and $(\sigma_4, \sigma_5, \sigma_6)$ denote shear stresses; $(\sigma_1, \sigma_2, \sigma_6)$ are the inplane stresses and $(\sigma_3, \sigma_4, \sigma_5)$ are the out–of–plane (i.e., transverse) stresses. Similar terminology is used for the strain components.