Worked examples in

Electrical and Electronic Measurement

D. F. A. EDWARDS

WORKED EXAMPLES IN ELECTRICAL AND ELECTRONIC MEASUREMENT

WORKED EXAMPLES IN ELECTRICAL AND ELECTRONIC MEASUREMENT

D. F. A. EDWARDS

HUTCHINSON EDUCATIONAL

HUTCHINSON EDUCATIONAL LTD
178–202 Great Portland Street, London W1

London Melbourne Sydney
Auckland Johannesburg Cape Town
and agencies throughout the world

First published 1970

This book has been set in Bookman by E. W. C. Wilkins and Associates Ltd., printed in Great Britain by litho by Anchor Press, and bound by Wm. Brendon, both of Tiptree, Essex

ISBN 0 09 105380 3 (cased)
0 09 105081 1 (paper)

CONTENTS

PREFACE

The object of this book is to show the student at H.N.C. and H.N.D. level how to solve numerical problems in the sphere of electrical and electronic measurements and to provide sufficient descriptive material to give him a working grasp of instrument and equipment construction and application. In this way he will become confident in not only solving the calculation parts of examination questions but also in the very necessary ability to give a good description of the test procedure which is also required in the professional examination. While measurements is now excluded as a special topic from professional engineering examinations there is every indication that this trend will shortly be reversed. The subject is still included in B.Sc. examinations and the questions included in this book should be of value to students taking University of London B.Sc. Parts I and II. It has been found that students are going into industry with no idea of fundamental measurements technique, and it is hoped that this book will help to overcome this state of affairs.

GENERAL PRINCIPLES

1. *Electrons are accelerated through a* p.d. V_0 *to a velocity* v. *They then form two long thin and parallel cylindrical beams at a distance x apart. A current I is carried by each beam. Derive an expression for the total force per unit length between the beams. If the accelerating voltage* $V = 20\,000$ *volt,* $x = 1$ cm *and* $I = 20$ mA, *calculate the force acting on each beam per metre length and indicate the direction of the force.*

(*I.E.E.*)

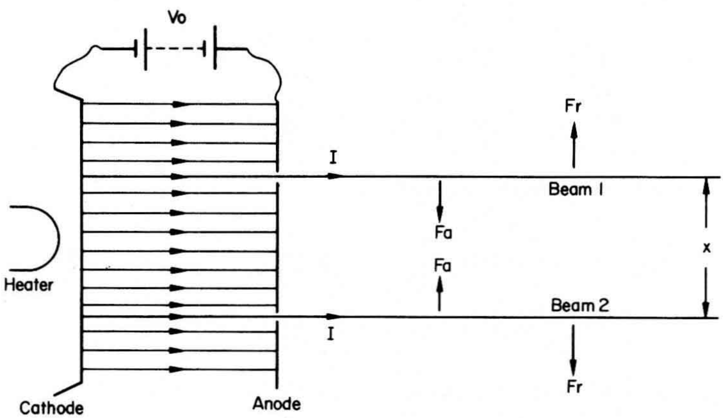

Fig. 1. Forces on parallel electron beams

The situation is as shown in Fig. 1. Each beam may be considered as a current-carrying conductor of length l metre.

Field at beam 2 due to beam 1 is

$$H_1 = \frac{I_1}{2\pi x} AT/m$$

Flux density B_1 at beam 2 due to beam 1

$$= \mu H_1 = \frac{\mu I_1}{2\pi x} \text{ tesla}$$

Force on beam 2

$$= B_1 I_2 l = \frac{\mu I_1 I_2 l}{2\pi x} \text{ newton } = \frac{\mu I_1 I_2}{2\pi x} \text{ newton per metre length.}$$

In the case under consideration, $I_1 = I_2$ and $\mu = \mu_0$, thus the attractive force F_a on one beam due to the other is given by

$$F_a = \frac{\mu_0}{2\pi}\frac{I^2}{x} \text{ newton.}$$

Now let the beams have charges $+q$ and $-q$ coulombs per metre.
Capacitance C between beams

$$= \frac{\pi\epsilon}{\log_e\left(\frac{x-a}{a}\right)} \text{ farad per metre.}$$

Energy stored per metre

$$J = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{q^2}{\pi\epsilon}\log_e\left(\frac{x-a}{a}\right) \text{ joule per metre.}$$

Now force F_r between beams

$$= \frac{-dJ}{dx} = -\frac{1}{2}\frac{q^2}{\pi\epsilon}\frac{a}{x-a}\frac{1}{a} = \frac{q^2}{2\pi\epsilon(x-a)} \text{ newton per metre,}$$

$$= \frac{q^2}{2\pi\epsilon x} \text{ if } x \gg a \text{ (as in the present case).}$$

Thus coulomb repulsive force F_r on each beam is given by

$$F_r = \frac{q^2}{2\pi\epsilon x} \text{ where } \epsilon = \epsilon_0 = \frac{1}{4\pi \times 9 \times 10^9} \text{ F/m.}$$

Substituting given values,

$$F_a = \frac{4\pi \times 10^{-7} \times 400 \times 10^{-6}}{2\pi \times 10^{-2}} = 800 \times 10^{-11} = 0{\cdot}008\,\mu\text{N/m.}$$

Now $qv = I$ where

$$v = \sqrt{2V_0\frac{e}{m}} = \sqrt{2 \times 20\,000 \times 1{\cdot}76 \times 10^{11}} = 10^7\sqrt{70{\cdot}4}$$

Thus

$$q = I/v = 20 \times 10^{-3}/10^7\sqrt{70{\cdot}4}$$

Therefore

$$F_r = \frac{\dfrac{400 \times 10^{-6}}{70{\cdot}4 \times 10^{14}}}{\dfrac{1}{2 \times 9 \times 10^{9}} \times 10^{-2}} = 0{\cdot}102\,\mu\text{N/m}$$

Thus force acting on each beam per metre length is $0{\cdot}102 - 0{\cdot}008$
$= \underline{0{\cdot}094\,\mu\text{N (repulsive)}}$

Note: Although the above question is not directly concerned with measurements it is an instructive one since it entails a knowledge of

(a) The attractive force between parallel current-carrying conductors (used in the definition of the ampere)

(b) The coulomb repulsive force between two charged bodies (used as the principle of some types of electrometers)

(c) The current in an electron beam (used in cathode-ray tube theory).

2. *Distinguish between 'error of method' and 'variable error' as applied to electrical measurements. Give a simple example of an error of method.*

In any measurement, there may be one or more factors which have the effect of making the quantity on test appear greater or less than it really is. Repeating the test any number of times yields exactly the same result. If the quantity causing the error could be estimated it could easily be allowed for; an addition to or subtraction from the apparent value resulting from the measurement would effect the necessary correction.

If such an error is suspected, the measurement should be repeated by another method, and even with different apparatus. If the new method gives a result which is different from that obtained by the first method, clearly there is an error of method that should be located and allowed for.

When a number of readings of the same quantity, taken by the same method and the same apparatus, do not exactly agree with one another the discrepancy is said to be due to a variable error. This kind of error frequently arises out of the human factor; readings such as the 'kick' of a galvanometer when the switch is closed cannot be made with absolute certainty by even the most skilled experimenter. A number of readings are taken and the mean of them used as the true value.

One simple example of error of method arises in the measurement of small resistances. The connecting leads and switch contacts have resistances which, being measured along with that which is under test, can give quite a serious error, unless special methods are used.

If resistances are measured by connecting a milliammeter in series and a voltmeter across both milliammeter and unknown resistance, the relation between the meter readings, $R = E/I$, gives the resistance of the unknown plus that of the milliammeter. If the latter is not allowed for, it will give rise to an error of method.

3. *Suggest a possible cause of variable error in a voltmeter. How would you attempt to correct for this error when using the instrument?*

Variable error is due to something that causes the pointer not always to come to rest in the same position for the same current through the instrument. The most likely cause is excessive friction at the pivots. Friction in a voltmeter has the same effect as friction on any moving body, namely, to bring it to rest before it would otherwise do so. Hence, if the instrument were connected to a circuit the pointer, coming to rest too soon, would read low. If, however, a higher voltage were applied and then reduced to normal value the pointer, again coming to rest too soon, would read high.

The only way to correct for this type of error is to take a number of readings and find the mean. If friction is the cause of the trouble readings should, if possible, be taken with the pointer rising from zero and with it falling from a higher value than that being tested. The mean of the upward and downward readings will be approximately correct.

Another possible cause of variable error is a poor contact, which gives rise to a constantly changing resistance. In this case it would be useless to take the mean of a set of readings for the highest – when the contact resistance is zero – is the correct one. If there is no way of avoiding the use of such an instrument in that condition the only course would be to tap it and note the highest reading obtainable. Normally, the time taken to find the bad contact and get it soldered would be more than repaid.

4. *A milliammeter has to be calibrated. The following four readings, for the actual current when the meter read* 2 mA, *are taken by potentiometer.*

1	2	3	4
1·96	2·04	1·98	1·99

Within what current limits will the meter read correctly and within what percentage limits would you guarantee a correct reading?

To find the actual current as nearly as possible take the mean of the four readings.

Mean value of current

$$= \frac{1{\cdot}96 + 2{\cdot}04 + 1{\cdot}98 + 1{\cdot}99}{4} = \frac{7{\cdot}97}{4} = 1{\cdot}993 \text{ mA.}$$

The meter, then, reads 0·007 too high. Now the actual values of current range between 1·96 and 2·04. Thus there is a maximum variation of ± 0·04 mA. As this variation is greater than the correction factor the meter can be declared as reading correctly within the limits of ± 0·04 mA. The meter is correct to within an uncertainty of 2·0%.

5. *The labels are worn off a number of resistors intended for anode decoupling in radio receivers. You have available a Post Office box type Wheatstone bridge with delicate galvanometer, a* 0–200 *voltmeter, a* 0–5 *milliammeter and a* d.c. *supply of up to* 100 *volt. State what method you would use to test the resistance values and give your reasons.*

Decoupling resistors may have values between say 5 000 and 50 000 ohm. As the resistance values for this purpose in a particular circuit are never very critical, it would be quite satisfactory to measure to within 10%. If a resistor changes its value it usually does so by much more than 10%, probably by several hundred percent, so that faulty components would be made evident by even a poorer accuracy.

The Wheatstone bridge would give results to within 1% at least but the measurements would take a long time to carry out. Apart from the time required to use this method it is bad practice to use delicate instruments for rough work, as this exposes them to unnecessary wear and risk of damage. If a test has to be done quickly there is always a tendency to be careless and so to damage a delicate galvanometer.

The ammeter-voltmeter method should be used as it will give the required accuracy and can be quickly carried out. The apparatus should be connected up as shown in Fig. 2.

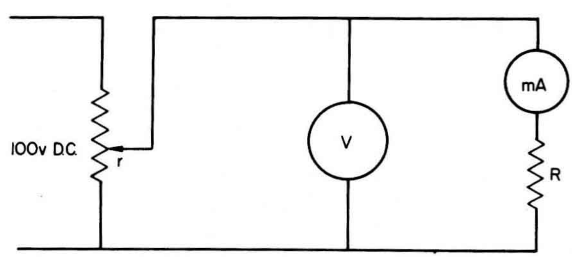

Fig. 2. Ammeter-voltmeter method of measuring resistance

R is the resistance to be measured, in series with the milliammeter. The voltmeter should be connected as shown and not directly across the unknown resistance because its own resistance is probably low enough to introduce an appreciable error, whereas the milliammeter resistance, being very small compared with R, would not affect the result in the circuit shown. A resistance r is shown connected across the supply and tapped to enable a convenient voltage to be obtained; to get 5 mA through a 5 000 ohm resistance would require only 25 volt (i.e. for maximum reading on the milliammeter) whereas in the case of a 50 000 ohm resistance 100 volt would only give 2 mA. If the d.c. supply were an h.t. battery r would be omitted and the battery tappings used. The value of R is given by $R = E/I$ where E and I are the voltmeter and milliammeter reading respectively.

6. *The resistances of a Wheatstone bridge are correct to within* $\pm 0{\cdot}1\%$. *To what accuracy can an unknown resistance be measured?*

The condition of balance in a Wheatstone bridge is $X = RP/Q$ where X is the unknown resistance, R the standard and P and Q the values of the ratio arms (or lengths between the ends and the moving contact, in the case of a metre bridge).

The question states that R, P and Q are known to within $\pm 0{\cdot}1\%$. The greatest error in X arises when P and R are high by $0{\cdot}1\%$ and Q is low by that amount. Then, if δX is the error in X,

$$X + \delta X = \frac{P(1{\cdot}001)}{Q(0{\cdot}999)} R(1{\cdot}001)$$

Hence

$$\delta X = \frac{P}{Q} R\left(\frac{1{\cdot}001^2}{0{\cdot}999} - 1\right)$$

Therefore the percentage error in X is

$$\frac{\delta X}{X} \times 100 = \frac{1{\cdot}001^2 - 0{\cdot}999}{0{\cdot}999} \times 100 = \underline{0{\cdot}3\% \text{ approximately}}$$

7. *A simple slide wire potentiometer is used to compare two* e.m.f.'s. *If it is possible to read the position of the sliding contact to within* 0·5 mm, *what is the highest accuracy obtainable for the ratio of the* e.m.f.'s? *How does the accuracy depend upon the length of potentiometer wire being actually used? (Assume the* e.m.f.'s *are nearly equal and that the potentiometer has a wire one metre long).*

By the principle of the potentiometer, the ratio of the e.m.f.'s being compared is equal to the ratio of the length of wire between the common connection and the sliding contact required to obtain a balance.

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

Now if l_1 has a positive error and l_2 a negative, the error in the ratio is given by

$$\frac{l_1 + 0{\cdot}5}{l_2 - 0{\cdot}5} - \frac{l_1}{l_2} = \frac{l_1(l_2 + 0{\cdot}5) - l_2(l_1 - 0{\cdot}5)}{l_2(l_2 - 0{\cdot}5)} = \frac{0{\cdot}5(l_2 + l_1)}{l_2(l_2 - 0{\cdot}5)}$$

where l_1 and l_2 are measured in millimetres.

If l_2 is large compared with 0·5 mm this may be written

$$\frac{0{\cdot}5(l_2 + l_1)}{l_2{}^2}$$

Putting $l_2 + l_1 = 2l_2$, then this becomes

$$\frac{0{\cdot}5 \times 2l_2}{l_2{}^2}$$

$$\therefore \quad \text{error} = \frac{1}{l_2}$$

Hence it is seen that the larger l_2, that is the longer the potentiometer wire, the less the error – the error is inversely proportional to the length of wire and the error in this case is $1/l_2$ and for $l_2 = 1\,000$ mm

<u>Error = 1 in 1 000</u>

8. *What are the chief differences between deflectional and null methods of measurement? Give the advantages and disadvantages of each.*

In deflectional methods the value of the quantity being measured is indicated directly on a scale by a moving pointer. The instrument has to be calibrated by the manufacturers and it is assumed that this calibration remains constant. Wear, friction of pivots, change of temperature, loss of magnetism by permanent magnets are all possible causes of the calibration changing. A pointer position cannot be read with very great accuracy, as one division is usually not less than one millimetre.

In null methods the circuit is balanced so that no current flows in a certain branch: this is observed by the absence of deflection on a delicate indicator. The measurement is then obtained from the settings of the various circuit constants. The bridge has to be balanced and the result calculated, so that the measurement takes longer than merely reading a pointer. It is possible, however, to obtain much more accurate results, provided the operator is sufficiently skilled. Resistances can be obtained guaranteed to a higher degree of accuracy than any deflectional meter, moreover it is easier to observe the absence of a deflection than accurately to make a pointer reading.

DIRECT-CURRENT INSTRUMENTS

9. *Describe, with the aid of diagrams, a type of moving-coil galvanometer with which you are familiar. Mention any means provided for altering the sensitivity to suit particular requirements. Indicate how the movement can be critically damped in the type you select.*

Either the iron-cored type or the coreless type may be described. The movement of an iron-cored galvanometer is shown in Fig. 3. The

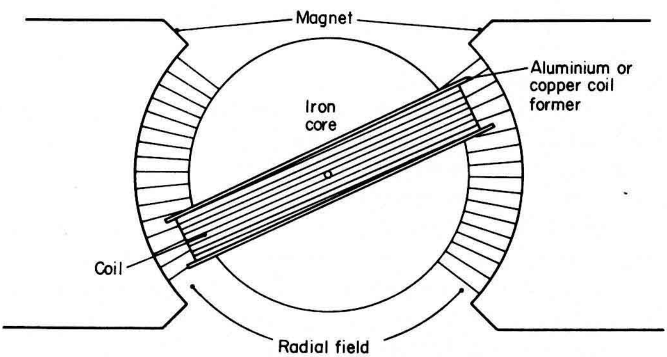

Fig. 3. Moving-coil Galvanometer Movement

permanent magnet provides a strong field, which is made radial by the soft iron core. The advantage of a radial field is that the turning moment exerted on the coil by the current is independent of the angle of deflection; if the field is not radial the number of lines of force threading the coil will vary according to coil position, and hence the couple acting will vary. With a radial field, then, the restoring couple alone governs the 'law' of the instrument, and, since this is of the nature of a spring, the deflection is directly proportional to the current. Phosphor–bronze strip is used as the suspension for the coil using a small mirror to deflect a beam of light on to the scale.

Figure 4 shows a coreless type of galvanometer: this is the Type 3038, made by H. Tinsley & Company Ltd. using the Ayrton and

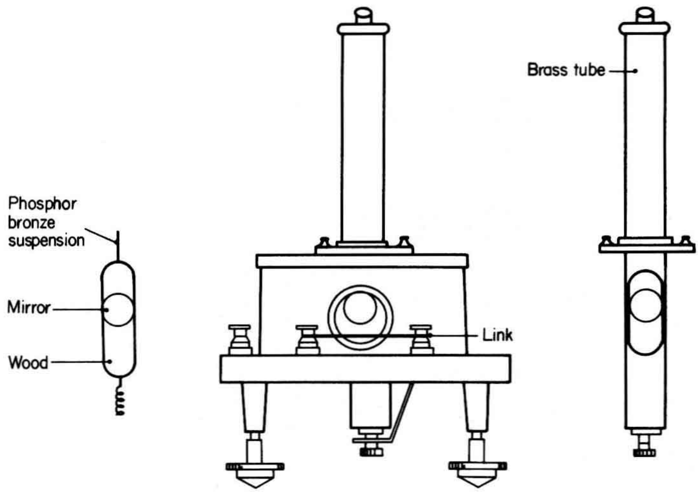

Fig. 4. Tinsley Type 3038 Galvanometer

Mather method of construction.

In instruments of this type the coil, complete with suspension, can be removed and replaced by another having a resistance suitable for the particular purpose for which the instrument is to be used. A magnetic shunt is also sometimes provided. These, as well as the use of an external shunt, are possible ways of altering the sensitivity.

Damping can be obtained solely by the use of suitable external resistances, such that the induced e.m.f. in the moving coil due to the motion of the coil in the magnetic field produces a circulating current of such a value that the movement is critically damped. In the iron-cored type the coil can be wound on an aluminium or copper frame which, constituting a shorted turn, will provide critical damping when the instrument is used in circuits of relatively high resistance. In the coreless type a special coil, in parallel with the moving operating coil, can be used, it being shorted by means of a special link.

10. *How are the temperature errors of a panel mounting type direct current ammeter compensated for?*

In a particular case the potential difference between the potential terminals of the shunt of a 1000 A *instrument is* 0·03 *volt. The connecting leads to the instrument are of copper and have a resistance of* 0·12 *ohm, the moving coil has a resistance of* 1·20 *ohm and requires a current of* 15 mA *to deflect it to the* 1000 A *point on the scale. The temperature coefficient of the alloy used for the shunt is* 0·00001, *and that of copper* 0·004 *per degree* C – *in terms of the resistance at* 15°C. *To what*