ANNOTATED INSTRUCTOR'S EDITION

ALGEBRA FOR COLLEGE STUDENTS

THIRD EDITION

MARGARET L. LIAL
E. JOHN HORNSBY, JR.
CHARLES D. MILLER

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HarperCollinsCollegePublishers

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Compositor: Interactive Composition Corporation
Printer and Binder: R.R. Donnelley & Sons Company
Cover Printer: Phoenix Color Corporation

Algebra for College Students, Third Edition

Library of Congress Cataloging-in-Publication Data
Lial, Margaret L.
Algebra for college students / Margaret L. Lial, E. John Hornsby, Jr., Charles D. Miller. -- 3rd ed.
p. cm.
Includes index.
ISBN 0-673-99061-3 (Student Edition)
ISBN 0-673-99546-1 (Annotated Instructor's Editon)
1. Algebra. I. Hornsby, E. John. II. Miller, Charles David. III. Title.
QA154.2.L495 1996

512.9--dc20 95–8117
CIP

95 96 97 98 9 8 7 6 5 4 3 2 1

PREFACE

This third edition of *Algebra for College Students* is designed to give a thorough treatment of those topics in algebra necessary for success in later courses. Although we assume that most students using this book will have had a previous course in algebra, all necessary ideas are introduced or reviewed as needed.

The text retains the successful features of previous editions: learning objectives for each section; careful exposition; fully developed examples; cautions and notes; and boxes that set off important definitions, formulas, rules, and procedures. In this new edition, we have made several content changes to follow the guidelines set forth in the *Curriculum and Evaluation Standards for School Mathematics,* published by the National Council of Teachers of Mathematics.

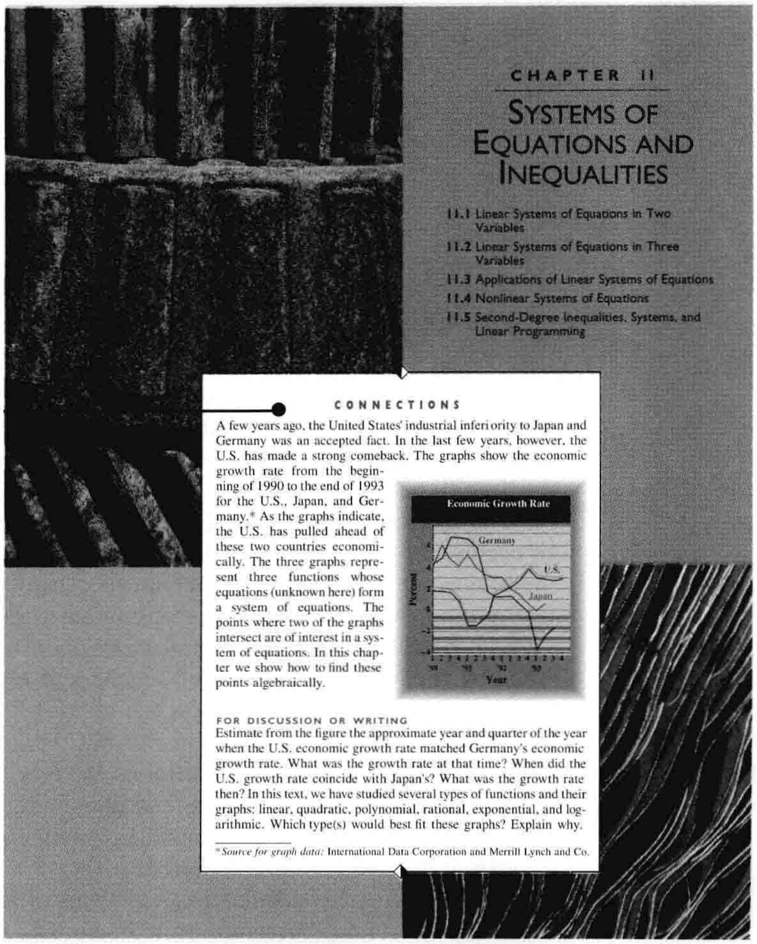
CHANGES IN CONTENT

- More than 80 percent of the exercises are new to this edition and many of these exercises now incorporate real data and graphics.
- Beginning with Chapter 2, cumulative reviews appear after each chapter, covering material learned up to that point.
- In this text we view calculators as a means of allowing students to spend more time on the conceptual nature of mathematics and less time on the mechanics of computation with paper and pencil. We have included an introduction to graphics calculators, and the use of both the scientific and graphics calculator is discussed throughout the book wherever appropriate. Some sections include exercises that require a calculator.
- Graphics calculator text and exercises are included as appropriate throughout the book. This material is designed so that it can easily be incorporated into the course, treated separately, or omitted, as the instructor chooses.

FEATURES

The following pages illustrate important features. These features are designed to assist students in the learning process and deepen their understanding of the underlying principles and interrelations between topics.

Connections boxes that provide connections to the real world or to other mathematical concepts or other disciplines open each chapter and appear throughout the text—almost one for every section. Most of these include thought-provoking questions for writing or class discussion. The Connections provide motivation for the topic under discussion, show how mathematics is used in many different aspects of life, give some historical background, and provide a larger context for the current material.



CHAPTER 11

SYSTEMS OF EQUATIONS AND INEQUALITIES

11.1 Linear Systems of Equations in Two Variables
11.2 Linear Systems of Equations in Three Variables
11.3 Applications of Linear Systems of Equations
11.4 Nonlinear Systems of Equations
11.5 Second-Degree Inequalities, Systems, and Linear Programming

CONNECTIONS

A few years ago, the United States' industrial inferi ority to Japan and Germany was an accepted fact. In the last few years, however, the U.S. has made a strong comeback. The graphs show the economic growth rate from the beginning of 1990 to the end of 1993 for the U.S., Japan, and Germany.* As the graphs indicate, the U.S. has pulled ahead of these two countries economically. The three graphs represent three functions whose equations (unknown here) form a system of equations. The points where two of the graphs intersect are of interest in a system of equations. In this chapter we show how to find these points algebraically.

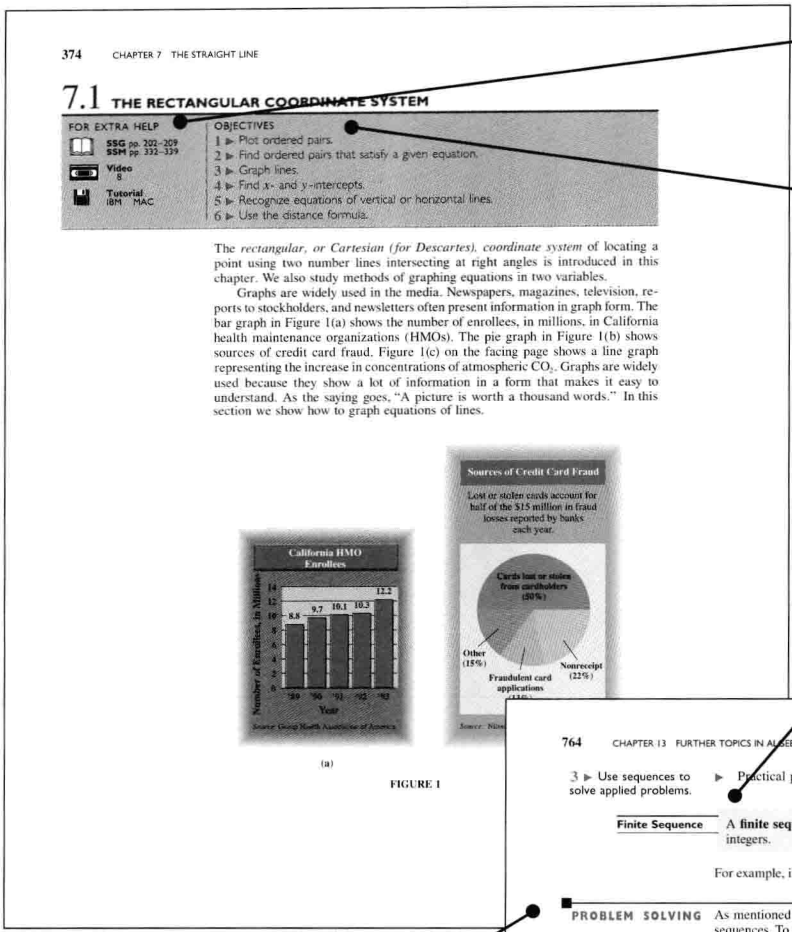
FOR DISCUSSION OR WRITING

Estimate from the figure the approximate year and quarter of the year when the U.S. economic growth rate matched Germany's economic growth rate. What was the growth rate at that time? When did the U.S. growth rate coincide with Japan's? What was the growth rate then? In this text, we have studied several types of functions and their graphs: linear, quadratic, polynomial, rational, exponential, and logarithmic. Which type(s) would best fit these graphs? Explain why.

*Source for graph data: International Data Corporation and Merrill Lynch and Co.

Student Resources provide cross-references to relevant student supplements in each section of the text and for the chapter review material.

Learning Objectives, found at the beginning of each section, offer students an excellent study aid by providing an overview of the section content that follows.


374 CHAPTER 7 THE STRAIGHT LINE

7.1 THE RECTANGULAR COORDINATE SYSTEM

FOR EXTRA HELP
SSG pp. 202–209
SSM pp. 332–339
Video 8
Tutorial IBM MAC

OBJECTIVES
1 ▶ Plot ordered pairs.
2 ▶ Find ordered pairs that satisfy a given equation.
3 ▶ Graph lines.
4 ▶ Find x- and y-intercepts.
5 ▶ Recognize equations of vertical or horizontal lines.
6 ▶ Use the distance formula.

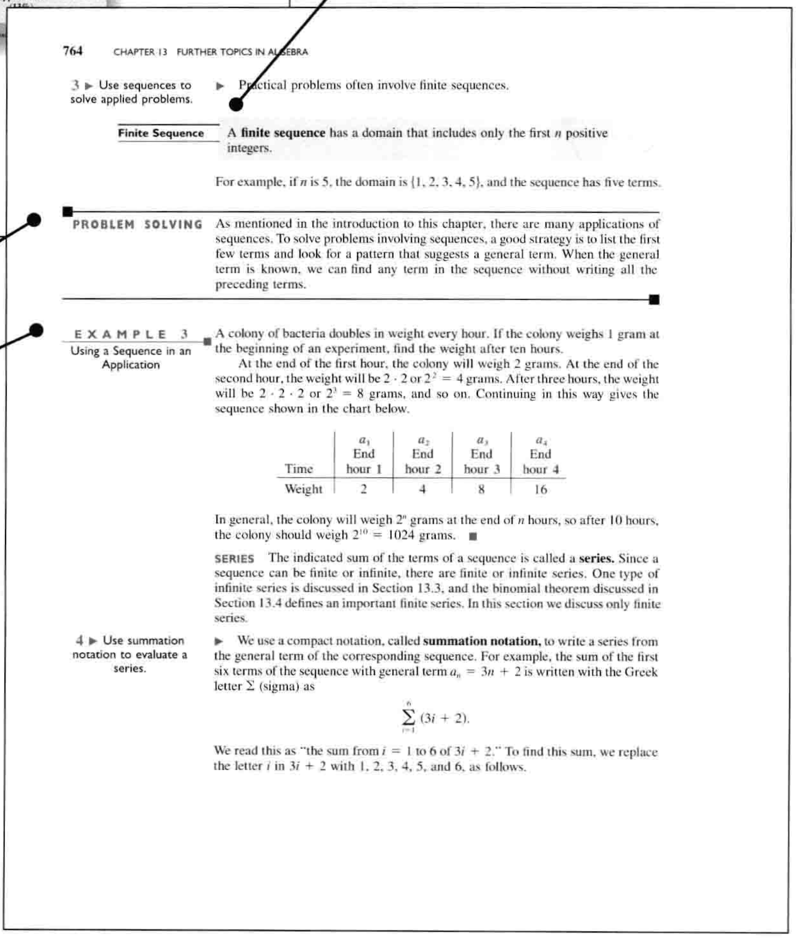
The *rectangular, or Cartesian (for Descartes), coordinate system* of locating a point using two number lines intersecting at right angles is introduced in this chapter. We also study methods of graphing equations in two variables.

Graphs are widely used in the media. Newspapers, magazines, television, reports to stockholders, and newsletters often present information in graph form. The bar graph in Figure 1(a) shows the number of enrollees, in millions, in California health maintenance organizations (HMOs). The pie graph in Figure 1(b) shows sources of credit card fraud. Figure 1(c) on the facing page shows a line graph representing the increase in concentrations of atmospheric CO_2. Graphs are widely used because they show a lot of information in a form that makes it easy to understand. As the saying goes, "A picture is worth a thousand words." In this section we show how to graph equations of lines.

(a)

FIGURE 1

Rules and Definitions highlight important words and procedures.


764 CHAPTER 13 FURTHER TOPICS IN ALGEBRA

3 ▶ Use sequences to solve applied problems.

▶ Practical problems often involve finite sequences.

Finite Sequence A **finite sequence** has a domain that includes only the first n positive integers.

For example, if n is 5, the domain is $\{1, 2, 3, 4, 5\}$, and the sequence has five terms.

PROBLEM SOLVING As mentioned in the introduction to this chapter, there are many applications of sequences. To solve problems involving sequences, a good strategy is to list the first few terms and look for a pattern that suggests a general term. When the general term is known, we can find any term in the sequence without writing all the preceding terms.

EXAMPLE 3 Using a Sequence in an Application

A colony of bacteria doubles in weight every hour. If the colony weighs 1 gram at the beginning of an experiment, find the weight after ten hours.

At the end of the first hour, the colony will weigh 2 grams. At the end of the second hour, the weight will be $2 \cdot 2$ or $2^2 = 4$ grams. After three hours, the weight will be $2 \cdot 2 \cdot 2$ or $2^3 = 8$ grams, and so on. Continuing in this way gives the sequence shown in the chart below.

Time	a_1 End hour 1	a_2 End hour 2	a_3 End hour 3	a_4 End hour 4
Weight	2	4	8	16

In general, the colony will weigh 2^n grams at the end of n hours, so after 10 hours, the colony should weigh $2^{10} = 1024$ grams. ■

SERIES The indicated sum of the terms of a sequence is called a **series.** Since a sequence can be finite or infinite, there are finite or infinite series. One type of infinite series is discussed in Section 13.3, and the binomial theorem discussed in Section 13.4 defines an important finite series. In this section we discuss only finite series.

4 ▶ Use summation notation to evaluate a series.

▶ We use a compact notation, called **summation notation,** to write a series from the general term of the corresponding sequence. For example, the sum of the first six terms of the sequence with general term $a_n = 3n + 2$ is written with the Greek letter Σ (sigma) as

$$\sum_{i=1}^{6} (3i + 2).$$

We read this as "the sum from $i = 1$ to 6 of $3i + 2$." To find this sum, we replace the letter i in $3i + 2$ with 1, 2, 3, 4, 5, and 6, as follows.

Problem Solving paragraphs include suggestions for successful problem-solving techniques.

Titled Examples include detailed, step-by-step solutions and descriptive side comments.

738 CHAPTER 12 MATRICES AND DETERMINANTS

Both numerators and the common denominator of these values for x and y can be written as determinants, since

$$c_1b_2 - c_2b_1 = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}, \quad a_1c_2 - a_2c_1 = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix},$$

$$\text{and} \quad a_1b_2 - a_2b_1 = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}.$$

Using these determinants, the solutions for x and y become

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} \quad \text{and} \quad y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}}, \quad \text{if} \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} \neq 0.$$

For convenience, we denote the three determinants in the solution as

$$\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = D, \quad \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix} = D_x, \quad \text{and} \quad \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} = D_y.$$

NOTE The elements of D are the four coefficients of the variables in the given system, the elements of D_x are obtained by replacing the coefficients of x in D by the respective constants, and the elements of D_y are obtained by replacing the coefficients of y in D by the respective constants.

Notes draw attention to important comments to help students solve problems and understand concepts.

These results are summarized as **Cramer's rule.**

Cramer's Rule for 2 × 2 Systems

Given the system

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2,$$

with $a_1b_2 - a_2b_1 \neq 0$,

then

$$x = \frac{D_x}{D} \quad \text{and} \quad y = \frac{D_y}{D},$$

where

$$D_x = \begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}, \quad D_y = \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}, \quad \text{and} \quad D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}.$$

2 ▶ Apply Cramer's rule to a linear system with two equations and two variables.

▶ Cramer's rule is used to solve a system of linear equations by evaluating the three determinants D, D_x, and D_y and then writing the appropriate quotients for x and y.

CAUTION As indicated above, Cramer's rule does not apply if $D = 0$. When $D = 0$, the system is inconsistent or has dependent equations. For this reason, it is a good idea to evaluate D first.

Cautions highlight common student errors and difficulties.

ve the problem. See Example 8.

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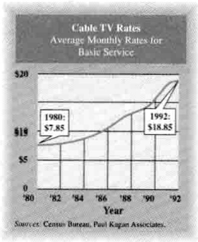
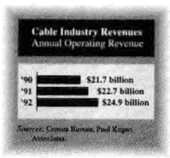
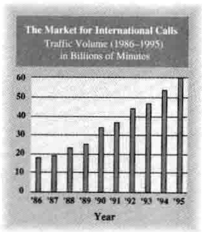


56. Assuming a linear relationship, what is the average rate of change for cable industry revenues over the period from 1990 to 1992?



57. The 1993 Annual Report of AT&T cites the following figures concerning the global growth of international telephone calls: From 47.5 billion minutes in 1993, traffic is expected to rise to 60 billion minutes in 1995. Assuming a linear relationship, what is the average rate of change for this time period? (*Source:* TeleGeography, 1993, Washington, D.C.)

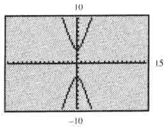
58. The market for international phone calls during the ten-year period from 1986 to 1995 is depicted in the accompanying bar graph. The tops of the bars approximate a straight line. Assuming that the traffic volume at the beginning of this period was 18 billion minutes and at the end was 60 billion minutes, what was the average rate of change for the ten-year period?



Graph Reading Activities A variety of graphs from both popular and professional media sources are used throughout the text to help students visualize mathematics and interpret real data.

514 CHAPTER 8 GRAPHING RELATIONS AND FUNCTIONS

34. Repeat Exercise 33 for the graph of $\frac{y^2}{9} - x^2 = 1$, shown in the accompanying figure.



Use a graphics calculator in function mode to graph the hyperbola. Use a square viewing window.

35. $\frac{x^2}{25} - \frac{y^2}{49} = 1$

36. $\frac{x^2}{4} - \frac{y^2}{16} = 1$

37. $y^2 - 9x^2 = 9$

38. $4y^2 - 36x^2 = 144$

MATHEMATICAL CONNECTIONS (Exercises 39–44)

From the discussion in this section, we know that the graph of $\frac{x^2}{4} - y^2 = 1$ *is a hyperbola. We know that the graph of this hyperbola approaches its asymptotes as x gets larger and larger. Work Exercises 39–44 in order to see the relationship between the hyperbola and one of its asymptotes.*

39. Solve $\frac{x^2}{4} - y^2 = 1$ for y, and choose the positive square root.

40. Find the equation of the asymptote with positive slope.

41. Use a calculator to evaluate the y-coordinate of the point where $x = 50$ on the graph of the portion of the hyperbola represented by the equation obtained in Exercise 39. Round your answer to the nearest hundredth.

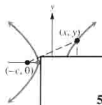
42. Find the y-coordinate of the point where $x = 50$ on the graph of the asymptote found in Exercise 40.

43. Compare your results in Exercises 41 and 42. How do they support the following statement? When $x = 50$, the graph of the function defined by the equation found in Exercise 39 lies *below* the graph of the asymptote found in Exercise 40.

44. What do you think will happen if we choose x-values larger than 50?

45. Suppose that a hyperbola has center at the origin, foci at $(-c, 0)$ and $(c, 0)$, and the absolute value of the difference between the distances from any point (x, y) of the hyperbola to the two foci is $2a$. See the figure. Let $b^2 = c^2 - a^2$, and show that an equation of the hyperbola is

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$



Mathematical Connections exercises tie together different topics and highlight the relationships among various concepts and skills.

Exercises are carefully paired (evens with odds) and are graded with regard to increasing difficulty. Exercises now include conceptual, writing, challenging, and calculator (both scientific and graphics calculators) types.

524 CHAPTER 8 GRAPHING RELATIONS AND FUNCTIONS

CHAPTER 8 SUMMARY

KEY TERMS

8.2 **quadratic function, parabola, axis of a parabola, vertex of a parabola, focus of a parabola, translation, directrix**

8.5 **circle, center, radius, ellipse, foci of an ellipse, center of an ellipse, vertices of an ellipse, major axis, minor axis**

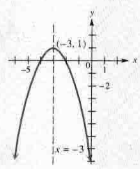
8.6 **hyperbola, asymptotes of a hyperbola, fundamental rectangle, conic sections, square root function**

8.7 **absolute value function, functions defined piecewise, greatest integer function, step function**

NEW SYMBOLS

$f \circ g$	composite function
$[\![x]\!]$	greatest integer function

QUICK REVIEW

CONCEPTS	EXAMPLES
8.1 COMBINING FUNCTIONS: THE ALGEBRA OF FUNCTIONS	
Operations on Functions $(f + g)(x) = f(x) + g(x)$ $(f - g)(x) = f(x) - g(x)$ $(fg)(x) = f(x) \cdot g(x)$ $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$	If $f(x) = 3x^2 + 2$ and $g(x) = \sqrt{x}$, then $(f + g)(x) = 3x^2 + 2 + \sqrt{x}$ $(f - g)(x) = 3x^2 + 2 - \sqrt{x}$ $(fg)(x) = (3x^2 + 2)(\sqrt{x})$ $\left(\frac{f}{g}\right)(x) = \frac{3x^2 + 2}{\sqrt{x}}, \quad x > 0.$
Composition of Functions If f and g are functions, then the composite function of g and f is $(g \circ f)(x) = g[f(x)].$	If $g(x) = \sqrt{x}$ and $f(x) = x^2 - 1$, then the composite function of g and f is $(g \circ f)(x) = \sqrt{x^2 - 1}.$
8.2 QUADRATIC FUNCTIONS; PARABOLAS	
1. The graph of the quadratic function $f(x) = a(x - h)^2 + k, \quad a \neq 0$ is a parabola with vertex at (h, k) and the vertical line $x = h$ as axis. 2. The graph opens upward if a is positive and downward if a is negative. 3. The graph is wider than $f(x) = x^2$ if $0 < \lvert a \rvert < 1$ and narrower if $\lvert a \rvert > 1$.	Graph $f(x) = -(x + 3)^2 + 1$. 

Chapter Summaries present students with a helpful, section-referenced overview of each chapter. The Chapter Summary is followed by the Chapter Review Exercises, the Chapter Test, and the Cumulative Review Exercises.

Connections

Connections boxes that provide connections to the real world or to other mathematical concepts or other disciplines open each chapter and appear throughout the text—almost one for every section. Most of these include thought-provoking questions for writing or class discussion. The Connections provide motivation for the topic under discussion, show how mathematics is used in many different aspects of life, give some historical background, and provide a larger context for the current material.

Calculator Coverage

Connections boxes that focus on scientific and graphics calculators address the growing interest in the use of calculator technology, as well as provide extrinsic motivation. These Connections boxes are included in addition to the calculator passages that are part of the regular text and are optional in nature. Each such Connections box is specially marked with a scientific or graphics calculator symbol. Corresponding exercises are included in the exercise sets.

Graph Reading Activities

A variety of graphs from both popular and professional media sources are used throughout the text to help students visualize mathematics and interpret real data.

EXERCISES

More than 80 percent of the exercises are new to this edition. Care has been taken to pair exercises (evens with odds) and to grade the exercises with regard to increasing difficulty. Exercises now include a number of special types:

Mathematical Connections exercises tie together different topics and highlight the relationships among various concepts and skills. These multiple-skill and multiple-concept exercises sharpen students' problem-solving techniques and improve students' critical thinking abilities. Mathematical Connections exercises are included in most exercise sets and are grouped under a special heading.

Conceptual and writing exercises are designed to require a deeper understanding of concepts. 270 of the more than 720 exercises require the student to respond by writing a few sentences. Answers are not given for the writing exercises because they are open-ended, and instructors may use them in different ways.

Challenging exercises require the student to go beyond the examples in the text.

Cumulative Reviews end each chapter. These begin with Chapter 2 and test the topics covered from the beginning of the text up to that point.

Calculator exercises are included in some sections and require the student to use a scientific calculator. Other sections include passages on the use of graphics calculators accompanied by appropriately designed exercises. These are grouped and identified with a special symbol so that they can be easily omitted if the instructor prefers.

Applications have been updated and rewritten to include interesting and realistic information. In many cases they use actual data from current events, sports, and other sources.

SUPPLEMENTS

Our extensive supplemental package includes an Annotated Instructor's Edition, testing materials, solutions, software, and videotapes.

For the Instructor

Annotated Instructor's Edition This edition provides instructors with immediate access to the answers to every exercise in the text, with the exception of writing exercises. Each answer is printed in color next to the corresponding text exercise.

Symbols are used to identify the conceptual ◉, writing ✍, and challenging ▲ exercises to assist in making homework assignments. Scientific ▦ and graphics ▦ calculator exercises are marked in both the student and instructor texts. Additional exercises, called Chalkboard Exercises, parallel almost every example, and Teaching Tips corresponding to the text discussions are also included in the Annotated Instructor's Edition.

Instructor's Test Manual The Instructor's Test Manual includes short-answer and multiple-choice versions of a placement test; six forms of chapter tests for each chapter, including four open response and two multiple-choice forms; short-answer and multiple-choice forms of a final examination; and an extensive set of additional exercises (including more "mixed" exercises) providing 10 to 20 extra exercises for each textbook objective that instructors may use as an additional source of questions for tests, quizzes, or student review of difficult topics. Finally, this manual also includes a list of all conceptual, writing, connection, challenging, and calculator exercises.

Instructor's Solution Manual This book includes detailed, worked-out solutions to each section exercise in the book, including conceptual *and* writing exercises. This manual also includes a list of all conceptual, writing, connection, challenging, and calculator exercises.

Instructor's Answer Manual This manual includes answers to all exercises and a list of conceptual, writing, connection, challenging, and calculator exercises.

HarperCollins Test Generator/Editor for Mathematics with QuizMaster Available in IBM (both DOS and Windows applications) and Macintosh versions, the Test Generator is fully networkable. The Test Generator enables instructors to select questions by objective, section, or chapter, or to use a ready-made test for each chapter. The Editor allows instructors to edit any preexisting data or to easily create their own questions. The software is algorithm driven, so the instructor may regenerate constants while maintaining problem type, providing a very large number of test or quiz items in multiple-choice and/or open response formats for one or more test forms. The system features printed graphics and accurate mathematics symbols. **QuizMaster** enables instructors to create tests and quizzes using the Test Generator/Editor and save them to disk so students can take the test or quiz on a stand-alone computer or network. **QuizMaster** then grades the test or quiz and allows the instructor to create reports on individual students or entire classes. CLAST and TASP versions of this package are also available for IBM and Mac machines.

For the Student

Student's Solution Manual This book contains solutions to every odd-numbered section exercise (including conceptual and writing exercises) as well as solutions to all Connections, chapter review exercises, chapter tests, and cumulative review exercises. (ISBN 0-673-99547-X)

Student's Study Guide This book provides additional practice problems and reinforcement for each learning objective in the book. Self-tests are included at the end of every chapter. (ISBN 0-673-99545-3)

Interactive Mathematics Tutorial Software with Management System This innovative package is also available in DOS, Windows, and Macintosh versions and is fully networkable. As with the Test Generator/Editor, this software is algorithm driven, which automatically regenerates constants so that the numbers rarely repeat in a problem type when students revisit any particular section. The tutorial is objective-based, self-paced, and provides unlimited opportunities to review lessons and to practice problem solving. If students give a wrong answer, they can ask to see the problem worked out and get a textbook page reference. Many problems include hints for first incorrect responses. Tools such as an on-line glossary and Quick Reviews provide definitions and examples, and an on-line calculator aids students in computation. The program is menu-driven for ease of use and on-screen help can be obtained at any time with a single keystroke. Students' scores are calculated at the end of each lesson and can be printed for a permanent record. The optional **Management System** lets instructors record student scores on disk and print diagnostic reports for individual students or classes. CLAST and TASP versions of this tutorial are also available for both IBM and Mac machines. This software may also be purchased by students for home use. Student versions include record-keeping and practice tests.

Videotapes A new videotape series has been developed to accompany *Algebra for College Students,* Third Edition. In a separate lesson for each section in the book, the series covers all objectives, topics, and problem-solving techniques discussed in the text.

Overcoming Math Anxiety This book, written by Randy Davidson and Ellen Levitov, includes step-by-step guides to problem solving, note taking, and applied problems. Students can discover the reasons behind math anxiety and ways to overcome those obstacles. The book also will help them learn relaxation techniques, build better math skills, and improve study habits. (ISBN 0-06-501651-3)

OTHER BOOKS IN THIS SERIES

Other textbooks in this series include: *Beginning Algebra,* Seventh Edition, *Intermediate Algebra,* Seventh Edition, *Intermediate Algebra with Early Graphs and Functions,* Seventh Edition, and *Beginning and Intermediate Algebra.*

ACKNOWLEDGMENTS

We appreciate the many contributions of users of the sixth edition of the book. We also wish to thank our reviewers for their insightful comments and suggestions:

Carol Adjemian, *Seaver College of Pepperdine University*
George W. Brewer, *Odessa College*
James Carolan, *Wharton County Junior College*
Li Chao, *Teikyo Loretto Heights University*
Allen Christian, *Eastfield College*
Janet Clark, *Hillsborough Community College*
Donald O. Clayton, *Madisonville Community College*
Margaret Coleman, *Louisiana Tech University*
Carol DeVille, *Louisiana Tech University*
Cleon Diers, *San Bernardino Valley College*
Mark Dillman, *Navarro College*

Karen Edwards, *Paul Smiths College of Arts and Sciences*
James Godfrey, *Wayne Community College*
Linda D. Green, *Santa Fe Community College*
Mary Henderson, *Okaloosa-Walton Community College*
Virgil Hinson, *Shoals Community College*
Gayle S. Kent, *Florida Southern College*
Parviz Khajeh-Khalili, *Christopher Newport University*
Stephen G. Landry, *Seton Hall University*
Richard Mabry, *Louisiana State University at Shreveport*
Calvin I. Owens, *Chowan College*
Kathy Powell, *Essex County College*
Kirby S. Pressly, *Wharton County Junior College*
Nancy Reding, *Bossier Parish Community College*
Sharon Schwenk, *Columbia Basin College*
Radha Shrinivas, *St. Louis Community College at Forest Park*
Ronald Smith, *Edison Community College*
L. Dwayne Snider, *Tarleton State University*
Judy Stahl, *Lake-Sumter Community College*
Ann Steen, *Sante Fe Community College*
Charles Sweatt, *Odessa College*
Karen R. Swick, *Palm Beach Atlantic College*
Lee Ullery, *St. Louis Community College at Forest Park*
Nancy Warden, *University of Science and Arts of Oklahoma*
Peggy Womble, *Wayne Community College*

As always, Paul Eldersveld, *College of DuPage,* has done an outstanding job of coordinating all the print supplements for us.

We also wish to thank those who did an excellent job checking all the answers:

Imelda Arce, *University of Texas at Brownsville*
Carol DeVille, *Louisiana Tech University*
Daniel P. Kuykendall, *Del Mar College*
Kathleen L. Pellissier
Radha Shrinivas, *St. Louis Community College at Forest Park*
Glynna Strait, *Odessa College*

Our appreciation also goes to Tommy Thompson, *Cedar Valley College,* for his suggestions for the feature "To the Student: Success in Algebra."

Special thanks go to the dedicated staff at HarperCollins who have worked so long and hard to make this book a success: Karin Wagner, Sandi Goldstein, Cathy Wacaser, Anne Kelly, George Duda, Ed Moura, and Linda Youngman.

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TO THE STUDENT: SUCCESS IN ALGEBRA

The main reason students have difficulty with mathematics is that they don't know how to study it. Studying mathematics *is* different from studying subjects like English or history. The key to success is regular practice.

This should not be surprising. After all, can you learn to play the piano or to ski well without a lot of regular practice? The same thing is true for learning mathematics. Working problems nearly every day is the key to becoming successful. Here is a list of things you can do to help you succeed in studying algebra.

1. *Attend class regularly.* Pay attention in class to what your teacher says and does, and make careful notes. In particular, note the problems the teacher works on the board and copy the complete solutions. Keep these notes separate from your homework to avoid confusion when you read them over later.
2. Don't hesitate to ask questions in class. It is not a sign of weakness, but of strength. There are always other students with the same question who are too shy to ask.
3. *Read your text carefully.* Many students read only enough to get by, usually only the examples. Reading the complete section will help you to be successful with the homework problems. Most exercises are keyed to specific examples or objectives that will explain the procedures for working them.
4. Before you start on your homework assignment, rework the problems the teacher worked in class. This will reinforce what you have learned. Many students say, "I understand it perfectly when you do it, but I get stuck when I try to work the problem myself."
5. Do your homework assignment only *after* reading the text and reviewing your notes from class. Check your work with the answers in the back of the book. If you get a problem wrong and are unable to see why, mark that problem and ask your instructor about it. Then practice working additional problems of the same type to reinforce what you have learned.
6. Work as neatly as you can. Write your symbols neatly, and make sure the problems are clearly separated from each other. Working neatly will help you to think clearly and also make it easier to review the homework before a test.
7. After you have completed a homework assignment, look over the text again. Try to decide what the main ideas are in the lesson. Often they are clearly highlighted or boxed in the text.
8. Use the chapter test at the end of each chapter as a practice test. Work through the problems under test conditions, without referring to the text or the answers until you are finished. You may want to time yourself to see

how long it takes you. When you have finished, check your answers against those in the back of the book and study those problems that you missed. Answers are referenced to the appropriate sections of the text.

9. Keep any quizzes and tests that are returned to you and use them when you study for future tests and the final exam. These quizzes and tests indicate what your instructor considers most important. Be sure to correct any problems on these tests that you missed, so you will have the corrected work to study.
10. Don't worry if you do not understand a new topic right away. As you read more about it and work through the problems, you will gain understanding. Each time you look back at a topic you will understand it a little better. No one understands each topic completely right from the start.

AN INTRODUCTION TO GRAPHICS CALCULATORS*

CAPABILITIES OF GRAPHICS CALCULATORS

Graphics calculators are a result of the amazingly rapid evolution in computer technology toward packaging more power into smaller "boxes." These machines have powerful graphing capabilities in addition to the full range of features found on programmable scientific calculators. Instead of a one-line display, graphics calculators typically can show up to eight lines of text. This makes it much easier to keep track of the steps of your work, whether you are doing routine computations, entering a long mathematical function, or writing a program. Like programmable scientific calculators, graphics calculators are capable of doing many things we have previously come to expect only from computers. Programs can be written relatively easily, and after they are stored in memory, the programs are always available. Like computers, graphics calculators can be programmed to include graphic displays as part of the program.

It takes some study to learn how to use graphics calculators, but they are much easier to master than most computers—and they can go wherever you do! New models with added features, more memory, greater ease of use, and other improvements are frequently being introduced. The most popular brand names at this time are Casio, Sharp, Texas Instruments, and Hewlett-Packard.

GRAPHICS CALCULATOR FEATURES

Every graphics calculator has keys for the usual operations of arithmetic and all commonly used functions (square root, x^2, log, ln, and so on). All of them can graph functions of the form $y = f(x)$ and have programming capabilities. Except for the cheapest ones, graphics calculators may have a variety of additional features. Before buying one, you should consider which features you are likely to need.

Many of the features of the typical graphics calculator do not play a role in the topics covered in this book, and further mathematics courses will be needed in order to fully appreciate their power. The more advanced models have the capability to perform some symbolic manipulations (such as factoring, adding polynomials, and performing operations with rational algebraic expressions). However, these calculators are more expensive and more difficult to learn to use because of their increased complexity and the fact that they do not use standard algebraic order of operations.

ADVICE ON USING A GRAPHICS CALCULATOR

1. **BASICS** Graphics calculators have forty-nine or more keys. Most modern desk-top computers have 101 keys on their keyboards. With fewer keys, each key must be used for more actions, so you will find special mode-changing keys such as **"2nd," "shift," "alpha,"** and **"mode".** Become familiar with

* Prepared by Jim Eckerman of *American River College.*

the capabilities of the machine, the layout of the keyboard, how to adjust the screen contrast, and so on. Remember that a graphics calculator is composed of two parts: the machinery *and* the owner's manual!

2. **EDITING** When keying in expressions, you can pause at any time and use the arrow keys, located at the upper right of the keyboard, to move the cursor to any point in the text. You can then make changes by using the **"DEL"** key to delete and the **"INS"** key to insert material. ["INS" is the "second function" of "DEL."] After an expression has been entered or a calculation made, it can still be edited by using the **edit/replay** feature, available on almost every calculator model. On TI calculators (except TI-81), use "2nd, ENTER" to return to the previously entered expression. On Casio calculators, use the left or right arrow key, and on Sharp models use "2nd, up arrow."
3. **SCIENTIFIC NOTATION** Learn how to enter and read data in **scientific notation** form. This form is used when the numbers become too large or too small (too many zeros between the decimal point and the first significant digit) for the machine's display.
4. **FUNCTION GRAPHING**
   A. **Setting the Range or Window** Learn to set **"RANGE"** or **"WINDOW"** values to delineate a window that is appropriate for the function you are graphing before using the **"Graph"** key. This involves keying in the minimum and maximum values of x and y that will be displayed on the screen, along with the distance to be used between tick marks along the axes. If you do not do this, you will often find your graph screen blank! Usually one can quickly find a point on the graph by substitution of either zero or one for x. Use the **"RANGE"** or **"WINDOW"** command to set the x-values to the left and right of the x-coordinate of this point and set the y-values above and below the y-coordinate of this point. Then the **"zoom out"** feature can be used to see more of the graph.
   B. **Using the Function Memory** Learn how to redraw graphs without reentering the function. Often you will need to change the **"RANGE"** settings several times before you get the "window" that is most appropriate for your function. All graphics calculators allow you to do this without reentering the function. If you plan to graph a particular function often, then you should either store it in the **function memory** (which is labeled "$y =$ " on TI and "EQTN" on Sharp models) or store it in **program memory.** Of course, you have to write a program with your function as part of the program in order to make use of the program memory. Once entered into the machine's memory, this function can be used at any time.
   C. **Using the Trace Feature** With the **"Trace"** feature, the left/right arrows can be used to move the cursor along the last curve plotted, and the values of x and y will be displayed for each point plotted on the screen. If more than one graph was plotted, one can move the cursor vertically between the different graphs by using the up/down arrows.
   D. **Using the Zoom Feature** The **"zoom"** feature allows a quick redrawing of your graph using smaller ranges of values for x and y (**"zoom in"**) or larger ranges of values (**"zoom out"**). Thus one can easily examine the behavior of the graph of a function within the close vicinity of a particular point or the general behavior as seen from farther away. Using **"zoom box,"** a box can be drawn for a particular region for closer inspection of the graph within that region.

SOLVING EQUATIONS AND SYSTEMS GRAPHICALLY

Some mathematical procedures can be quite difficult or even impossible to do algebraically but can be done easily and to a very high degree of accuracy using a graphics calculator. Listed below are some examples.

1. **SOLVING EQUATIONS OF THE FORM $f(x) = k$** The quickest way to solve this type of equation is to form the new equation $y = f(x) - k$ and then use the built-in Equation Solver or Root Finder feature. If your calculator does not have this capability or if you prefer to see the solution graphically, you can find the roots of $y = f(x) - k$ by locating the points where the graph crosses the x-axis.
2. **SOLVING SYSTEMS OF TWO EQUATIONS IN TWO UNKNOWNS** Some graphics calculators have built-in programs for solving systems of linear equations, but all of them can be used to solve systems of two equations (linear or not) by finding intersection points. If both equations can be solved explicitly for one variable in terms of the other, then the two equations can be put into the forms $y = f(x)$ and $y = g(x)$. Then we can graph both in the same window and zoom in on the point or points where the two curves intersect.

PROGRAMMING

Many formulas are used often enough to justify automating the process of evaluating them. The distance formula and the quadratic formula are two examples of this. Graphics calculators are able to store programs that will perform such tasks. The realm of *programming* is much different from that of *using* a graphics calculator. Many programs are available from the manufacturers and the literature that they support. Some students like to experiment and program their own calculators. The old adage "The sky's the limit" is certainly applicable to programming, and it is up to the user's imagination as to how far he or she wishes to take the programming capability of the calculator.

SOME SUGGESTIONS FOR REDUCING FRUSTRATION

We all find ways to make even the simplest machines do the wrong things without even trying. One of the more common problems with graphics calculators is getting a blank screen when a graph was expected. This usually results from not setting the **"WINDOW"** values appropriately before graphing the function, although it could also easily result from incorrectly entering the function.

A common problem that is particularly annoying is interpreting cryptic error messages such as **"Syn ERROR"** and **"Ma ERROR."** (Keep that manual handy!) The most common mistakes are made entering formulas and using special functions. For example, on the Casio machines **"Syn ERROR"** means that a mistake was made when entering the function or operation, such as entering "Graph $Y = \log x^y$" instead of "Graph $Y = \log x^2$".

Another common error is having more right parentheses than left parentheses. To confuse us further, these same machines think it is perfectly OK to have more left parentheses than right parentheses. For instance, the expression $5(3 - 4(2 + 7)$ has two left parentheses and one right parenthesis, but it will be evaluated as $5(3 - 4(2 + 7))$. The message **"Ma ERROR"** appears when a number is too large or when a number is not allowed. If you try to find the 1000th power of ten or divide a number by zero you will most certainly see some kind of error message. By pressing one of the **"cursor"** keys on the Casio you will see the cursor blinking at the location of the error in your expression. When the Texas Instruments models detect an error, they display a special menu that lists a code number and a name for the type of error. For certain types of errors the choice **"Go to error"** is