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Peter Eklund (Ed.)

Concept Lattices

Second International Conference on Formal Concept Analysis, ICFCA 2004
Sydney, Australia, February 2004
Proceedings

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Preface

This volume contains the Proceedings of ICFCA 2004, the 2nd International Conference on Formal Concept Analysis. The ICFCA conference series aims to be the premier forum for the publication of advances in applied lattice and order theory and in particular scientific advances related to formal concept analysis.

Formal concept analysis emerged in the 1980s from efforts to restructure lattice theory to promote better communication between lattice theorists and potential users of lattice theory. Since then, the field has developed into a growing research area in its own right with a thriving theoretical community and an increasing number of applications in data and knowledge processing including data visualization, information retrieval, machine learning, data analysis and knowledge management.

In terms of theory, formal concept analysis has been extended into attribute exploration, Boolean judgment, contextual logic and so on to create a powerful general framework for knowledge representation and reasoning. This conference aims to unify theoretical and applied practitioners who use formal concept analysis, drawing on the fields of mathematics, computer and library sciences and software engineering. The theme of the 2004 conference was 'Concept Lattices" to acknowledge the colloquial term used for the line diagrams that appear in almost every paper in this volume.

ICFCA 2004 included tutorial sessions, demonstrating the practical benefits of formal concept analysis, and highlighted developments in the foundational theory and standards. The conference showcased the increasing variety of formal concept analysis software and included eight invited lectures from distinguished speakers in the field. Seven of the eight invited speakers submitted accompanying papers and these were reviewed and appear in this volume.

All regular papers appearing in this volume were refereed by at least two referees. In almost all cases three (or more) referee reports were returned. Long papers of approximately 14 pages represent substantial results deserving additional space based on the recommendations of reviewers. The final decision to accept the papers (as long, short or at all) was arbitrated by the Program Chair based on the referee reports. As Program Chair, I wish to thank the Program Committee and the additional reviewers for their involvement which ensured the high scientific quality of these proceedings. I also wish to particularly thank Prof. Paul Compton and Rudolf Wille, Dr. Richard Cole and Peter Becker for their support and enthusiasm.

January 2004

Peter Eklund

Organization

The International Conference on Formal Concept Analysis (ICFCA) is intended as a regular conference and principal research forum in the theory and practice of formal concept analysis. The inaugural International Conference on Formal Concept Analysis was held at the Technical University of Darmstadt, Germany in 2003 and ICFCA 2004 in Sydney, Australia.

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Preconcept Algebras and Generalized Double Boolean Algebras

Rudolf Wille

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Abstract. *Boolean Concept Logic* as an integrated generalization of Contextual Object Logic and Contextual Attribute Logic can be substantially developed on the basis of preconcept algebras. The main results reported in this paper are the *Basic Theorem on Preconcept Algebras* and the Theorem characterizing the equational class generated by all preconcept algebras by the equational axioms of the *generalized double Boolean algebras.*

1 Preconcepts in Boolean Concept Logic

Concepts are the basic units of thought wherefore a concept-oriented mathematical logic is of great interest. G. Boole has offered the most influential foundation for such a logic which is based on a general conception of *signs* representing *classes* of objects from a given *universe of discourse* [Bo54]. In the language of Formal Concept Analysis [GW99a], Boole's basic notions can be explicated:

- for a "universe of discourse", by the notion of a *"formal context"* defined as a mathematical structure $\mathbb{K} := (G, M, I)$ where G is a set whose elements are called *"objects"*, M is a set whose elements are called *"attributes"*, and I is a subset of $G \times M$ for which gIm (i.e. $(g, m) \in I$) is read: the object g *has* the attribute m,
- for a "sign", by the notion of an *"attribute"* of a formal context and,
- for a "class", by the notion of an *"extent"* defined in a formal context $\mathbb{K} := (G, M, I)$ as a subset $Y' := \{g \in G \mid \forall m \in Y : gIm\}$ for some $Y \subseteq M$.

How Boole's logic of signs and classes may be developed as a *Contextual Attribute Logic* by means of Formal Concept Analysis is outlined in [GW99b]. The dual *Contextual Object Logic*, which is for instance used to determine conceptual contents of information [Wi03a], can be obtained from Contextual Attribute Logic by interchanging the role of objects and attributes so that, in particular, the notion of an "extent" is replaced by

- the notion of an *"intent"* defined in a formal context $\mathbb{K} := (G, M, I)$ as a subset $X' := \{m \in M \mid \forall g \in X : gIm\}$ for some $X \subseteq G$.

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Since a concept, as a unit of thought, combines an extension consisting of objects and an intension consisting of attributes (properties, meanings) (cf. [Sch90], p.83ff.), a concept-oriented mathematical logic should be an integrated generalization of a mathematical attribute logic and a mathematical object logic. In our contextual approach based on Formal Concept Analysis, such an integrated generalization can be founded on

- the notion of a *"formal concept"* defined, in a formal context $\mathbb{K} := (G, M, I)$, as a pair (A, B) with $A \subseteq G$, $B \subseteq M$, $A = B'$, and $B = A'$ [Wi82],

and its generalizations:

- the notions of a *"⊓-semiconcept"* (A, A') with $A \subseteq G$ and a *"⊔-semiconcept"* (B', B) with $B \subseteq M$ [LW91], the notion of a *"protoconcept"* (A, B) with $A \subseteq G$, $B \subseteq M$, and $A'' = B'$ ($\Leftrightarrow B'' = A'$) [Wi00a], and the notion of a *"preconcept"* (A, B) with $A \subseteq G$, $B \subseteq M$, and $A \subseteq B'$ ($\Leftrightarrow B = A'$) [SW86].

Clearly, formal concepts are always semiconcepts, semiconcepts are always protoconcepts, and protoconcepts are always preconcepts. Since, for $X \subseteq G$ and $Y \subseteq M$, we always have $X''' = X'$ and $Y''' = Y'$, formal concepts can in general be constructed by forming (X'', X') or (Y', Y''). The basic logical *derivations* $X \mapsto X'$ and $Y \mapsto Y'$ may be naturally generalized to the conceptual level by

- $(X, Y) \mapsto (X, X') \mapsto (X'', X')$ and $(X, Y) \mapsto (Y', Y) \mapsto (Y', Y'')$ for an arbitrary preconcept (X, Y) of $\mathbb{K} := (G, M, I)$.

It is relevant to assume that (X, Y) is a preconcept because otherwise we would obtain $Y \not\subseteq X'$ and $X \not\subseteq Y'$, i.e., (X, X') and (Y', Y) would not be extensions of (X, Y) with respect to the order $\sqsubseteq^2$ defined by

- $(X_1, Y_1) \sqsubseteq^2 (X_2, Y_2) :\Longleftrightarrow X_1 \subseteq X_2$ and $Y_1 \subseteq Y_2$.

Notice that, in the ordered set $(\mathfrak{V}(\mathbb{K}), \sqsubseteq^2)$ of all preconcepts of a formal context $\mathbb{K} := (G, M, I)$, the formal concepts of $\mathbb{K}$ are exactly the maximal elements and the protoconcepts of $\mathbb{K}$ are just the elements which are below exactly one maximal element (formal concept).

For contextually developing a *Boolean Concept Logic* as an integrated generalization of the Contextual Object Logic and the Contextual Attribute Logic, *Boolean operations* have to be introduced on the set $\mathfrak{V}(\mathbb{K})$ of all preconcepts of a formal context $\mathbb{K} := (G, M, I)$. That shall be done in the same way as for semiconcepts [LW91] and for protoconcepts [Wi00a]:

$$\begin{aligned}
(A_1, B_1) \sqcap (A_2, B_2) &:= (A_1 \cap A_2, (A_1 \cap A_2)') \\
(A_1, B_1) \sqcup (A_2, B_2) &:= ((B_1 \cap B_2)', B_1 \cap B_2) \\
\neg(A, B) &:= (G \setminus A, (G \setminus A)') \\
\lrcorner(A, B) &:= ((M \setminus B)', M \setminus B) \\
\bot &:= (\emptyset, M) \\
\top &:= (G, \emptyset)
\end{aligned}$$

The set $\mathfrak{V}(\mathbb{K})$ together with the operations $\sqcap, \sqcup, \neg, \lrcorner, \bot$, and $\top$ is called the *preconcept algebra* of $\mathbb{K}$ and is denoted by $\underline{\mathfrak{V}}(\mathbb{K})$; the operations are named *"meet"*, *"join"*, *"negation"*, *"opposition"*, *"nothing"*, and *"all"*. For the structural analysis of the preconcept algebra $\underline{\mathfrak{V}}(\mathbb{K})$, it is useful to define additional operations on $\mathfrak{V}(\mathbb{K})$:

$$\mathfrak{a} \mathbin{\bar{\sqcup}} \mathfrak{b} := \neg(\neg\mathfrak{a} \sqcap \neg\mathfrak{b}) \text{ and } \mathfrak{a} \mathbin{\bar{\sqcap}} \mathfrak{b} := \lrcorner(\lrcorner\mathfrak{a} \sqcup \lrcorner\mathfrak{b}),$$
$$\bar{\top} := \neg\bot \text{ and } \bar{\bot} := \lrcorner\top.$$

The semiconcepts resp. protoconcepts of $\mathbb{K}$ form subalgebras $\underline{\mathfrak{H}}(\mathbb{K})$ resp. $\underline{\mathfrak{P}}(\mathbb{K})$ of $\underline{\mathfrak{V}}(\mathbb{K})$ which are called the *semiconcept algebra* resp. *protoconcept algebra* of $\mathbb{K}$. The set $\mathfrak{H}_\sqcap(\mathbb{K})$ of all $\sqcap$-semiconcepts is closed under the operations $\sqcap$, $\bar{\sqcup}$, $\neg$, $\bot$, and $\bar{\top}$; therefore, $\underline{\mathfrak{H}}_\sqcap(\mathbb{K}) := (\mathfrak{H}_\sqcap(\mathbb{K}), \sqcap, \bar{\sqcup}, \neg, \bot, \bar{\top})$ is a Boolean algebra isomorphic to the Boolean algebra of all subsets of G. Dually, the set $\mathfrak{H}_\sqcup(\mathbb{K})$ of all $\sqcup$-semiconcepts is closed under the operations $\bar{\sqcap}$, $\sqcup$, $\lrcorner$, $\bar{\bot}$, and $\top$; therefore, $\underline{\mathfrak{H}}_\sqcup(\mathbb{K}) := (\mathfrak{H}_\sqcup(\mathbb{K}), \bar{\sqcap}, \sqcup, \lrcorner, \bar{\bot}, \top)$ is a Boolean algebra antiisomorphic to the Boolean algebra of all subsets of M. Furthermore, $\mathfrak{B}(\mathbb{K}) = \mathfrak{H}_\sqcap(\mathbb{K}) \cap \mathfrak{H}_\sqcup(\mathbb{K})$, and $(\mathfrak{B}(\mathbb{K}), \wedge, \vee)$ is the so-called *concept lattice* of $\mathbb{K}$ with the operations $\wedge$ and $\vee$ induced by the operations $\sqcap$ and $\sqcup$, respectively. The general order relation $\sqsubseteq$ of $\underline{\mathfrak{P}}(\mathbb{K})$, which coincides on $\mathfrak{B}(\mathbb{K})$ with the subconcept-superconcept-order $\leq$, is defined by

$$(A_1, B_1) \sqsubseteq (A_2, B_2) :\Longleftrightarrow A_1 \subseteq A_2 \text{ and } B_1 \supseteq B_2.$$

The introduced notions found a Boolean Concept Logic in which the Contextual Object Logic and the Contextual Attribute Logic can be integrated by transforming any object sets X to the $\sqcap$-semiconcept (X, X') and any attribute set Y to the corresponding $\sqcup$-semiconcept (Y', Y). In the case of Contextual Attribute Logic [GW99b], this integration comprises a transformation of the Boolean compositions of attributes which is generated by the following elementary assignments:

$$m \wedge n \mapsto (\{m\}', \{m\}) \sqcap (\{n\}', \{n\}),$$
$$m \vee n \mapsto (\{m\}', \{m\}) \mathbin{\bar{\sqcup}} (\{n\}', \{n\}),$$
$$\neg m \mapsto \neg(\{m\}', \{m\}).$$

In the dual case of Contextual Object Logic, the corresponding transformation uses the operations $\bar{\sqcap}$, $\sqcup$, and $\lrcorner$.

Preconcept algebras can be illustrated by *line diagrams* which shall be demonstrated by using the small formal context in Fig.1. The line diagram of the preconcept algebra of that formal context is shown in Fig.2: the formal concepts are represented by the black circles, the proper $\sqcap$-semiconcepts by the circles with only a black lower half, the proper $\sqcup$-semiconcepts by the circles with only a black upper half, and the proper preconcepts (which are even not protoconcepts) by the unblackened circles. An object (attribute) belongs to a preconcept if and only if its name is attached to a circle representing a subpreconcept (superpreconcept) of that preconcept. The regularity of the line diagram in Fig.2 has a general reason which becomes clear by the following proposition:

	male	female	old	young
father	×		×	
mother		×	×	
son	×			×
daughter		×		×

Fig. 1. A context $\mathbb{K}^f$ of family members

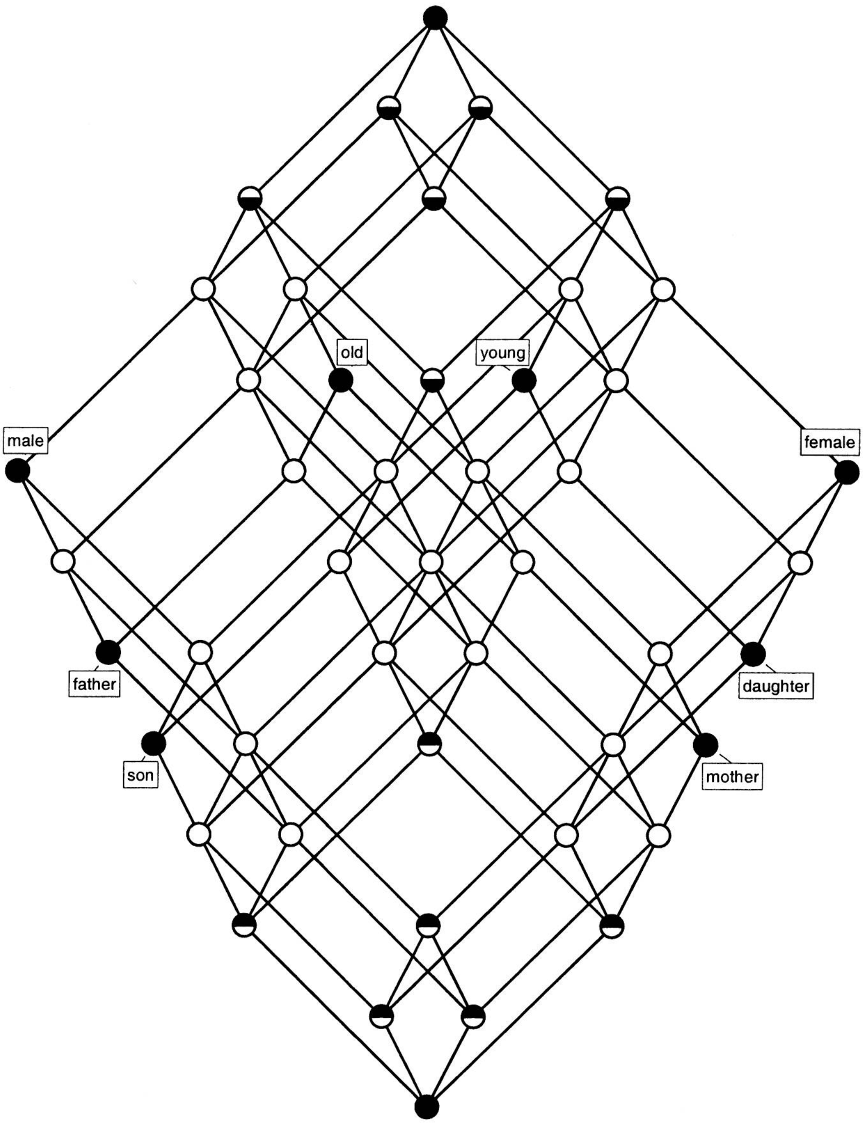

Fig. 2. Line diagram of the preconcept algebra of the formal context $\mathbb{K}^f$ in Fig.1

Proposition 1 *For a formal context* $\mathbb{K} := (G, M, I)$, *the ordered set* $(\mathfrak{V}(\mathbb{K}), \sqsubseteq)$ *is a completely distributive complete lattice, which is isomorphic to the concept lattice of the formal context* $\mathbb{V}(\mathbb{K}) := (G\dot\cup M, G\dot\cup M, I \cup (\neq \backslash G \times M))$.

Proof For $(A_t, B_t) \in \mathfrak{V}(\mathbb{K})$ $(t \in T)$, we obviously have

$$inf_{t\in T}(A_t, B_t) = (\bigcap_{t\in T} A_t, \bigcup_{t\in T} B_t) \text{ and } sup_{t\in T}(A_t, B_t) = (\bigcup_{t\in T} A_t, \bigcap_{t\in T} B_t);$$

hence $\mathfrak{V}(\mathbb{K})$ is a complete sublattice of the completely distributive complete lattice $(\mathfrak{P}(G), \subseteq) \times (\mathfrak{P}(M), \supseteq)$ which proves the first assertion. For proving the second assertion, we consider the assignment $(A, B) \overset{\iota}{\mapsto} (A \cup (M \setminus B), (G \setminus A) \cup B)$. It can be easily checked that $(A \cup (M \setminus B), (G \setminus A) \cup B)$ is a formal concept of $\mathbb{V}(\mathbb{K})$. Let (C, D) be an arbitrary formal concept of $\mathbb{V}(\mathbb{K})$. Obviously, $C = (C \cap G) \cup (M \setminus D)$, $D = (G \setminus C) \cup (D \cap M)$, and $(C \cap G, D \cap M)$ is a preconcept of $\mathbb{K}$. Therefore we obtain $(C \cap G, D \cap M) \overset{\iota}{\mapsto} (C, D)$. Thus, ι is a bijection from $\mathfrak{V}(\mathbb{K})$ onto $\mathfrak{B}(\mathbb{V}(\mathbb{K}))$. Since $(A_1, B_1) \sqsubseteq (A_2, B_2) \iff A_1 \subseteq A_2$ and $B_1 \supseteq B_2 \iff A_1 \cup (M \setminus B_1) \subseteq A_2 \cup (M \setminus B_2) \iff (A_1 \cup (M \setminus B_1), (G \setminus A_1) \cup B_1) \leq (A_2 \cup (M \setminus B_2), (G \setminus A_2) \cup B_2)$, the bijection ι is even an isomorphism from $\underline{\mathfrak{V}}(\mathbb{K})$ onto $\underline{\mathfrak{B}}(\mathbb{V}(\mathbb{K}))$. □

2 The Basic Theorem on Preconcept Algebras

A detailed understanding of the *structure of preconcept algebras* is basic for Boolean Concept Logic. To start the necessary structure analysis, we first determine basic equations valid in all preconcept algebras and study abstractly the class of all algebras satisfying those equations. The aim is to prove a characterization of preconcept algebras analogously to the *Basic Theorem on Concept Lattices* [Wi82].

Proposition 2 *In a preconcept algebra* $\underline{\mathfrak{V}}(\mathbb{K})$ *the following equations are valid:*

1a)	$(x \sqcap x) \sqcap y = x \sqcap y$	*1b)*	$(x \sqcup x) \sqcup y = x \sqcup y$
2a)	$x \sqcap y = y \sqcap x$	*2b)*	$x \sqcup y = y \sqcup x$
3a)	$x \sqcap (y \sqcap z) = (x \sqcap y) \sqcap z$	*3b)*	$x \sqcup (y \sqcup z) = (x \sqcup y) \sqcup z$
4a)	$x \sqcap (x \sqcup y) = x \sqcap x$	*4b)*	$x \sqcup (x \sqcap y) = x \sqcup x$
5a)	$x \sqcap (x \dot\sqcup y) = x \sqcap x$	*5b)*	$x \sqcup (x \dot\sqcap y) = x \sqcup x$
6a)	$x \sqcap (y \dot\sqcup z) = (x \sqcap y) \dot\sqcup (x \sqcap z)$	*6b)*	$x \sqcup (y \dot\sqcap z) = (x \sqcup y) \dot\sqcap (x \sqcup z)$
7a)	$\neg\neg(x \sqcap y) = x \sqcap y$	*7b)*	$\lrcorner\lrcorner(x \sqcup y) = x \sqcup y$
8a)	$\neg(x \sqcap x) = \neg x$	*8b)*	$\lrcorner(x \sqcup x) = \lrcorner x$
9a)	$x \sqcap \neg x = \bot$	*9b)*	$x \sqcup \lrcorner x = \top$
10a)	$\neg\bot = \top \sqcap \top$	*10b)*	$\lrcorner\top = \bot \sqcup \bot$
11a)	$\neg\top = \bot$	*11b)*	$\lrcorner\bot = \top$
12a)	$x_{\sqcap\sqcup\sqcap} = x_{\sqcap\sqcup}$	*12b)*	$x_{\sqcup\sqcap\sqcup} = x_{\sqcup\sqcap}$

where $t_\sqcap := t \sqcap t$ *and* $t_\sqcup := t \sqcup t$ *is defined for every term* t.

Proof The equations of the proposition can be easily verified in preconcept algebras. This shall only be demonstrated by proving 4a) and 12a): $(A, B) \sqcap ((A, B) \sqcup (C, D)) = (A, B) \sqcap ((B \cap D)', B \cap D) = (A \cap (B \cap D)', (A \cap (B \cap D)')') = (A, A') = (A, B) \sqcap (A, B)$ and $(A, B)_{\sqcap\sqcup\sqcap} = (A, A')_{\sqcup\sqcap} = (A'', A')_{\sqcap} = (A'', A') = (A, A')_{\sqcup} = (A, B)_{\sqcap\sqcup}$. □