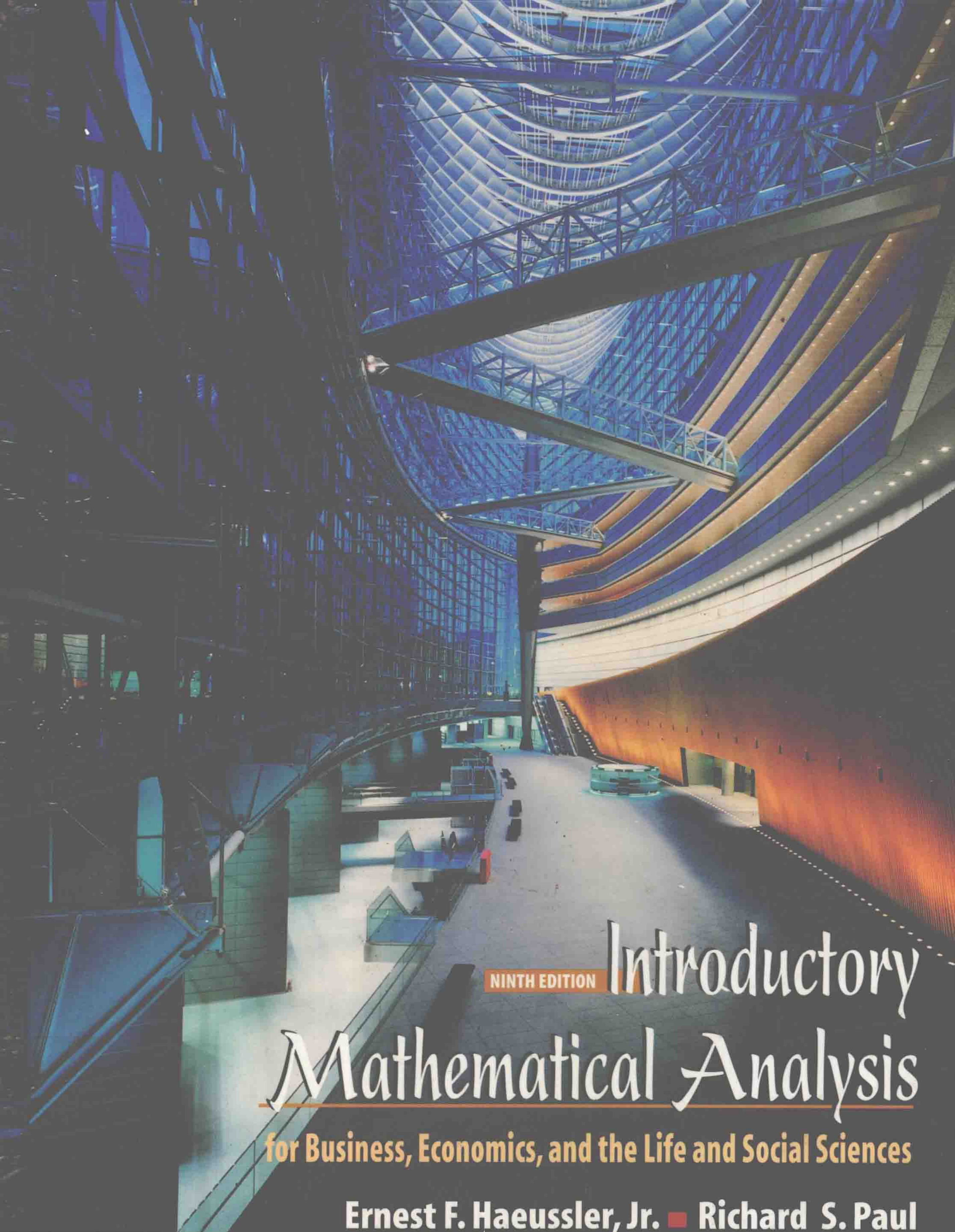

NINTH EDITION

Introductory Mathematical Analysis

for Business, Economics, and the Life and Social Sciences

Ernest F. Haeussler, Jr. ■ Richard S. Paul

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Introductory Mathematical Analysis

FOR BUSINESS, ECONOMICS, AND THE LIFE AND SOCIAL SCIENCES

Ernest F. Haeussler, Jr.
The Pennsylvania State University

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with Contributions by Laurel Technical Services

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Preface

This ninth edition of *Introductory Mathematical Analysis* continues to provide a mathematical foundation for students in business, economics, and the life and social sciences. It begins with noncalculus topics such as equations, functions, matrix algebra, linear programming, mathematics of finance, and probability. Then it progresses through both single-variable and multivariable calculus, including continuous random variables. Technical proofs, conditions, and the like, are sufficiently described, but are not overdone. At times, informal intuitive arguments are given to preserve clarity.

Applications

An abundance and variety of applications for the intended audience appear throughout the book; students continually see how the mathematics they are learning can be used. These applications cover such diverse areas as business, economics, biology, medicine, sociology, psychology, ecology, statistics, earth science, and archaeology. Many of these real-world situations are drawn from literature and are documented by references. In some, the background and context are given in order to stimulate interest. However, the text is virtually self-contained, in the sense that it assumes no prior exposure to the concepts on which the applications are based.

Changes to the Ninth Edition

Principles in Practice

This new element provides students with even more applications. Located in the margin of the text, these additional exercises give students real-world applications and more opportunities to see the chapter material put into practice. *Principles in Practice* applications that can be solved using a graphing calculator are indicated by an icon . Answers to Principles in Practice applications appear at the end of the text.

Concepts for Calculus Appendix

New to the ninth edition, this useful end-of-text appendix features additional calculus concepts for student review. Such topics include: Slopes and Equations of Lines, Secant Lines and Average Rate of Change, and Slope of a Curve and Derivative.

Updated Mathematical Snapshots

Included at the end of many chapters, this popular feature has been revised for the ninth edition. Each Snapshot provides an interesting, and at times, novel application involving the mathematics of the chapter in which it occurs. Many of the Snapshots also include exercises, reinforcing the text's strong emphasis on hands-on practice.

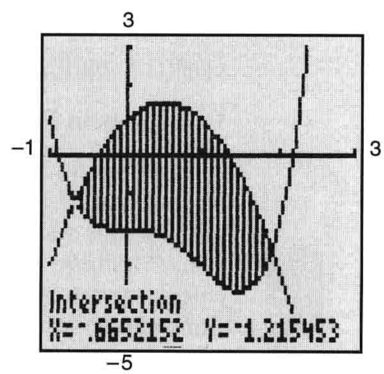
Retained Features

Interspersed throughout the text are many warnings to the student that point out commonly made errors. These warnings are indicated under the heading **Pitfall**. Definitions are clearly stated and displayed. Key concepts, as well as important rules and formulas, are boxed to emphasize their importance. Throughout the text, notes to the student are placed in the margin. They reflect passing comments which supplement discussions.

More than 850 examples are worked out in detail. Some include a **strategy** that is specifically designed to guide the student through the logistics of the solution before the solution is obtained.

An abundant number of diagrams (almost 500) and exercises (more than 5,000) are included. In each exercise set, grouped problems are given in increasing order of difficulty. In many exercise sets the problems progress from the basic mechanical-drill type to more interesting thought-provoking problems. Many real-world type problems with real data are included. Considerable effort has been made to produce a proper balance between the drill-type exercises and the problems requiring the integration of the concepts learned.

In order that a student appreciates the value of current **technology**, optional graphics-calculator material appears throughout the text both in the exposition and exercises. It appears for a variety of reasons: as a mathematical tool, to visualize a concept, as a computing aid, and to reinforce concepts. Although calculator displays (see below) for a TI-82 accompany the corresponding technology discussion, our approach is general enough so that it can be applied to other fine graphics calculators.



In the exercise sets, graphics-calculator problems are indicated by an icon. To provide flexibility for an instructor in planning assignments, these problems are placed at the end of an exercise set.

Each chapter (except Chapter 0) has a review section that contains a list of important terms and symbols, a chapter summary, and numerous review problems.

Answers to odd-numbered problems appear at the end of the book. For many of the differentiation problems, the answers appear in both unsimplified and simplified forms. This allows students to readily check their work.

Course Planning

Because instructors plan a course outline to serve the individual needs of a particular class and curriculum, we shall not attempt to provide sample outlines. However, depending on the background of the students, some instructors will choose to omit Chapter 0, *Algebra Refresher,* or Chapter 1, *Equations.* Others may exclude the topics of matrix algebra and linear programming. Certainly there are other sections that may be omitted at the discretion of the instructor. As an aid to planning a course outline, perhaps a few comments may be helpful. Section 2.1 introduces some business terms, such as total revenue, fixed cost, variable cost and profit. Section 4.2 introduces the notion of supply and demand equations, and Section 4.6 discusses the equilibrium point. Optional sections, which will not cause problems if they are omitted, are: 7.3, 7.5, 15.4, 17.1, 17.2, 19.4, 19.6, 19.9 and 19.10. Section 17.8 may be omitted if Chapter 18 is not covered.

Supplements

For Instructors

Instructor's Solution Manual. Worked out solutions to all exercises and Principles-in-Practice applications.

Test Item File. Provides over 1,700 test questions, keyed to chapter and section.

Prentice Hall Custom Test. Allows the instructor to access from the computerized Test Item File and personally prepare and print out tests. Includes an editing feature which allows questions to be added or changed.

For Students

Student Solutions Manual with Visual Calculus and Explorations in Finite Mathematics Software. Worked out solutions for every odd-numbered exercise and all Principles-in-Practice applications. Software includes unique programs which enhance the fundamental concepts of calculus and finite mathematics visually, and include exercises taken directly from the text.

For Instructors and Students

PH Companion Website. Designed to complement and expand upon the text, the PH Companion Website offers a variety of interactive learning tools, including: links to related websites, practice work for students, and the ability for instructors to monitor and evaluate students' work on the website. For more information, contact your local Prentice Hall representative.
www.prenhall.com/Haeussler

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Ernest F. Haeussler, Jr.
Richard S. Paul

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Appendix A Concepts for Calculus A1

CHAPTER 0

Algebra Refresher

0.1 Purpose

This chapter is designed to give you a brief review of some terms and methods of manipulative mathematics. No doubt you have been exposed to much of this material before. However, because these topics are important in handling the mathematics that comes later, perhaps an immediate second exposure to them would be beneficial. Devote whatever time is necessary to the sections in which you need review.

OBJECTIVE

To become familiar with sets, the classification of real numbers, and the real-number line.

0.2 Sets and Real Numbers

In simplest terms, a *set* is a collection of objects. For example, we can speak of the set of even numbers between 5 and 11, namely, 6, 8, and 10. An object in a set is called an *element* or *member* of that set.

One way to specify a set is by listing its elements, in any order, inside braces. For example, the previous set is {6, 8, 10}, which we can denote by a letter such as A. A set A is said to be a subset of a set B if and only if every element of A is also an element of B. For example, if $A = \{6, 8, 10\}$ and $B = \{6, 8, 10, 12\}$, then A is a subset of B.

Certain sets of numbers have special names. The numbers 1, 2, 3, and so on form the set of **positive integers** (or **natural numbers**):

$$\begin{matrix}\textit{set of} \\ \textit{positive integers}\end{matrix} = \{1, 2, 3, \ldots\}.$$

The three dots mean that the listing of elements is unending, although we know what the elements are.

The positive integers, together with 0 and the **negative integers** $-1, -2, -3, \ldots$, form the set of **integers:**

$$\begin{matrix}\textit{set of} \\ \textit{integers}\end{matrix} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}.$$

The reason for $q \neq 0$ is that we cannot divide by zero.

The set of **rational numbers** consists of numbers, such as $\frac{1}{2}$ and $\frac{5}{3}$ that can be written as a ratio (quotient) of two integers. That is, a rational number is a number that can be written as p/q, where p and q are integers and $q \neq 0$. (The symbol "$\neq$" is read "is not equal to.") For example, the numbers $\frac{19}{20}, \frac{-2}{7}$, and $\frac{-6}{-2}$, are rational. We remark that $\frac{2}{4}, \frac{1}{2}, \frac{3}{6}, \frac{-4}{-8}$, and 0.5 all represent the same rational number. The integer 2 is rational, since $2 = \frac{2}{1}$. In fact, every integer is rational.

Every integer is a rational number.

All rational numbers can be represented by decimal numbers that *terminate,* such as $\frac{3}{4} = 0.75$ and $\frac{3}{2} = 1.5$, or by *nonterminating repeating decimal numbers* (composed of a group of digits that repeats without end), such as $\frac{2}{3} = 0.666\ldots, \frac{-4}{11} = -0.3636\ldots$, and $\frac{2}{15} = 0.1333\ldots$. Numbers represented by *nonterminating nonrepeating* decimals are called **irrational numbers.** An irrational number cannot be written as an integer divided by an integer. The numbers π (pi) and $\sqrt{2}$ are irrational.

The real numbers consist of all decimal numbers.

Together, the rational numbers and irrational numbers form the set of **real numbers.** Real numbers can be represented by points on a line. First we choose a point on the line to represent zero. This point is called the *origin.* (See Fig. 0.1.) Then a standard measure of distance, called a "unit distance," is chosen and is successively marked off both to the right and to the left of the origin. With each point on the line we associate a directed distance, or *signed number,* which depends on the position of the point with respect to the origin. Positions to the right of the origin are considered positive (+) and positions to the left are negative (−). For example, with the point $\frac{1}{2}$ unit to the right of the origin there corresponds the signed number $\frac{1}{2}$, which is called the **coordinate** of that point. Similarly, the coordinate of the point 1.5 units to the left of the origin is −1.5. In Figure 0.1, the coordinates of some points are marked. The arrowhead indicates that the direction to the right along the line is considered the positive direction.

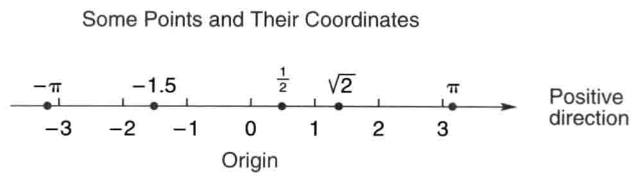

FIGURE 0.1 The real-number line.

To each point on the line there corresponds a unique real number, and to each real number there corresponds a unique point on the line. For this reason, we say that there is a *one-to-one correspondence* between points on the line and real numbers. We call this line a **coordinate line** or the **real-number line.** We feel free to treat real numbers as points on a real-number line and vice versa.

■ Exercise 0.2

In Problems* 1–12, *classify the statement as either true or false. If false, give a reason.

1. -7 is an integer.

2. $\frac{1}{6}$ is rational.

3. -3 is a natural number.

4. 0 is not rational.

5. 5 is rational.

6. $\frac{7}{0}$ is a rational number.

7. $\frac{4}{2}$ is not a positive integer.

8. π is a real number.

9. $\frac{0}{6}$ is rational.

10. 0 is a natural number.

11. -3 is to the right of -4 on the real number line.

12. Every integer is positive or negative.

OBJECTIVE

To state and illustrate the following properties of real numbers: transitive, commutative, associative, inverse, and distributive. To define subtraction and division in terms of addition and multiplication, respectively.

0.3 Some Properties of Real Numbers

We now state a few important properties of the real numbers. Let a, b, and c be real numbers.

1. The Transitive Property of Equality

If $a = b$ and $b = c$, then $a = c$.

Thus, two numbers that are both equal to a third number are equal to each other. For example, if $x = y$ and $y = 7$, then $x = 7$.

2. The Commutative Properties of Addition and Multiplication

$$a + b = b + a \qquad \text{and} \qquad ab = ba.$$

This means that two numbers can be added or multiplied in any order. For example, $3 + 4 = 4 + 3$ and $7(-4) = (-4)(7)$.

3. The Associative Properties of Addition and Multiplication

$$a + (b + c) = (a + b) + c \qquad \text{and} \qquad a(bc) = (ab)c.$$

This means that in addition or multiplication, numbers can be grouped in any order. For example, $2 + (3 + 4) = (2 + 3) + 4$; in both cases, the sum is 9. Similarly, $2x + (x + y) = (2x + x) + y$ and $6(\frac{1}{3} \cdot 5) = (6 \cdot \frac{1}{3}) \cdot 5$.

4. The Inverse Properties

For each real number a, there is a unique real number denoted $-a$ such that

$$a + (-a) = 0.$$

The number $-a$ is called the **additive inverse,** or **negative,** of a.

For example, since $6 + (-6) = 0$, the additive inverse of 6 is -6. The additive inverse of a number is not necessarily a negative number. For example, the additive inverse of -6 is 6, since $(-6) + (6) = 0$. That is, the negative of -6 is 6, so we can write $-(-6) = 6$.

For each real number a, except 0, there is a unique real number denoted a^{-1} such that

$$a \cdot a^{-1} = 1.$$

The number a^{-1} is called the **multiplicative inverse** of a.

Zero does not have a multiplicative inverse because there is no number that, when multiplied by 0, gives 1.

Thus, all numbers except 0 have a multiplicative inverse. You may recall that a^{-1} can be written $\frac{1}{a}$ and is also called the *reciprocal* of a. For example, the multiplicative inverse of 3 is $\frac{1}{3}$, since $3(\frac{1}{3}) = 1$. Hence, $\frac{1}{3}$ is the reciprocal of 3. The reciprocal of $\frac{1}{3}$, is 3, since $(\frac{1}{3})(3) = 1$. **The reciprocal of 0 is not defined.**

5. The Distributive Properties

$$a(b + c) = ab + ac \quad \text{and} \quad (b + c)a = ba + ca.$$

For example, although $2(3 + 4) = 2(7) = 14$, we can write

$$2(3 + 4) = 2(3) + 2(4) = 6 + 8 = 14.$$

Similarly,

$$(2 + 3)(4) = 2(4) + 3(4) = 8 + 12 = 20,$$

$$\text{and} \quad x(z + 4) = x(z) + x(4) = xz + 4x.$$

The distributive property can be extended to the form

$$a(b + c + d) = ab + ac + ad.$$

In fact, it can be extended to sums involving any number of terms.

Subtraction is defined in terms of addition:

$$a - b \quad \text{means} \quad a + (-b),$$

where $-b$ is the additive inverse of b. Thus, $6 - 8$ means $6 + (-8)$.

In a similar way, we define **division** in terms of multiplication. If $b \neq 0$, then $a \div b$, or $\frac{a}{b}$, is defined by

$$\frac{a}{b} = a(b^{-1}).$$

Since $b^{-1} = \frac{1}{b}$,

$$\frac{a}{b} = a(b^{-1}) = a\left(\frac{1}{b}\right).$$

$\frac{a}{b}$ means a times the reciprocal of b.

Thus, $\frac{3}{5}$ means 3 times $\frac{1}{5}$, where $\frac{1}{5}$ is the multiplicative inverse of 5. Sometimes we refer to $a \div b$ or $\frac{a}{b}$ as the *ratio* of a to b. We remark that since 0 does not have a multiplicative inverse, **division by 0 is not defined.**

The following examples show some manipulations involving the preceding properties.

EXAMPLE 1 Applying Properties of Real Numbers

a. $x(y - 3z + 2w) = (y - 3z + 2w)x$, by the commutative property of multiplication.