INTERACTIVE MATHEMATICS

College Algebra

Personal Academic Notebook

ACADEMIC SYSTEMS

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LESSON EIII.A – ALGEBRA BUILDING BLOCKS

OVERVIEW

In this lesson, you will review some of the building blocks of algebra: real numbers, exponents, and radicals. These concepts will form the foundation for your continued study of algebra.

Here's what you'll learn in this lesson:

Real Numbers and Notation

a. *Number line and notation: sets, exponents, ordering symbols, absolute value and distance*

b. *Operations on signed numbers*

c. *Properties of real numbers: commutative property, associate property, distributive property, additive and multiplicative identities, inverses*

Exponents and Radicals

a. *Nonnegative integer exponents*

b. *Negative integer exponents*

c. *Roots and rational exponents*

d. *Properties of exponents*

e. *Operations on radicals*

EXPLAIN

REAL NUMBERS AND NOTATION

Summary

Sets of Numbers and the Number Line

Some of the main building blocks of algebra are the different sets of numbers that you will use to solve problems. All these numbers are contained in the set of real numbers.

Here are the different sets of numbers contained in the real numbers:

Counting numbers or Natural numbers	1, 2, 3, 4, 5, 6, 7,...
Whole numbers	0, 1, 2, 3, 4,...
Integers	..., −4, −3, −2, −1, 0, 1, 2, 3, 4, 5, ...
Rational numbers	Numbers which can be written as fractions $\frac{a}{b}$, where *a* and *b* are integers and $b \neq 0$. Some examples are: $\frac{8}{13}, \frac{-5}{39}, \frac{17}{1}$
Irrational numbers	Numbers which cannot be written as fractions $\frac{a}{b}$, where *a* and *b* are integers and $b \neq 0$. Some examples are: π, e, $-\sqrt{7}, \sqrt[3]{19}$

Now look at some numbers and the sets they belong to.

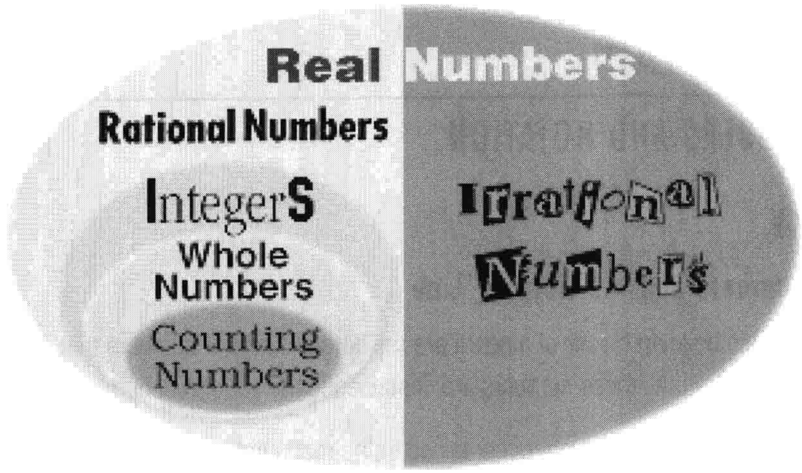
The number −13 is an integer, a rational number, and a real number, but it is not a counting number, a whole number, or an irrational number.

The number $\sqrt[4]{27}$ is an irrational number and a real number, but it is not a counting number, a whole number, an integer, or a rational number.

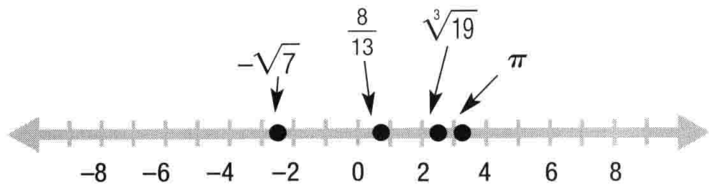
The number $\frac{9}{37}$ is a rational number and a real number, but it is not a counting number, a whole number, an integer, or an irrational number.

The rational numbers and the irrational numbers taken together are the real numbers.

Here is one way to picture this:



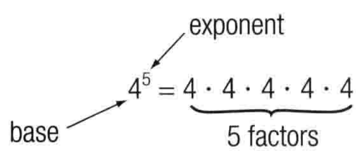
Real numbers can also be represented as points on the number line, like this:



Exponents

Exponents are used to indicate repeated multiplication of the same number.

For example: $4^5 = 4 \cdot 4 \cdot 4 \cdot 4 \cdot 4 = 1024$



In this case, the number 4 is called the base and the number 5 is called the exponent. The exponent 5 indicates that there are 5 factors of 4.

Here is another example of an exponent: $(-2)^3 = (-2)(-2)(-2) = -8$

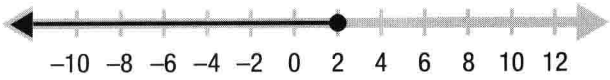
Ordering Symbols

The symbols $=$, $\neq$, $<$, $>$, $\leq$ or $\geq$ are used to compare and order numbers.

For example, $-6 < -2$ means that the number -6 is less than the number -2. That is, -6 lies to the left of -2 on the number line.

Similarly, $8 > 4$ means that 8 is greater than 4. That is, 8 lies to the right of 4 on the number line.

The statement $x \leq 2$ means that the variable x is any real number less than 2 or equal to 2. You can picture this on a number line as follows.



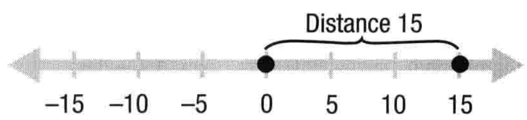
Absolute Value

The absolute value of a number is the distance of that number from zero on the number line. Since absolute value represents a distance, the absolute value of a number is never negative. Vertical bars enclosing a number are used to denote the absolute value of that number.

If a number is positive or zero, then the absolute value of that number is just the original number.

For example: $|15| = 15$

$|0| = 0$

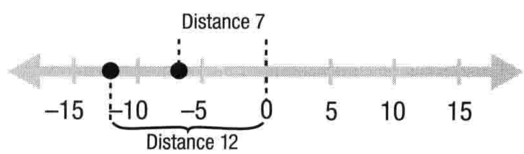


In general, if $x \geq 0$, then $|x| = x$.

If a number is negative, then the absolute value of that number is obtained by multiplying it by negative one. This makes the answer positive.

For example: $|-7| = -1 \cdot (-7) = 7$

$|-12| = -1 \cdot (-12) = 12$



In general, if $x < 0$, then $|x| = -1 \cdot x = -x$.

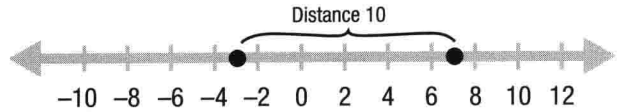
Absolute value can also be used to represent the distance between any two points on the number line.

For example, the distance between –3 and 7 on the number line is given by the absolute value of the difference between –3 and 7. So the distance between –3 and 7 is:

$|(-3) - 7| = |-10| = 10$

Notice that you get the same answer if you take the difference in the opposite order. That is:

$|7 - (-3)| = |10| = 10$



In general, the distance between two numbers a and b on the number line is given by $|a - b|$, which is equal to $|b - a|$.

Operations on Signed Numbers

Signed numbers can be combined using the operations of addition, subtraction, multiplication and division. For example:

$3 - 19 = -16$ $\qquad \frac{5}{6} + \frac{1}{4} = \frac{13}{12}$

$14 \cdot (-7) = -98$ $\qquad (-48) \div (-4) = 12$

To add signed numbers:

If both numbers are positive, their sum is positive.	$2 + 13 = 15$
If both numbers are negative, their sum is negative.	$-8 + (-4) = -12$
If the numbers have different signs, ignore their signs and subtract the smaller number from the larger number. Attach the original sign of the larger number.	$-3 + 7 = 4$ $3 + (-7) = -4$

To subtract signed numbers:

1. Change the subtraction sign (–) to an addition sign (+) and change the sign of the number being subtracted.
2. Add according to the rules for addition of signed numbers.

For example, to find $2 - 6$:

1. Change – to + and change the sign of the number being subtracted. $\qquad 2 + (-6)$
2. Add according to the rules for addition of signed numbers. $\qquad = -4$

To multiply or divide signed numbers:

If both numbers have the same sign, then the answer is positive.	$-3 \cdot (-7) = 21$ $3 \cdot 7 = 21$	$-8 \div (-2) = 4$ $8 \div 2 = 4$
If the numbers have different signs, then the answer is negative.	$-4 \cdot 7 = -28$ $4 \cdot (-7) = -28$	$(-54) \div 9 = -6$ $54 \div (-9) = -6$

Properties of Real Numbers

Below are some properties that will help you simplify calculations with real numbers.

Property	Examples	General Rule (a, b, and c are real numbers)
Commutative Property of Addition	$2 + 13 = 13 + 2$	$a + b = b + a$
Commutative Property of Multiplication	$5 \cdot (-8) = (-8) \cdot 5$	$a \cdot b = b \cdot a$
Associative Property of Addition	$(2 + 7) + 4 = 2 + (7 + 4)$	$(a + b) + c = a + (b + c)$
Associative Property of Multiplication	$(2 \cdot 8) \cdot 5 = 2 \cdot (8 \cdot 5)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Distributive Property	$8 \cdot (3 + 9) = 8 \cdot 3 + 8 \cdot 9$ $7 \cdot (3 - 9) = 7 \cdot 3 - 7 \cdot 9$	$a \cdot (b + c) = a \cdot b + a \cdot c$ $a \cdot (b - c) = a \cdot b - a \cdot c$
Additive Identity Multiplicative Identity	$37 + 0 = 37$ $-29 \cdot 1 = -29$	$a + 0 = a$ $a \cdot 1 = a$
Additive Inverse	$17 + (-17) = 0$ The additive inverse of 17 is –17.	$a + (-a) = 0$
Multiplicative Inverse	$18 \cdot \frac{1}{18} = 1$ The multiplicative inverse of 18 is $\frac{1}{18}$.	$a \cdot \frac{1}{a} = 1$ for $a \neq 0$

The Commutative Property does not work for subtraction or division.

The Associative Property does not work for subtraction or division.

The additive identity is 0.
The multiplicative identity is 1.

The additive inverse of a number is its opposite.

The multiplicative inverse of a nonzero number is its reciprocal.

Order of Operations

When you are simplifying expressions that involve several operations you must perform these operations in the correct order. Here is the order you should use:

1. Perform all operations inside parentheses or brackets.
2. Simplify terms with exponents.
3. Multiply or divide, working from left to right.
4. Add or subtract, working from left to right.

A simple example is: $5 - 3 \cdot 4$

Here you multiply before you add. $= 5 - 12$

$= -7$

*One way to remember the order of operations is to remember the phrase "**P**lease **E**xcuse **M**y **D**ear **A**unt **S**ally." The first letter of each word corresponds to an operation – **P**arentheses, **E**xponents, **M**ultiply or **D**ivide, **A**dd or **S**ubtract.*

Here's another example.

To simplify this expression:	$5 - 4^2 \div 8 - (7 - 5) \cdot 3$
1. Work within the parentheses.	$= 5 - 4^2 \div 8 - \mathbf{2} \cdot 3$
2. Simplify the term with the exponent.	$= 5 - \mathbf{16} \div 8 - 2 \cdot 3$
3. Multiply and divide from left to right.	$= 5 - \mathbf{2} - 2 \cdot 3$
	$= 5 - 2 - \mathbf{6}$
4. Add and subtract from left to right.	$= \mathbf{3} - 6$
	$= \mathbf{-3}$

Sample Problems

1. For each number below, check all of the sets of numbers to which it belongs:

$-7, 0, -\frac{1}{8}, \sqrt[3]{10}, 2, \sqrt{16}, \pi$

☑ a. -7

✓ real numbers
____ irrational numbers
✓ rational numbers
✓ integers
____ whole numbers
____ counting numbers

☐ b. 0

X real numbers
____ irrational numbers
X rational numbers
X integers
X whole numbers
____ counting numbers

☐ c. $-\frac{1}{8}$

X real numbers
____ irrational numbers
X rational numbers
____ integers
____ whole numbers
____ counting numbers

☐ d. $\sqrt[3]{10}$

X real numbers
X irrational numbers
____ rational numbers
____ integers
____ whole numbers
____ counting numbers

Answers to Sample Problems

b. The number 0 belongs to real numbers, rational numbers, integers, and whole numbers.

c. The number $-\frac{1}{8}$ belongs to real numbers and rational numbers.

d. The number $\sqrt[3]{10}$ belongs to real numbers and irrational numbers.

☐ e. 2

_X_ real numbers
____ irrational numbers
_X_ rational numbers
_X_ integers
_X_ whole numbers
_X_ counting numbers

☐ f. $\sqrt{16}$

_X_ real numbers
____ irrational numbers
_X_ rational numbers
_X_ integers
_X_ whole numbers
_X_ counting numbers

☐ g. π

_X_ real numbers
_X_ irrational numbers
____ rational numbers
____ integers
____ whole numbers
____ counting numbers

2. Find the distance on the number line between the two numbers 6 and –5.

☑ a. The distance between a and b is given by $|a - b|$. Let $a = 6$ and $b = -5$.

$|a - b|$
$= |6 - (-5)|$

☑ b. Simplify. $=$ 11

3. Find: $-7 \cdot 8 - 32 \div (-4) + (-7)$

☑ a. Start with the multiplication.

$-7 \cdot 8 - 32 \div (-4) + (-7)$
$= -56 - 32 \div (-4) + (-7)$

☑ b. Do the division. $= -56 -$ 8 $+ (-7)$

☑ c. Do the addition and subtraction from left to right. $=$ –55

Answers to Sample Problems

e. The number 2 belongs to real numbers, rational numbers, integers, whole numbers, and counting numbers.

f. The number $\sqrt{16} = 4$ belongs to real numbers, rational numbers, integers, whole numbers, and counting numbers.

g. The number π belongs to real numbers and irrational numbers.

b. 11

b. (–8)

c. –55