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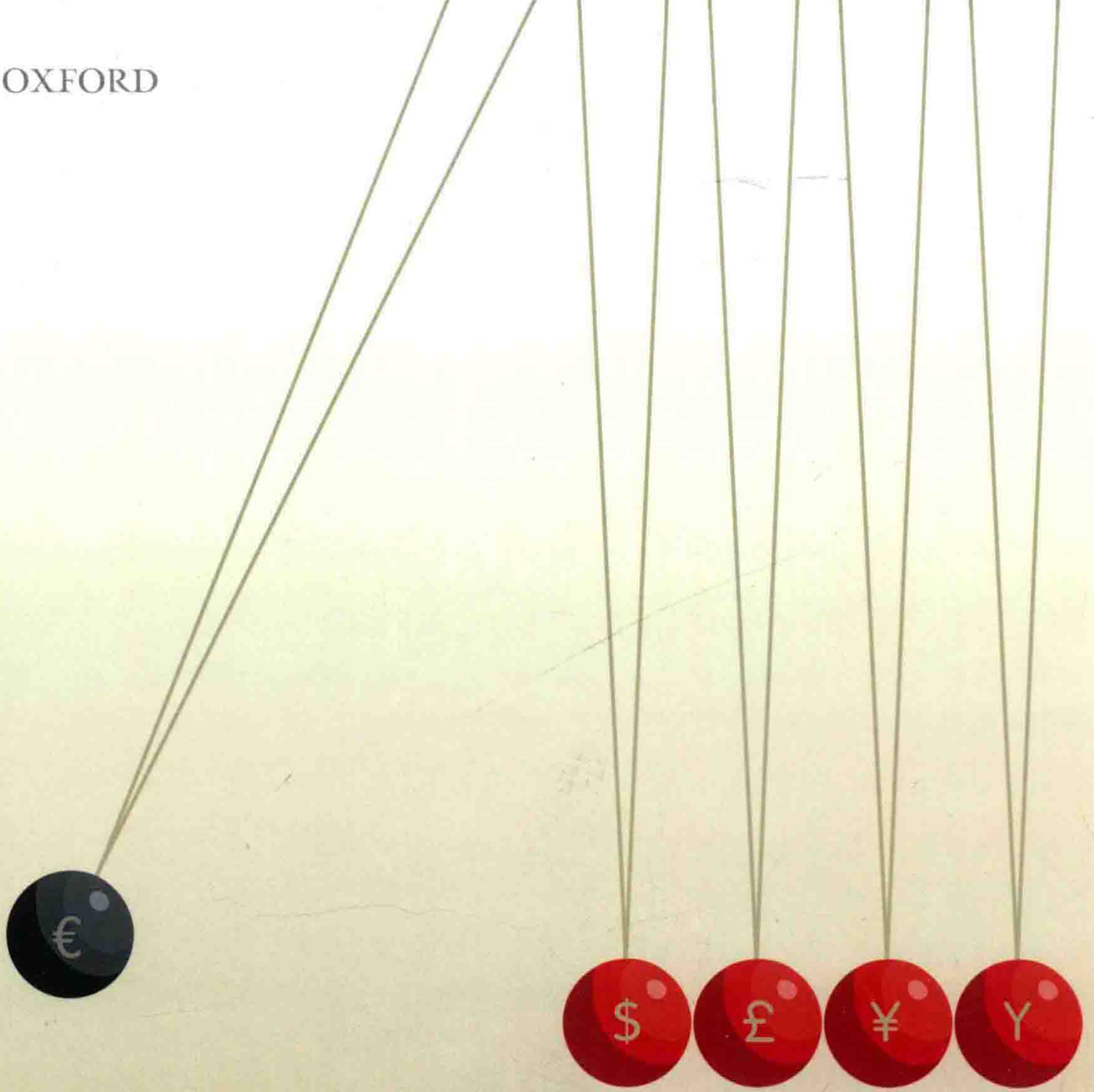

ECONOPHYSICS & PHYSICAL ECONOMICS

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Econophysics and Physical Economics

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Econophysics and Physical Economics

Preface

Financial assets are vital parts of the global economy, and an understanding of the origin and nature of price movements is crucial to management of risk. The sums of money involved can be measured in trillions of euros, dollars, or yen—choose any currency you wish, the figures are huge. Time investments correctly and it is possible to make millions of dollars. Robert Merton of Harvard has pointed out that a dollar invested in 1926 in US Treasury bills would have become 14 dollars by 1996. The same dollar invested in the S&P index would have grown to 1,370 dollars. However if perfect timing each month had been possible and the money switched between these two investment routes the total by 1996 would have become 2,296,183,456 dollars! Undoubtedly, and despite the ongoing financial crisis, the total in 2012 would have become at least ten times this value, as may be estimated by considering only the main three peaks and troughs in the index since 1996. Really clever people might have achieved much more by riding the smaller waves within the booms and crashes!

One should not therefore be surprised that people dream about such gains. Birth dates are widely used to fill out lottery tickets and football pools (and it is probably true that the people who spend money in this way are the people who are least able to afford to lose their stake).

During a recent visit to the London School of Economics, Queen Elizabeth II of England is reported to have asked of the economists why they were not able to predict the financial crash of 2008–9. The news item did not give details of any answer. One suspects no answer was provided. Given this situation it is not surprising that a frequently told joke is that economists disagree with each other so often that they make astrologists look good.

For some years now, many economists have agreed over economic theory. US President Richard Nixon, defending deficit spending against the conservative charge that it was 'Keynesian', is reported to have replied, 'We're all Keynesians now.' According to E. Roy Weintraub, economics professor at Duke University and associate editor of *History of Political Economy,* Nixon should have said 'We're all neoclassical economists now, even the Keynesians, because mainstream economics, the subject taught to students today, is neoclassical economics.' This theory is built on assumptions, such as the rationality of economic agents, the invisible hand, and market efficiency, that have become dogma. Physicist J-P Bouchaud has recently recounted the tale of an economist who once told him, to his bewilderment: 'These concepts are so strong that they supersede any empirical observation.'

That being said, some economists are now beginning to question these approaches and recognize that an economy is better described as a complex system whose properties emerge from the interactions of its individual agents. These agents do not always have complete information on which to make decisions; they cannot always be assumed to be the same ubiquitous 'rational agent' of the neoclassical theory. But there remain strong schools committed to the neoclassical theory which is still widely taught in our universities.

It is impossible to imagine such a situation arising in the natural sciences. Physics and the other natural sciences are empirical disciplines. If empirical observation is incompatible with any model, it is assumed that the model or basic principles are wrong and so must be modified or even abandoned. So many accepted ideas have been proven wrong in the history of physics that physicists can be intensely critical of both their own models and those of others. But why are physicists interested in economics and finance?

Physicists, as we shall see in the introductory chapter, have since the renaissance period, been involved in the development and application of the subject, and if one includes the writings of Aristotle, even since the beginning of science itself in ancient Greece. Leaving it around the time of the industrial revolution, a few returned at the turn of the twentieth century to study the subject as the tools of thermodynamics and statistical mechanics were sharpened. The direct involvement of many physicists in finance really got underway in the 1990s as computers took over the recording of financial activity giving rise to vast amounts of data for analysis and, in the earlier chapters, we shall present some of the work that has been done.

In the second part of the book we outline an approach to economics in a manner analogous to thermodynamics and statistical physics, developed by the nineteenth century scientist Josiah Willard Gibbs and others. The resulting laws apply to assemblies of large numbers of interacting molecules for which experiments may be not be repeated sufficiently enough to obtain any but the most probable results. We argue that similar laws apply to assemblies of interacting economic agents for which repeatable experiments are also not always possible. The theory leads naturally to an understanding of a range of financial and economic phenomena. One central issue, namely that of non-equilibrium, is also discussed by drawing on recent ideas developed to explore the phenomenon in physical systems, which leads to new insights into the distribution functions of the interacting agents. It is our view that this approach, which combines both theory and empiricism, offers scope for further development and application.

There have been many books published by the economic and mathematical community on quantitative finance in recent years. However, these can be difficult to read for a person trained in natural science; the mathematics can also sometimes seem to obscure the basic science involved.

Some important books and numerous articles have been published by members of the physics community in the decade or so since the very first conference concerned with the 'Application of Physics to Financial Analysis' (APFA) was held in Dublin in 1999. In particular, books by Rosario Mantegna and Gene Stanley (2000), Jean-Philip Bouchaud and Marc Potters (2000), and Johannes Voit (2001) sought to bring 'econophysics'—a name coined by Gene Stanley—into the mainstream. The recent book by Sinha *et al.* (2011) extends the scope of these books by also addressing progress in understanding the nature of wealth distributions and topics such as game and network theory.

It was at the time of the first APFA conference that one of us (PR) whilst in Trinity College Dublin, gave a few lectures on the topic, which over the next few years developed into a short course given to undergraduates as an option in their final year. This was supplemented by a short project for some of the course members. As lecture notes were prepared, we became aware that none of the existing books on the subject proved ideal in the sense of being a pedagogic text for students new to the topic. Our overall aim here was to provide a basic introduction to the application of physics to economic and financial systems for such young students, based on both the course given over the past decade in Dublin and complementary lectures delivered by JM at the University of Paderborn and SH at Trinity College Dublin.

The Dublin lectures contributed to a new generation of Irish physics students choosing to take up a career in the financial service industry as the 'Celtic Tiger' began to roar and the Irish economy was booming. Sadly, policy mistakes within government and the banking community and affecting the construction industry ensured the scale of the current crisis in Ireland was greater than it otherwise might have been. However, the nation, and indeed the rest of the world, will in time come out of this crisis. Hopefully by then—ready to face up to other future crises— we shall have a properly trained group of expert advisers within both banking and government, not just in Ireland but across the world.

Acknowledgments

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P. Richmond, J. Mimkes, S. Hutzler
Norwich, Paderborn, Dublin, June 2013

Frequently used symbols

The numbers refer to the *chapter* section where a symbol is first introduced.

k_B	1.4	Boltzmann constant
S	1.4	entropy
$s(t)$	2.1	price as a function of time t
$r(t, \delta t)$	2.1	log price return as a function of time t and time interval δt
$\sigma_r(t, \delta t)$	3.1	instantaneous volatility of log returns
$G(\tau)$	2.1	autocorrelation function
$R(\tau)$	2.1	normalized autocorrelation function
$p(x, t)$	3.1	probability density function
$C_<(X)$	3.1	cumulative distribution function
$C_>(X)$	3.1	complementary cumulative distribution function
$m_1 = m$	3.1	first moment, mean
m_n	3.1	n-th moment
σ	3.1	standard deviation
σ^2	3.1	variance
v_d	3.2	drift velocity
D	3.2	diffusion coefficient
D_n	7.1	generalized diffusion coefficients
P_u	15.1	profit/surplus
Y	15.1	income
C	15.1	costs
S_e	15.3	economic entropy
λ	15.4	dither

Contents

1 Introduction

Send your grain across the seas, and in time you will get a return. Divide your merchandise among seven ventures, eight maybe, since you do not know what disasters may occur on earth.

Ecclesiastes 11:1-2

1.1 Physicists, finance, and economics

Is physics what physicists do? Perhaps, but of all the sciences, physics is also arguably the only science that takes upon itself to explore those areas of the universe where the fundamental laws are unknown.

People are frequently surprised to learn that physicists are applying their subject to finance and economics. One young school leaver who was thinking about studying physics said to one of us a couple of years ago that her 'dad would be relieved to know that the subject I want to study has some practical application'. Many colleagues of physicists who are currently looking at problems in economics, finance, and sociology still sometimes comment that these subjects do not fall within the domain of physics. History teaches us otherwise. Whilst Aristotle, Copernicus, Newton, Huygens, Pascal, and Halley are all well known for their studies of natural phenomena, they are also remembered for important contributions that led to greater understanding and advances in economics and finance, as we shall see below.

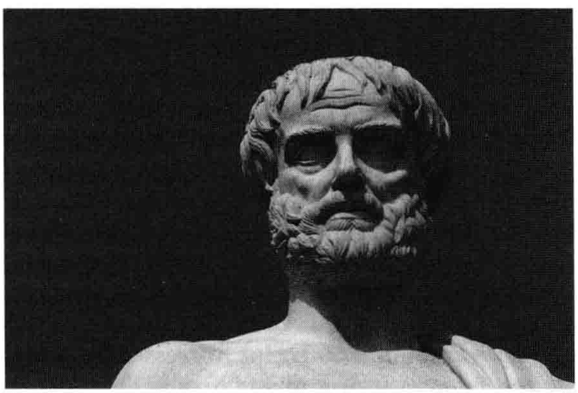

Figure 1.1 The writings of Aristotle (384–322 BC) were important not only for the development of the natural sciences but also for theories of goods exchange and wealth creation. (©iStockphoto.com/sneska).

Figure 1.2 Nicolaus Copernicus (1473–1543) is famous for his development of the heliocentric theory, placing the sun at the centre of our planetary system. He also made important contributions to a theory of money.
(©iStockphoto.com/labsas).

Aristotle (384–322 BC, Figure 1.1) is known for many things. He laid the foundations of logic. He also laid the foundations of physics by suggesting that nature is composed of things that change and that studying these changes is useful. Indeed his treatise was called $\varphi\upsilon\sigma\iota\kappa\alpha$ [physica] from which the word physics originated! His thoughts on economics arose from his ideas on politics and society and these ideas prevailed into the twentieth century (Schumpeter, 1954).

Aristotle argued that in most societies, goods and services are exchanged initially via bartering. A problem arises when one person wants what another has, but does not possess anything that this other person wants in return. The introduction of money, initially in the form of metals such as gold and silver, with an intrinsic value, helps to overcome this situation. The exchange now is not between two different products, but between one product and something of equivalent value. The idea of precious metals as a store of value was firmly entrenched in the minds of the Spanish as they plundered South America and shipped gold back to Spain. Even in 2012 the value of gold benefits from a global financial crisis!

Nicolaus Copernicus (1473–1543, Figure 1.2) is universally acknowledged as the founder of modern astronomy. *De Revolutionibus Orbium Coelestium Libri VI* (Six Books Concerning the Revolutions of the Heavenly Spheres), published in 1543, is one of the great books of science, such as Newton's *Principia* and Darwin's *Origin of Species*.

But Copernicus also achieved great reputation for his work on economics as an adviser to both the Prussian State Parliament and the King of Poland. Copernicus was acutely aware of the economic and social distress caused by wartime inflation. In 1522, he prepared the first draft of a report on the economic problems that arise from debasing the currency. As a result he set out a few basic rules for the issue and maintenance of a sound currency. The final report was used by King Sigismund I of Poland during negotiations for a monetary union between Prussia and Poland in 1528.

Figure 1.3 Sir Isaac Newton (1642–1727) is undoubtedly one of the most influential figures in physics. As Master of the Mint he wrote several reports on the value of gold and silver in various European coins.
(©iStockphoto.com/Tony Bagget).

Prior to this time, most contributions to economics and finance were based on moral interpretations by medieval theologians of previous work of Aristotle. They were concerned with what ought to happen rather than with what actually did happen. Copernicus chose to take a purely empirical and pragmatic approach. Rather than appealing to a priori principles or being guided by a philosophical argument, Copernicus developed his ideas in the manner of a physicist, entirely from empirical studies of economic effects.

It is acknowledged today that the work of Copernicus could have been used as a basis for the assessment of the social and economic problems that occurred even during the mid-twentieth century as well as those of the early sixteenth century (Taylor, 1955).

An interesting assessment of the contributions of Sir Isaac Newton (1642-1727, Figure 1.3) to currency issues and finance can be found in the paper by Shirras and Craig (1945). For thirty one years, Newton was first Warden and then Master of the Royal Mint in London. Writing in 1896, W.A. Shaw notes 'Mathematician and philosopher as he was, it is still a matter for genuine admiration to note the unobtrusive skill and lucidity and modesty with which Newton put himself abreast of the keenest coin traffickers of his time' (Shaw, 1896).

Christiaan Huygens (1629–95, Figure 1.4) is widely known for important work in optics. Less well referenced is his work on games of chance. Using John Graunt's (London) book, *Natural and Political Observations Made upon the Bills of Mortality* published in 1662, Huygens constructed a mortality curve. A detailed discussion of issues of life expectancy is found in an exchange of letters with his younger brother Lodewijk. Christiaan Huygens' work may be seen as setting the foundation for the application of the theory of probability to insurance and annuities (Dahlke *et al.*, 1989).

In 1662 Huygens also published on the mechanics of 'Newton's cradle' (Hutzler *et al.* 2004), pictured on the front cover of this book, and discussed it in terms of

Figure 1.4 Christiaan Huygens (1629–95) established wave optics. He also wrote on probability theory and mortality statistics.
(©iStockphoto.com/ZU_09).

conservation of momentum and kinetic energy. However, it was left to Mandelbrot, 300 years later, to show that a collision process of balls or molecules may serve as a metaphor for the exchange of money between interacting people, or agents. We will return to this in detail when discussing modelling wealth distributions in Section 21.3.

Blaise Pascal (1623–62, Figure 1.5) was a French mathematician, physicist, and religious philosopher who made numerous contributions to science, mathematics, and literature. His earliest work was concerned with the construction of mechanical calculators and he also was interested in fluids, clarifying the concepts of pressure, and vacuum. However, arguably his most important contribution to science arose out of his collaboration with Pierre de Fermat that began in 1654. Together they laid the foundations of probability theory (perhaps motivated by correspondence in 1654 with the writer and gambler Chevalier de Méré, concerning the outcome of throws of dice), now widely used not only in quantum theory but also in economics and social science.

In 1654 Pascal was almost killed when horses pulling his carriage bolted. Convinced that it was God who had saved him, he became a committed Christian and gave up gambling. It was after this life-changing event that he proposed his famous wager that is sometimes used to illustrate game theory: 'How can anyone lose who chooses to become a Christian? If, when he dies, there turns out to be no God and his faith was in vain, he has lost nothing—in fact, he has been happier in life than his non-believing friends. If, however, there is a God and a heaven and hell, then he has gained heaven and his skeptical friends will have lost everything in hell!' (taken from Morris, 1982). In the seventeenth century it would have been difficult, if not impossible, to argue against this wager, and even today, many who believe in God would subscribe to Pascal's view. As the former UK Prime Minister, Margaret Thatcher once said, 'with Christianity one is offered a choice'. Pascal codified the choice in terms of odds and probabilities.

Edmond Halley (1656–1742, Figure 1.6) is well known for his work in astronomy and the discovery of the comet that now bears his name. He is less well-known within the physics community for his work in the area of mortality and life expectancy. In

Figure 1.5 Blaise Pascal (1623–62) made important contributions to the understanding of hydrodynamics. Together with Pierre de Fermat he also established probability theory. (©iStockphoto.com/GoergiosArt).

Figure 1.6 Edmond Halley (1656–1742) was a physicist and astronomer, now best known for the identification of 'Halley's' comet. He also studied life expectancies for the pricing of life annuities.
(©iStockphoto.com/picture).

1693 Halley published an article on life annuities based on an analysis of the age-at-death of citizens in Breslau, a town then in Germany but now renamed Wroclaw and located in Poland. At the time of Halley, the city comprised a relatively closed community with little or no immigration and emigration and the associated data was accurate and complete. Halley's article (Halley, 1693) subsequently enabled the British government to sell life annuities in what was then a novel manner. The price of the annuity could be based on the age of the purchaser rather than being simply a constant! Not surprisingly, Halley's work proved to be a key driver for the later development of actuarial science.