FOURTH EDITION

CALCULUS

WITH ANALYTIC GEOMETRY

EARLY TRANSCENDENTALS VERSION

EDWARDS & PENNEY

FOURTH EDITION

Calculus

with Analytic Geometry

Early Transcendentals Version

C. H. Edwards, Jr.
The University of Georgia, Athens

David E. Penney
The University of Georgia, Athens

PRENTICE HALL Inc.
Englewood Cliffs, New Jersey 07632

Acquisitions Editor: Jacqueline Wood
Developmental Editor: Maurice Esses
Production Editor: Kelly Dickson
Cover Design: Carole Giguère
Cover Image: Michael Portman
Page Layout: Andrew Zutis, Karen Noferi
Text Composition: Interactive Composition Corporation

Printed and bound in the U. S. A.
1 2 3 4 5 98 97 96 95 94

ISBN 0-13-300575-5

Prentice-Hall Inc., Englewood Cliffs, New Jersey
Prentice-Hall International (UK) Limited, London
Prentice-Hall of Australia Pty. Limited, Sydney
Prentice-Hall Hispanoamericana, S.A., Mexico
Prentice-Hall of India Private Limited, New Delhi
Prentice-Hall of Japan, Inc., Tokyo
Simon & Schuster Asia Private Limited., Singapore
Editora Prentice-Hall do Brasil, Ltda., Rio de Janeiro

Acknowledgments

All experienced textbook authors know the value of critical reviewing during the preparation and revision of a manuscript. In our work on various editions of this book, we have benefited greatly from the advice (and frequently the consent) of the following exceptionally able reviewers:

Leon E. Arnold, Delaware County Community College
H. L. Bentley, University of Toledo
Michael L. Berry, West Virginia Wesleyan College
William Blair, Northern Illinois University
George Cain, Georgia Institute of Technology
Wil Clarke, Atlantic Union College
Peter Colwell, Iowa State University
James W. Daniel, University of Texas at Austin
Robert Devaney, Boston University
Dan Drucker, Wayne State University
William B. Francis, Michigan Technological University
Dianne H. Haber, Westfield State College
John C. Higgins, Brigham Young University
W. Cary Huffman, Loyola University of Chicago
Calvin Jongsma, Dordt College
Louise E. Knouse, Le Tourneau College
Morris Kalka, Tulane University
Catherine Lilly, Westfield State College
Joyce Longman, Villanova University
E. D. McCune, Stephen F. Austin State University
Arthur L. Moser, Illinois Central College
Barbara Moses, Bowling Green University
Barbara L. Osofsky, Rutgers University at New Brunswick
John Petro, Western Michigan University
Wayne B. Powell, Oklahoma State University
James P. Qualey, Jr., University of Colorado
Thomas Roe, South Dakota State University
Lawrence Runyan, Shoreline Community College
William L. Siegmann, Rensselaer Polytechnic Institute
John Spellman, Southwest Texas State University
Virginia Taylor, University of Lowell
Samuel A. Truitt, Jr., Middle Tennessee State University
Robert Urbanski, Middlesex County College
Robert Whiting, Villanova University
Cathleen M. Zucco, LeMoyne College

Many of the best improvements that have been made must be credited to colleagues and users of the first three editions throughout the United States, Canada, and abroad. We are grateful to all those, especially students, who have written to us, and hope they will continue to do so. We thank Terri Bittner of Laurel Tutoring (San Carlos, CA) who with her staff checked the accuracy of every example solution and odd-numbered answer. We also believe that the quality of the finished book itself is adequate testimony of the skill, diligence, and talent of an exceptional staff at Prentice Hall; we owe special thanks to George Lobell, mathematics editor; Karen Karlin, developmental editor, Ed Thomas, production editor; Andrew Zutis, designer; and Network Graphics who did the illustrations. Finally, we again are unable to thank Alice Fitzgerald Edwards and Carol Wilson Penney adequately for their continued assistance, encouragement, support, and patience.

C.H.E., Jr. D.E.P.

Athens, Georgia

ACCURACY

Our coverage of calculus is complete (though we hope it is somewhat less than encyclopedic). Still more than its predecessors, this edition was subjected to a comprehensive reviewing process to help ensure accuracy. For example, essentially every problem answer appearing in the Answers Section at the back of the book in this edition has been verified using *Mathematica*. With regard to the selection and sequence of mathematical topics, our approach is traditional. However, close examination of the treatment of standard topics may betray our own participation in the current movement to revitalize the teaching of calculus. We continue to favor an intuitive approach that emphasizes both conceptual understanding and care in the formulation of definitions and key concepts of calculus. Some proofs that may be omitted at the discretion of the instructor are placed at the ends of sections, and others are deferred to the book's appendices. In this way we leave ample room for variation in seeking the proper balance between rigor and intuition.

Supplementary Material

Answers to most of the odd-numbered problems appear in the back of the book. Solutions to most problems (other than those odd-numbered ones for which an answer alone is sufficient) are available in the *Instructor's Solutions Manual.* A subset of this manual, containing solutions to problems numbered 1, 4, 7, 10, · · · is availabe as a *Student's Solutions Manual.* A collection of some 1700 additional problems suitable for use as test questions, the *Calculus Test Item File,* is available (in both electronic and hard copy form) for use by instructors. Finally, an *Instructor's Edition* including section-by-section teaching outlines and suggestions is available to those who are using this book to teach calculus.

STUDENT LAB MANUALS

A variety of additional supplements are provided by the publisher, including author-written project manuals keyed to specific calculators and computer systems. These include

Calculus Projects Using Derive
Calculus Projects Using HP Graphics Calculators
Calculus Projects Using Maple
Calculus Projects Using *Mathematica*
Calculus Projects Using MATLAB
Calculus Projects Using TI Graphics Calculators
Calculus Projects Using X(PLORE)

Each of these manuals consists of versions of the text's 48 projects that have been expanded where necessary to take advantage of specific computational technology in the teaching of calculus. Each project is designed to provide the basis for an outside class assignment that will engage students for a period of several days (or perhaps longer). The Derive, Maple, *Mathematica,* and MATLAB manuals are accompanied by command-specific diskettes that will relieve students of much of the burden of typing (by providing "templates" for the principal commands used in each project). In many cases, the diskettes also contain additional discussion and examples.

Maintaining Traditional Strengths

While many new features have been added, five related objectives remained in constant view: concreteness, readability, motivation, applicability, and accuracy.

CONCRETENESS

The power of calculus is impressive in its precise answers to realistic questions and problems. In the necessary conceptual development of the subject, we keep in sight the central question: How does one actually *compute* it? We place special emphasis on concrete examples, applications, and problems that serve both to highlight the development of the theory and to demonstrate the remarkable versatility of calculus in the investigation of important scientific questions.

READABILITY

Difficulties in learning mathematics often are complicated by language difficulties. Our writing style stems from the belief that crisp exposition, both intuitive and precise, makes mathematics more accessible—and hence more readily learned—with no loss of rigor. We hope our language is clear and attractive to students and that they can and actually will read it, thereby enabling the instructor to concentrate class time on the less routine aspects of teaching calculus.

MOTIVATION

Our exposition is centered around examples of the use of calculus to solve real problems of interest to real people. In selecting such problems for examples and exercises, we took the view that stimulating interest and motivating effective study go hand in hand. We attempt to make it clear to students how the knowledge gained with each new concept or technique will be worth the effort expended. In theoretical discussions, especially, we try to provide an intuitive picture of the goal before we set off in pursuit of it.

APPLICATIONS

Its diverse applications are what attract many students to calculus, and realistic applications provide valuable motivation and reinforcement for all students. This book is well-known for the broad range of applications that we include, but it is neither necessary nor desirable that the course cover all the applications in the book. Each section or subsection that may be omitted without loss of continuity is marked with an asterisk. This provides flexibility for each instructor to determine his or her own flavor and emphasis.

functions is available for use in examples and problems throughout the balance of differential calculus (Chapters 3 and 4) and in the study of integral calculus (Chapters 5 and 6). Chapter 7 returns to exponential and logarithmic functions, offering a more complete and rigorous treatment plus further applications. But Section 7.2—based on the formal definition of the logarithm as an integral—actually can be covered anytime after the integral has been defined early in Chapter 5 (along with as much of the remainder of Chapter 7 as the instructor desires). Thus this version of the text is designed to support a course syllabus that includes exponential functions early in differential calculus, as well as logarithmic functions (defined as integrals) early in integral calculus.

The remaining transcendental functions—inverse trigonometric and hyperbolic—are now treated in Chapter 8. This newly organized chapter now includes also indeterminate forms and l'Hopital's rule (much earlier than in the third edition).

STREAMLINING TECHNIQUES OF INTEGRATION Chapter 9 is organized to accommodate those instructors who feel that methods of formal integration now require less emphasis, in view of modern techniques for both numerical and symbolic integration. Presumably everyone will want to cover the first four sections of the chapter (through integration by parts in Section 9.4.). The method of partial fractions appears in Section 9.5, and trigonometric substitutions and integrals involving quadratic polynomials follow in Section 9.6 and 9.7. Improper integrals now appear in Section 9.8, and the more specialized rationalizing substitutions have been relegated to the Chapter 9 miscellaneous problems. This rearrangement of Chapter 9 makes it more convenient to stop wherever the instructor desires.

INFINITE SERIES After the usual introduction to convergence of infinite sequences and series in Section 11.2 and 11.3, a combined treatment of Taylor polynomials and Taylor series appears in Section 11.4. This makes it possible for the instructor to experiment with a much briefer treatment of infinite series, but still offer exposure to the Taylor series that are so important for applications.

DIFFERENTIAL EQUATIONS Many calculus instructors now believe that differential equations should be seen as early and as often as possible. The very simplest differential equations (of the form $y' = f(x)$) appear in a subsection at the end of Section 5.2 (Antiderivatives). Section 6.5 illustrates applications of integration to the solution of separable differential equations. Section 9.5 includes applications of the method of partial fractions to population problems and the logistic equation. In such ways we have distributed enough of the spirit and flavor of differential equations throughout the text that it seemed expeditious to eliminate the (former) final chapter devoted solely to differential equations. However, those who so desire can arrange with the publisher to obtain for supplemental use appropriate sections of Edwards and Penney, *Elementary Differential Applications with Boundary Value Problems*, third edition (Englewood Cliffs, N.J.: Prentice-Hall, 1993).

INTRODUCTORY CHAPTERS Chapters 1 and 2 have been streamlined for a leaner and quicker start on calculus. Chapter 1 concentrates on functions and graphs. It now includes a section cataloging the elementary functions of calculus, and provides a foundation for an earlier emphasis on transcendental functions. Chapter 1 now concludes with a section addressing the question "What *is* calculus?" Chapter 2 on limits begins with a section on tangent lines to motivate the official introduction of limits in Section 2.2. In contrast with the third edition, trigonometric limits now are treated throughout Chapter 2, in order to encourage a richer and more visual introduction to the limit concept.

DIFFERENTIATION CHAPTERS The sequence of topics in Chapters 3 and 4 varies a bit from the most traditional order. We attempt to build student confidence by introducing topics more nearly in order of increasing difficulty. The chain rule appears quite early (in Section 3.3) and we cover the basic techniques for differentiating algebraic functions before discussing maxima and minima in Sections 3.5 and 3.6. The appearance of inverse functions is now delayed until Chapter 7. Section 3.7 now treats the derivatives of all six trigonometric functions. Implicit differentiation and related rates are combined in a single section (Section 3.8). The mean value theorem and its applications are deferred to Chapter 4. Sections 4.4 on the first derivative test and 4.6 on higher derivatives and concavity have been simplified and streamlined. A great deal of new graphic material has been added in the curve-sketching sections that conclude Chapter 4.

INTEGRATION CHAPTERS New and simpler examples have been inserted throughout Chapters 5 and 6. Antiderivatives (formerly at the end of Chapter 4) now begin Chapter 5. Section 5.4 (Riemann sums) has been simplified greatly, with upper and lower sums eliminated and endpoint and midpoint sums emphasized instead. Many instructors now believe that first applications of integration ought not be confined to the standard area and volume computations; Section 6.5 is an optional section that introduces separable differential equations. To eliminate redundancy, the material on centroids and the theorems of Pappus is delayed to Chapter 15 (Multiple Integrals) where it can be treated in a more natural context.

EARLY TRANSCENDENTAL FUNCTIONS OPTIONS In the "regular version" of this book the appearance of exponential and logarithmic functions is delayed until Chapter 7 (after integration). In the present "early transcendental version" these functions are introduced at the earliest practical opportunity in differential calculus—in Section 3.8, immediately following the differentiation of trigonometric functions in Section 3.7. Section 3.8 begins with an intuitive approach to exponential functions regarded as variable powers of a constant base, followed by the elementary idea of a logarithm as "the power to which the base a must be raised to get the number x". On this basis, the section includes a low-key review of the laws of exponents and of logarithms, and investigates somewhat informally the differentiation of exponential and logarithmic functions. Consequently, a diverse collection of transcendental

Fourth Edition Features

In preparing this edition, we have benefited from many valuable comments and suggestions from users of the first three editions. This revision was so pervasive that the individual changes are too numerous to be detailed in a preface, but the following paragraphs summarize those that may be of widest interest.

ADDITIONAL PROBLEMS The number of problems has steadily increased since the first edition, and now totals well over 6000. In the third edition we inserted many additional practice exercises near the beginnings of problem sets to insure that students gain sufficient confidence and computational skill before moving on to the more conceptual problems that constitute the real goal of calculus. In this edition we have added graphics-based problems that emphasize conceptual understanding and accommodate student use of graphics calculators.

NEW EXAMPLES AND COMPUTATIONAL DETAILS In many sections throughout this edition, we have inserted a simpler first example or have replaced existing examples with ones that are computationally simpler. Moreover, we have inserted an additional line or two of computational detail in many of the worked-out examples to make them easier for student readers to follow. The purpose of these computational changes is to make the computations themselves less of a barrier to conceptual understanding.

PROJECT MATERIAL Several supplementary projects have been inserted in each chapter—a total of four dozen in all. Each project employs some aspect of modern computational technology to illustrate the principal ideas of the preceding section, and typically contains additional problems intended for solution with the use of a graphics calculator or computer. Figures and data illustrate the use of graphics calculators and computer systems such as Derive, Maple, and *Mathematica.* This project material is suitable for use in a computer/calculator lab that is conducted in association with a standard calculus course, perhaps meeting weekly. It can also be used as a basis for graphics calculator or computer assignments that students will complete outside of class, or for individual study.

COMPUTER GRAPHICS Now that graphics calculators and computers are here to stay, an increased emphasis on graphical visualization along with numeric and symbolic work is possible as well as desirable. About 300 new MATLAB-generated figures illustrate the kind of figures students using graphics calculators can produce for themselves. Many of these are included with new graphical problem material. *Mathematica*-generated color graphics are included to highlight all sections involving 3-dimensional material.

HISTORICAL MATERIAL Historical and biographical chapter openings have been inserted to give students a sense of the development of our subject by real, live human beings. Both authors are fond of the history of mathematics, and believe that it can favorably influence our teaching of mathematics. For this reason, numerous historical comments appear in the text itself.

Preface

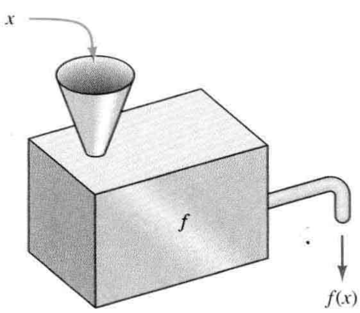
The role and practice of mathematics in the world at large is now undergoing a revolution that is driven largely by computational technology. Calculators and computer systems provide students and teachers with mathematical power that no previous generation could have imagined. We read even in daily newspapers of stunning current events like the recently announced proof of Fermat's last theorem. Surely *today* is the most exciting time in all history to be mathematically alive! So in preparing this new edition of ***CALCULUS with Analytic Geometry***, we wanted first of all to bring at least some sense of this excitement to the students who will use it.

We also realize that the calculus course is a principal gateway to technical and professional careers for a still increasing number of students in an ever widening range of curricula. Wherever we look—in business and government, in science and technology—almost every aspect of professional work in the world involves mathematics. We therefore have re-thought once again the goal of providing calculus students the solid foundation for their subsequent work that they deserve to get from their calculus textbook.

For the first time since the original version was published in 1982, the text for this fourth edition has been reworked from start to finish. Discussions and explanations have been rewritten throughout in language that (we hope) today's students will find more lively and accessible. Seldom-covered topics have been trimmed to accommodate a leaner calculus course. Historical and biographical notes have been added to show students the human face of calculus. Graphics calculator and computer lab projects (with Derive, Maple, and *Mathematica* options) for key sections throughout the text have been added. Indeed, a new spirit and flavor reflecting the prevalent interest in graphics calculators and computer systems will be discernible throughout this edition. Consistent with the graphical emphasis of the current calculus reform movement, the number of figures in the text has been almost doubled, with must new computer-generated artwork added. Many of these additional figures serve to illustrate a more deliberative and exploratory approach to problem-solving. Our own teaching experience suggests that use of contemporary technology can make calculus more concrete and accessible to many students.

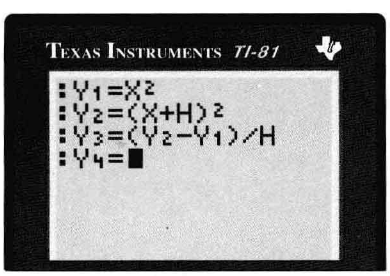
Contents

Preface x



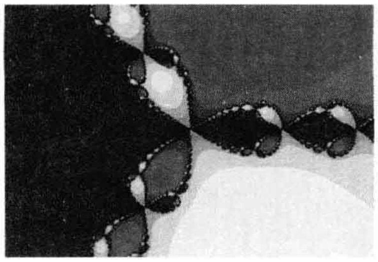
CHAPTER *1* **Functions and Graphs** 1

1.1 Functions and Real Numbers 2
PROJECTS 13
1.2 The Coordinate Plane and Straight Lines 14
1.3 Graphs of Equations and Functions 23
PROJECTS 31
1.4 A Brief Catalog of Functions 33
PROJECTS 42
1.5 A Preview: What *Is* Calculus? 42
REVIEW: DEFINITIONS, CONCEPTS, RESULTS 46



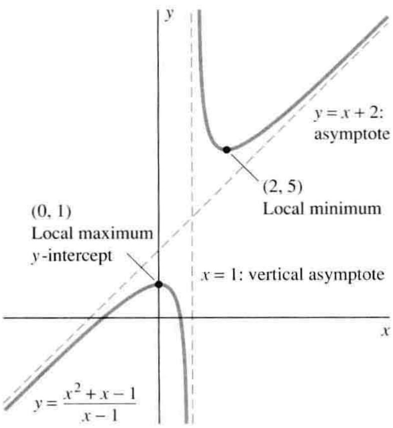
CHAPTER *2* **Prelude to Calculus** 49

2.1 Tangent Lines and the Derivative—A First Look 50
PROJECT 59
2.2 The Limit Concept 59
PROJECT 70
2.3 More About Limits 71
2.4 The Concept of Continuity 81
PROJECTS 91
REVIEW: DEFINITIONS, CONCEPTS, RESULTS 92



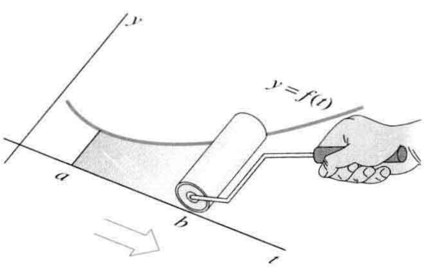
CHAPTER 3 The Derivative 94

3.1 The Derivative and Rates of Change 95
PROJECT 106
3.2 Basic Differentiation Rules 107
3.3 The Chain Rule 118
3.4 Derivatives of Algebraic Functions 125
3.5 Maxima and Minima of Functions on Closed Intervals 131
PROJECT 139
3.6 Applied Maximum-Minimum Problems 140
PROJECTS 154
3.7 Derivatives of Trigonometric Functions 155
3.8 Exponential and Logarithmic Functions 164
PROJECT 174
3.9 Differentiation and Related Rates 175
3.10 Successive Approximations and Newton's Method 183
PROJECTS 194
REVIEW: FORMULAS, CONCEPTS, DEFINITIONS 196



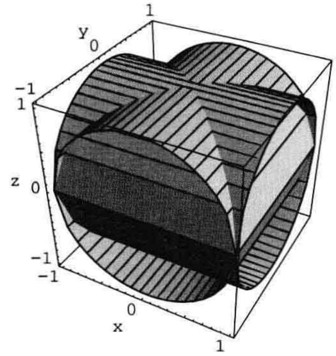
CHAPTER 4 Additional Applications of the Derivative 201

4.1 Introduction 202
4.2 Increments, Differentials, and Linear Approximation 202
4.3 Increasing and Decreasing Functions and the Mean Value Theorem 209
4.4 The First Derivative Test 220
PROJECTS 230
4.5 Simple Curve Sketching 231
PROJECTS 238
4.6 Higher Derivatives and Concavity 239
PROJECTS 253
4.7 Curve Sketching and Asymptotes 254
REVIEW: DEFINITIONS, CONCEPTS, RESULTS 263



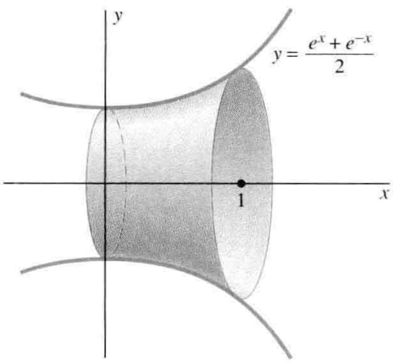
CHAPTER 5 The Integral 266

5.1 Introduction 267
5.2 Antiderivatives and Initial Value Problems 267
5.3 Elementary Area Computations 280
5.4 Riemann Sums and the Integral 291
PROJECTS 299
5.5 Evaluation of Integrals 301
5.6 Average Values and the Fundamental Theorem of Calculus 308
5.7 Integration by Substitution 318
5.8 Areas of Plane Regions 325
PROJECTS 334
5.9 Numerical Integration 335
PROJECTS 347
REVIEW: DEFINITIONS, CONCEPTS, RESULTS 348



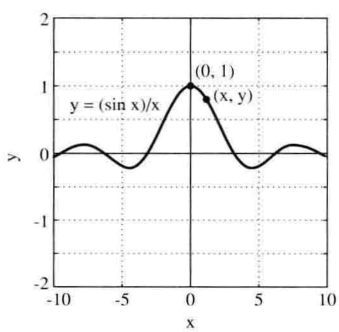
CHAPTER 6 Applications of the Integral 352

6.1 Setting Up Integral Formulas 353
6.2 Volumes by the Method of Cross Sections 360
PROJECT 371
6.3 Volume by the Method of Cylindrical Shells 372
PROJECT 379
6.4 Arc Length and Surface Area of Revolution 379
PROJECT 387
6.5 Separable Differential Equations 388
6.6 Force and Work 395
REVIEW: Definitions, Concepts, Results 405



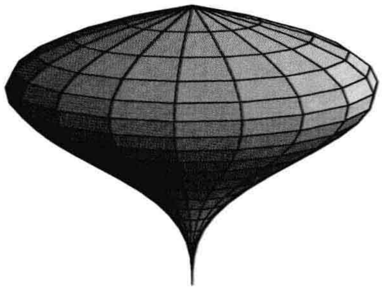
CHAPTER 7 More Exponential and Logarithmic Functions 409

7.1 Exponentials, Logarithms, and Inverse Functions 410
7.2 The Natural Logarithm 410
PROJECT 419
7.3 The Exponential Function 420
PROJECT 427
7.4 General Exponential and Logarithmic Functions 428
PROJECT 433
7.5 Natural Growth and Decay 434
*7.6 Linear First-Order Differential Equations and Applications 442
REVIEW: Definitions, Concepts, Results 448



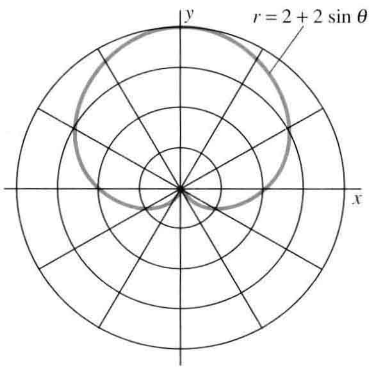
CHAPTER 8 Further Calculus of Transcendental Functions 452

8.1 Introduction 453
8.2 Inverse Trigonometric Functions 453
8.3 Indeterminate Forms and L'Hôpital's Rule 462
PROJECT 467
8.4 Additional Indeterminate Forms 468
8.5 Hyperbolic Functions and Inverse Hyperbolic Functions 472
PROJECT 481
REVIEW: Definitions and Formulas 482



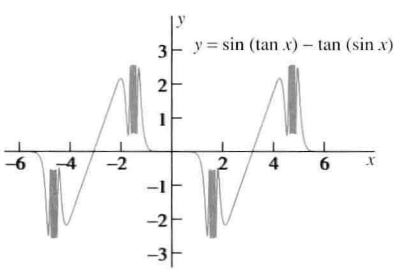
CHAPTER *9* Techniques of Integration 484

9.1 Introduction 485
9.2 Integral Tables and Simple Substitutions 485
PROJECT 488
9.3 Trigonometric Integrals 489
9.4 Integration by Parts 496
9.5 Rational Functions and Partial Fractions 503
PROJECT 511
9.6 Trigonometric Substitution 512
9.7 Integrals Containing Quadratic Polynomials 518
9.8 Improper Integrals 523
PROJECT 531
SUMMARY 532



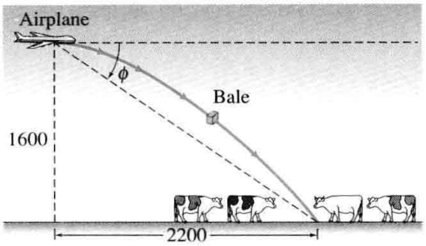
CHAPTER *10* Polar Coordinates and Conic Sections 537

10.1 Analytic Geometry and the Conic Sections 538
10.2 Polar Coordinates 543
PROJECTS 550
10.3 Area Computations in Polar Coordinates 551
PROJECT 555
10.4 The Parabola 556
10.5 The Ellipse 560
10.6 The Hyperbola 565
10.7 Rotation of Axes and Second-Degree Curves 571
REVIEW: PROPERTIES OF CONIC SECTIONS 576



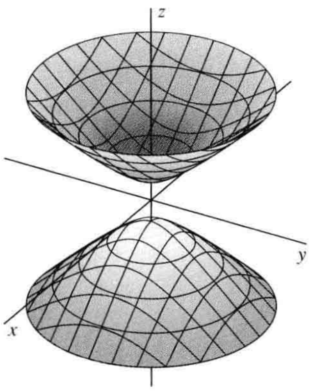
CHAPTER *11* Infinite Series 579

11.1 Introduction 580
11.2 Infinite Sequences 580
11.3 Infinite Series and Convergence 588
PROJECTS 597
11.4 Taylor Series and Taylor Polynomials 599
PROJECT 612
11.5 The Integral Test 612
PROJECT 618
11.6 Comparison Tests for Positive-Term Series 619
11.7 Alternating Series and Absolute Convergence 624
11.8 Power Series 632
PROJECT 642
11.9 Power Series Computations 643
PROJECT 648
REVIEW: DEFINITIONS, CONCEPTS, RESULTS 650



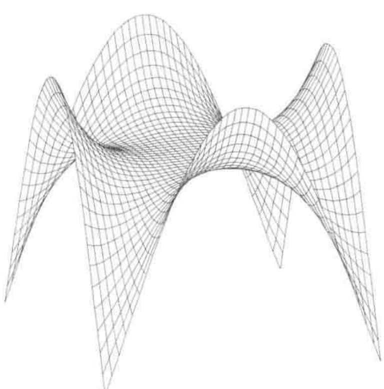
CHAPTER *12* Parametric Curves and Vectors in the Plane 653

12.1 Parametric Curves 654
PROJECTS 661
12.2 Integral Computations with Parametric Curves 663
PROJECTS 668
12.3 Vectors in the Plane 668
12.4 Motion and Vector-Valued Functions 676
*__12.5__ Orbits of Planets and Satellites 684
REVIEW: DEFINITIONS AND CONCEPTS 690



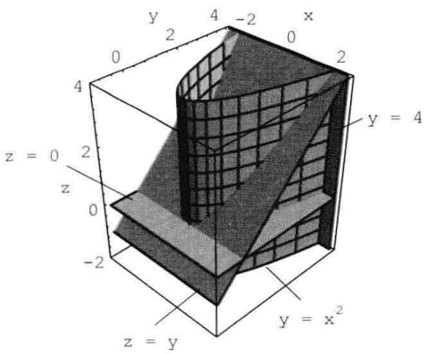
CHAPTER *13* Vectors, Curves, and Surfaces in Space 692

13.1 Rectangular Coordinates and Three-Dimensional Vectors 693
13.2 The Vector Product of Two Vectors 701
13.3 Lines and Planes in Space 710
13.4 Curves and Motion in Space 716
PROJECT 721
13.5 Curvature and Acceleration 722
13.6 Cylinders and Quadric Surfaces 733
13.7 Cylindrical and Spherical Coordinates 742
PROJECT 748
REVIEW: DEFINITIONS, CONCEPTS, RESULTS 749



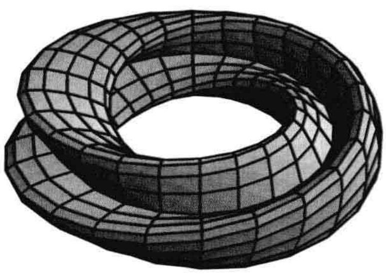
CHAPTER *14* Partial Differentiation 753

14.1 Introduction 754
14.2 Functions of Several Variables 754
PROJECTS 762
14.3 Limits and Continuity 763
14.4 Partial Derivatives 768
14.5 Maxima and Minima of Functions of Several Variables 776
PROJECT 786
14.6 Increments and Differentials 788
14.7 The Chain Rule 794
14.8 Directional Derivatives and the Gradient Vector 802
14.9 Lagrange Multipliers and Constrained Maximum-Minimum Problems 811
PROJECTS 820
14.10 The Second Derivative Test for Functions of Two Variables 822
PROJECT 829
REVIEW: DEFINITIONS, CONCEPTS, RESULTS 830



CHAPTER *15* Multiple Integrals 833

15.1 Double Integrals 833
PROJECT 839
15.2 Double Integrals over More General Regions 840
15.3 Area and Volume by Double Integration 846
15.4 Double Integrals in Polar Coordinates 852
15.5 Applications of Double Integrals 860
PROJECT 870
15.6 Triple Integrals 871
15.7 Integration in Cylindrical and Spherical Coordinates 878
PROJECT 885
15.8 Surface Area 886
PROJECTS 892
*15.9 Change of Variables in Multiple Integrals 893
REVIEW: DEFINITIONS, CONCEPTS, RESULTS 901



CHAPTER *16* Vector Analysis 904

16.1 Vector Fields 905
16.2 Line Integrals 910
16.3 Independence of Path 919
16.4 Green's Theorem 926
16.5 Surface Integrals 934
PROJECTS 942
16.6 The Divergence Theorem 944
16.7 Stokes' Theorem 951
REVIEW: DEFINITIONS, CONCEPTS, RESULTS 958

Appendices A-1

A Review of Trigonometry A-1
B Proofs of the Limit Laws A-7
C The Completeness of the Real Number System A-12
D Proof of the Chain Rule A-17
E Existence of the Integral A-18
F Approximations and Riemann Sums A-24
G L'Hôpital's Rule and Cauchy's Mean Value Theorem A-28
H Proof of Taylor's Formula A-30
I Units of Measurement and Conversion Factors A-31
J Formulas from Algebra, Geometry, and Trigonometry A-32
K The Greek Alphabet A-34

Answers to Odd-Numbered Problems A-35

References for Further Study A-70

Index A-71

Photo Credits

p. 1 Giraudon **p. 49** Hulton/Bettman; David Exton/The Science Museum, London; U.S. Navy **p. 94** Culver Pictures **p. 153** Richard Megna/Fundamental Photographs **p. 266** Culver Pictures **p. 276** Richard Megna/Fundamental Photographs **p. 352** Culver Pictures **p. 397** Focus on Sports **p. 400** Luis Villota/Stock Market **p. 409** courtesy of Ross Karlin **p. 436** Jean-Mark Barey, Agence Vandystadt/Photo Researchers **p. 452** Granger Collection **p. 482** Wayne Eastep/Stock Market **p. 484** New York Public Library Picture Collection **p. 537** Stephen Gerard/Photo Researchers; Stephen Gerard/Photo Researchers **p. 559** Paul Silverman **p. 579** University of Cambridge **p. 653** New York Public Library Picture Collection; Granger Collection **p. 656** Richard Megna/Fundamental Photographs **p. 692** New York Public Library Picture Collection; Gene Bayers/Focus on Sports **p. 753** New York Public Library Picture Collection **p. 757** courtesy of Trails Illustrated **p. 833** Bibliotheque Nationale; Tony Tomsis/Sports Illustrated **p. 860** Richard Megna/Fundamental Photographs **p. 904** Bettmann; Jullian Baum, Science Photo/Photo Researchers **p. 905** Takeshi Takahara/Photo Researchers **p. 940** NASA, Science Source/Photo Researchers