FUNDAMENTALS TECHNICAL MATHEMATICS WITH CALCULUS

SECOND EDITION

ARTHUR D. KRAMER

Fundamentals of Technical Mathematics with Calculus

Second Edition

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Dedicated to my dad, Joseph

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Preface

Fundamentals of Technical Mathematics with Calculus is primarily designed for students who are preparing for technical or scientific careers. This new edition has been substantially improved over the first edition. There is a more logical arrangement of chapters and a new chapter on matrices. Many new useful examples and exercises have been added, and more illustrations to clarify ideas have been included. Most important, the presentation and format for the text, the examples, and the exercises have been made more effective for the instructor to work with and for the student to understand.

While it is desirable for the student to have some background in elementary algebra, it is not essential. Chapter 2 includes a thorough treatment that can be used as a review or an introduction to algebra. The text provides comprehensive coverage of the mathematics necessary for the beginning student, starting with a review of arithmetic in Chapter 1, up to the calculus level student. The approach stresses a working knowledge of mathematics and the application of ideas to solving technical and practical problems.

Clearly, mathematics is best learned by doing many exercises and problems. The text now contains over 1100 worked-out examples and over 5500 exercises and problems. As much as possible, I have used meaningful applications taken from various scientific, technical, and practical areas, with an emphasis on electronics and computers. The applications do not require any prior knowledge of a specific subject and serve to develop an understanding of where and how mathematics is used in many fields. Many of the exercises relate back to the worked-out examples, which help the student to do the problems and reinforce the ideas in the text.

Each odd-numbered problem is similar to the following even-numbered problem, and the odd-numbered answers are given in Appendix C so they may be used for self-checking. Even-numbered answers are available in the *Instructor's Manual and Key*. Every chapter is followed by a series of review questions which not only cover the important chapter ideas but also strive to integrate several of them within some of the problems.

The chapters and chapter sections are arranged in a carefully thought-out, logical order so that ideas flow smoothly from one topic to another. Chapters 1, 2, 3, and 4 cover, respectively, basic material in arithmetic, algebra, varia-

tion, and geometry. Any topics in these chapters can be integrated into the course or used for review or reference. The metric system is presented in Chapter 4 and used equally throughout the text with the U.S. Customary system.

Chapters 5, 6, 7, 8, and 9 form a sequence of algebraic topics progressing from linear equations to logarithms. Chapters 10, 11, and 12 form a sequence of trigonometric topics from the basic trigonometric functions through graphs and identities. Chapters 13 to 16 contain further material in preparation for the calculus sequence, Chapters 19 to 25. Chapter 17 introduces inequalities and linear programming, topics of increasing importance in the technical world. Chapter 18, an introduction to computers and BASIC, is designed to augment other BASIC material; it presents applications to technology not usually found in introductory BASIC texts.

Many of the chapters can be studied independently, as there are cross-references to aid in their understanding. The chapter sequence is therefore flexible and can be arranged to meet various student needs. Calculator sections are presented at the end of those chapters which introduce calculator functions: Chapters 1, 2, 4, 9, and 10. These sections use examples in the chapter and are designed to reinforce understanding and strengthen the ability to estimate results and determine errors. They can be studied separately or integrated into the chapter material. In addition to these sections, many problems which lend themselves to calculator usage are marked with the symbol [7].

I would like to thank the many users of the first edition whose suggestions have helped shape this second edition. Particular thanks to my colleagues Ken Stelzig of Chippewa Valley Technical College, Sandra Beken of Horry Georgetown Technical College, and Gary Simundza of Wentworth Institute of Technology for their thorough reviews of the manuscript and many useful comments. Last but not least, thanks again to my many students who are an inspiration to me and whose needs provide an invaluable guide in the writing of the text.

ARTHUR D. KRAMER

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Review of Arithmetic

1-1 Laws of Arithmetic

Two types of laws apply to the operations of addition and multiplication. The first type states that it makes no difference in what order you add or multiply

A thorough understanding of arithmetic is essential for a good grasp of basic mathematics including algebra and geometry. Sections 1-1 to 1-4 are designed to reinforce your ability in arithmetic. Section 1-5 discusses arithmetic operations on the calculator and will help you estimate answers, interpret results, and identify errors. Section 1-6 explains precision and accuracy, which are important in technical applications. Chapters 2, 4, 9, and 10 introduce other calculator functions (sine, cosine, log, etc.) as the need arises. Finally, Section 1-7, Review Questions, contains many useful exercises which integrate the ideas in the chapter.

two numbers. For example, 2 + 5 = 5 + 2 and $3 \times 4 = 4 \times 3$. These are called the *commutative laws:*

$$A + B = B + A$$

and (1-1)
 $A \times B = B \times A$

The second type states that if three numbers are to be added or multiplied, it makes no difference whether you start the operations with the first and second numbers or with the second and third. For example, in addition, (2 + 3) + 5 = 2 + (3 + 5), or 5 + 5 = 2 + 8. In multiplication, $(3 \times 4) \times 5 = 3 \times 10^{-2}$

 (4×5) , or $12 \times 5 = 3 \times 20$. These are called the associative laws:

$$(A+B) + C = A + (B+C)$$
and
$$(A \times B) \times C = A \times (B \times C)$$

$$(1-2)$$

When we apply these two types of laws together, it follows that three or more numbers can be added or multiplied in any order. For example, 2 + 3 + 4 (or $2 \times 3 \times 4$) can be added (or multiplied) in six different ways:

$$2+3+4$$
 $2+4+3$ $3+2+4$ $3+4+2$ $4+2+3$ $4+3+2$

Another important law of arithmetic which combines multiplication and addition is the distributive law. This law says that multiplication distributes over addition:

$$A \times (B+C) = A \times B + A \times C \tag{1-3}$$

The distributive law is important in algebra and is applied in Chap. 2.

The order of operations in arithmetic, when there are no parentheses, is multiplication or division first, addition or subtraction second. Computers and most scientific calculators are programmed to perform the operations in this order. It is called algebraic logic, or the algebraic operating system.

Example 1-1

Calculate the following:

$$5 \times 21 - 36 + 4 \div 2$$

Solution

Multiply and divide first:

105 - 36 + 269 + 2 = 71

Then subtract and add:

You can test if your calculator uses algebraic logic as follows. Enter this example exactly as it appears above and see if you get 71 when you press the equals key. See Sec. 1-5 for further discussion of calculator operation.

Example 1-2

Calculate the following: $5 \times 21 - (36 + 4) \div 2$

$$5 \times 21 - (36 + 4) \div 2$$

Solution

Perform the operation in parentheses first:

$$5 \times 21 - 40 \div 2 = 105 - 20 = 85$$

See Example 1-20 for the calculator solution of Example 1-2.

Example 1-3

Calculate the following:

$$\frac{5 \times 21 \times 12}{15 \times 7 \times 3}$$

Solution

You can multiply and divide in any order. One way is to multiply across the top and bottom and then to divide:

$$\frac{5 \times 21 \times 12}{15 \times 7 \times 3} = \frac{1260}{315} = 4$$

An easier way is to divide common factors in the top and bottom first and then to multiply:

$$\frac{\frac{1}{\cancel{3}} \times \cancel{2} + \cancel{4}}{\cancel{4} \times \cancel{2} \times \cancel{4} \times \cancel{4}} = \frac{4}{1} = 4$$

See Example 1-21 for the calculator solution of Example 1-3.

Exercise 1-1

In problems 1 to 16, test your understanding of arithmetic by mentally calculating the result. Check by doing the problem by hand. (You can check further with the calculator.)

1.
$$6+5+7+3+5+4$$

2.
$$8+2-3+9-1$$

3.
$$5 \times 2 \times 3 \times 4$$

4.
$$12 \div 3 \div 2 \div 2$$

5.
$$(800 + 20) \div 20$$

6.
$$10 \div (5 + 40) \times 9$$

7.
$$8 + 13 \times 2 - 4$$

8.
$$7-6 \div 3 + 8 \div 4$$

9.
$$5 + (8 - 1) \times 6 \div 2$$

10.
$$(5-1) \div 2 + 3 \times 4$$

11.
$$\frac{9\times 4}{3\times 6} + \frac{18}{6}$$

12.
$$\frac{8\times 9}{4} - \frac{15}{3}$$

13.
$$\frac{12 \times 15}{5 \times 3 \times 2}$$

14.
$$\frac{8\times7\times6}{4\times28}$$

15.
$$\frac{(3+5)\times 2}{13-11}$$

16.
$$\frac{6+8\times(4-1)}{4-1\times2}$$

In problems 17 to 24, solve each applied problem by hand. (You can check for accuracy with the calculator.)

- 17. One car travels 228 miles (mi) on 12 gallons (gal) of gasoline, and a second car travels 336 mi on 16 gal. How many more miles per gallon does the car with the better gas mileage get?
- **18.** An experienced electronics technician earns \$330 for a 40-hour week, and a civil engineer earns \$287 for a 35-hour week. Who earns more per hour and how much more?
- **19.** A Mariner space probe traveling at an average speed of 6000 miles per hour (mi/h) takes 400 days to reach Mars. What is the total distance traveled by the space probe?

- 20. A bus route is 22 kilometers (km) long. It takes the bus 50 minutes (min) to complete the route in one direction and 70 min to complete it in the other direction. What is the average rate of speed of the bus in kilometers per hour for the total trip back and forth? (Average rate = total distance/total time.)
- **21.** The formula N(N+1)/2 can be used to calculate the sum of the first N numbers. Check the formula for the first 12 numbers by (a) adding 1 through 12 directly; (b) letting N=12 in the formula and calculating the result.
- 22. A number is divisible by 9 if the sum of its digits is divisible by 9. Otherwise it is not. Test this with (a) the eight-digit serial number on a dollar or another bill and (b) your nine-digit social security number.
- 23. In BASIC and other computer languages, the following symbols are used for the arithmetic operations:

Addition +

Subtraction -

Multiplication *

Division /

Applying the order of operations, calculate the following example written in BASIC:

$$6 - 4/2 + 3 * 5$$

24. Do as in problem 23 for:

$$6 * 8/(3 + 1) - 4$$

1-2 | Fractions

Calculations with fractions, decimals, and percentages lead to mistakes because of a misunderstanding of the concepts involved. The calculator can prevent some of these mistakes, but it is not a substitute for clear understanding. The following examples review the basic arithmetic of fractions. Each example is designed to be done by hand. (You can check the results with the calculator.)

Example 1-4

Simplify (reduce to lowest terms):

$$\frac{28}{42}$$

Solution

Divide out any common factors (divisors) in the top and bottom:

$$\frac{28}{42} = \frac{\cancel{\cancel{1}} \times 2 \times \cancel{\cancel{1}}}{\cancel{\cancel{1}} \times 3 \times \cancel{\cancel{1}}} = \frac{2}{3}$$

It is not necessary to show all the factors. This is done to clearly illustrate the procedure. You can simply divide top and bottom by 14. The numbers ²⁸/₄₂ and ²/₃ are called *equivalent fractions*. A fraction can be changed to an equivalent fraction by dividing out common factors or by multiplying top and bottom by the same factor. For example:

$$\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16}$$
 and so on

Example 1-5

Calculate each of the following:

1.
$$\frac{3}{8} \times \frac{2}{9}$$

2.
$$\frac{5}{12} \div \frac{15}{16}$$

3.
$$4 \times \frac{3}{14} \times \frac{5}{9}$$

4.
$$\frac{5}{4} \times 8 \div \frac{1}{4}$$

Solution

1.
$$\frac{3}{8} \times \frac{2}{9}$$

To *multiply fractions*, first divide out common factors that occur in any numerator and denominator. Then multiply the numerators and the denominators:

$$\frac{\frac{1}{\cancel{3}}}{\cancel{3}} \times \frac{\cancel{\cancel{1}}}{\cancel{\cancel{3}}} = \frac{1}{12}$$

2.
$$\frac{5}{12} \div \frac{15}{16}$$

To divide fractions, invert the divisor, that is, the fraction after the division sign, and multiply:

$$\frac{\cancel{5}}{\cancel{12}} \times \frac{\cancel{4}}{\cancel{15}} = \frac{4}{9}$$

3.
$$4 \times \frac{3}{14} \times \frac{5}{9} = \frac{\cancel{2}}{\cancel{1}} \times \frac{\cancel{1}}{\cancel{1}} \times \frac{5}{\cancel{9}} = \frac{10}{21}$$

4.
$$\frac{5}{4} \times 8 \div \frac{1}{4} = \frac{5}{4} \times \frac{8}{1} \times \frac{1}{4} = 40$$

Notice in (3) and (4) that a whole number can be written with a denominator of 1. To multiply a fraction by a whole number, multiply the numerator by the whole number:

$$3 \times \frac{1}{4} = \frac{3}{4}$$

Example 1-6

Combine the fractions:

$$\frac{2}{3} + \frac{5}{6}$$

Solution

To combine fractions, that is, to add or subtract, first change each fraction to an equivalent fraction so that the denominators are the same. The easiest denominator to use is the lowest common denominator (lcd), which is the smallest number that each denominator divides into. Then combine the numerators over the lcd:

$$\frac{2(2)}{3(2)} + \frac{5}{6} = \frac{4}{6} + \frac{5}{6} = \frac{4+5}{6} = \frac{9}{6} = \frac{3}{2}$$

The lcd = 6, and only the first fraction needs to be changed. Note that the result % is reduced to $\frac{3}{2}$.

Example 1-7

Combine:

$$\frac{7}{15} + \frac{5}{12} - \frac{1}{6}$$

Solution

The lcd is 60. This can be found by taking multiples of the largest denominator—15, 30, and so on—until each denominator divides into a multiple. Another way is to factor each denominator:

$$\frac{7}{(3)(5)} + \frac{5}{(2)(2)(3)} - \frac{1}{(2)(3)}$$

and to make up the lcd so that it contains all the factors appearing in each denominator: (2)(2)(3)(5) = 60. The solution is then:

$$\frac{7(4)}{15(4)} + \frac{5(5)}{12(5)} - \frac{1(10)}{6(10)} = \frac{28 + 25 - 10}{60} = \frac{43}{60}$$

Note that parentheses or a dot (\cdot) indicates multiplication. These signs are used in algebra to avoid confusing the multiplication sign (\times) with the letter x.

Example 1-8

Calculate:

$$\frac{13}{8} - \frac{7}{5} \times \frac{15}{14} + 2 \div \frac{8}{15}$$

Solution

Invert the last fraction, and change the operation of division to multiplication. Then divide common factors and multiply:

$$\frac{13}{8} - \frac{\cancel{7}}{\cancel{5}} \times \frac{\cancel{3}}{\cancel{4}\cancel{5}} + \cancel{\cancel{2}} \times \frac{15}{\cancel{\cancel{6}}} = \frac{13}{8} - \frac{3}{2} + \frac{15}{4}$$

Now combine over the lcd, 8:

$$\frac{13}{8} - \frac{3(4)}{2(4)} + \frac{15(2)}{4(2)} = \frac{13 - 12 + 30}{8} = \frac{31}{8}$$

See Example 1-22 for the calculator solution of Example 1-8.

Exercise 1-2

In problems 1 to 6, simplify each fraction (reduce to lowest terms).

- 1. $\frac{6}{10}$ 2. $\frac{12}{36}$
- 3. $\frac{28}{35}$ 4. $\frac{27}{54}$

5.
$$\frac{39}{52}$$

6.
$$\frac{34}{51}$$

In problems 7 to 32, calculate each by hand. (You can check with the calculator.)

7.
$$\frac{5}{9} \times \frac{6}{25}$$

8.
$$\frac{2}{21} \times \frac{7}{16}$$

9.
$$\frac{8}{9} \div \frac{2}{3}$$

10.
$$\frac{3}{11} \div \frac{1}{22}$$

11.
$$\frac{3}{5} \times \frac{15}{7} \times \frac{14}{9}$$

12.
$$6 \times \frac{4}{5} \div \frac{8}{15}$$

13.
$$\frac{3}{17} \div \left(\frac{1}{34} \times \frac{1}{2}\right)$$

14.
$$\left(\frac{9}{8} \div \frac{3}{4}\right) \div \frac{3}{2}$$

15.
$$\frac{3}{8} + \frac{1}{4}$$

16.
$$\frac{4}{15} + \frac{5}{6}$$

17.
$$\frac{3}{4} - \frac{1}{2} + \frac{7}{10}$$

18.
$$\frac{1}{6} - \frac{2}{3} + \frac{11}{20}$$

19.
$$2 + \frac{7}{8} + \frac{2}{3}$$

20.
$$\frac{5}{2} + \frac{5}{3} + \frac{5}{6}$$

21.
$$\frac{16}{9} \times \frac{1}{2} + \frac{1}{4}$$

22.
$$\frac{1}{6} + \frac{3}{8} \div \frac{1}{4}$$

23.
$$3 \times \frac{1}{6} + \frac{7}{2} - \frac{4}{5} \div 8$$

23.
$$3 \times \frac{1}{6} + \frac{7}{2} - \frac{4}{5} \div 8$$
 24. $\frac{3}{100} + \frac{7}{10} \times \frac{2}{35} - \frac{1}{50}$

25.
$$\left(\frac{1}{2} + \frac{1}{3}\right) \times \left(8 \div \frac{4}{3}\right)$$
 26. $\left(1 + \frac{3}{8}\right) \div \left(1 - \frac{3}{8}\right)$

26.
$$\left(1+\frac{3}{8}\right) \div \left(1-\frac{3}{8}\right)$$

- 27. A \$30,000 inheritance is distributed as follows: one-half to the spouse, two-thirds of what is left to the children, and the remainder to charity. How much money is given to charity?
- 28. A bookcase is to be 8 feet (ft) 3½ inches (in.) high and to contain six equally spaced shelves and a top, each ½ in. thick (seven pieces total). How many feet and inches apart should each shelf be?
- 29. Calculate the resistance of a series-parallel circuit given by:

$$R = \frac{1}{\frac{1}{12} + \frac{1}{4}} + 3$$

30. Calculate the focal length of a lens given by:

$$f = \frac{\frac{1}{10} \times \frac{3}{20}}{\left(\frac{3}{2} - 1\right)\left(\frac{1}{10} + \frac{3}{20}\right)}$$