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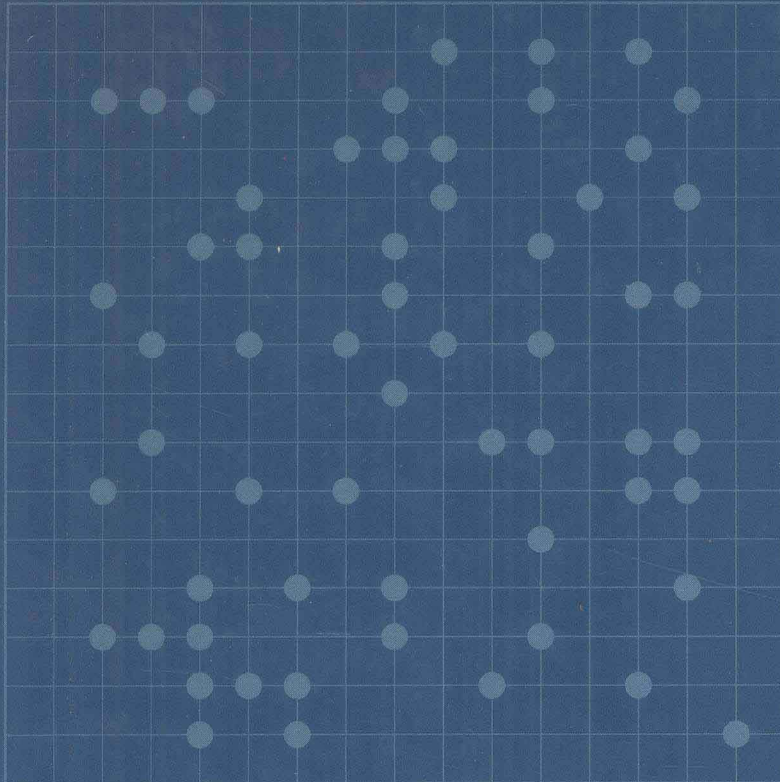

Statistical Mechanics of Disordered Systems

Mathematical Perspective

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Statistical Mechanics of Disordered Systems

A Mathematical Perspective

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and

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Statistical Mechanics of Disordered Systems

Our mathematical understanding of the statistical mechanics of disordered systems is going through a period of stunning progress. This self-contained book is a graduate-level introduction for mathematicians and for physicists interested in the mathematical foundations of the field, and can be used as a textbook for a two-semester course on mathematical statistical mechanics. It assumes only some basic knowledge of classical physics; on the mathematics side, the reader should have a good working knowledge of graduate-level probability theory.

Part I gives a concise introduction to thermodynamics and statistical mechanics, which provides the tools and concepts needed later. The main topics treated here are the classical ensembles of statistical mechanics, lattice gases and spin systems, the rigorous setting of the theory of infinite-volume Gibbs states (DLR theory), and cluster expansions for high- and low-temperature phases. Part II proceeds to disordered lattice models. It presents the general theory of random Gibbs states and metastates in the spirit of Newman–Stein and the random-field Ising model. Part III is devoted to disordered mean-field models. It begins with the random energy model as a toy example and then explains in depth the geometric structures arising in the description of the infinite-volume limit of the Gibbs states in the generalized random energy models. Finally, it presents the latest developments in the mathematical understanding of mean-field spin-glass models. In particular, recent progress towards a rigorous understanding of the replica symmetry breaking solutions of the Sherrington-Kirkpatrick spin-glass models, due to Guerra, Aizenman-Sims-Starr, and Talagrand, is reviewed in some detail. The last two chapters treat applications to non-physical systems: the Hopfield neural network model and the number partitioning problem.

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To the memory of my mother
Elisabeth Pee

Preface

Statistical mechanics is the branch of physics that attempts to understand the laws of the behaviour of systems that are composed of very many individual components, such as gases, liquids, or crystalline solids. The statistical mechanics of disordered systems is a particularly difficult, but also particularly exciting, branch of the general subject, that is devoted to the same problem in situations when the interactions between these components are very irregular and inhomogeneous, and can only be described in terms of their statistical properties. From the mathematical point of view, statistical mechanics is, in the spirit of Dobrushin, a 'branch of probability theory', and the present book adopts this point of view, while trying not to neglect the fact that it is, after all, also a branch of physics.

This book grew out of lecture notes I compiled in 2001 for a Concentrated Advanced Course at the University of Copenhagen in the framework of the MaPhySto programme and that appeared in the MaPhySto Lecture Notes series [39] in the same year. In 2004 I taught a two-semester course on Statistical Mechanics at the Technical University of Berlin within the curriculum of mathematical physics for advanced undergraduate students, both from the physics and the mathematics departments. It occurred to me that the material I was going to cover in this course could indeed provide a suitable scope for a book, in particular as the mathematical understanding of the field was going through a period of stunning progress, and that an introductory textbook, written from a mathematical perspective, was maybe more sought after than ever. I decided to include a considerable amount of basic material on statistical mechanics, in order to make it reasonably self-contained.

Thus, Part I gives a brief introduction to statistical mechanics, starting from the basic notions of thermodynamics and the fundamental concepts of statistical mechanics. It then introduces the theory of lattice spin systems, the Gibbsian formalism in the spirit of Dobrushin, Lanford, and Ruelle, as well as some of the main tools for the analysis of the Gibbs measures, including cluster expansions.

Part II of the book deals with disordered spin systems on the lattice. It starts out with a comprehensive introduction to the formalism of *random Gibbs measures* and *metastates*. Then I discuss the extensions and limitations of the methods introduced in the first part. The bulk of this part is devoted to the random field Ising model and the question of how uniqueness and non-uniqueness can be analyzed in this case. I present the proof of uniqueness in $d = 2$ due to Aizenman and Wehr. A large section is also devoted to the renormalization group approach of Bricmont and Kupiainen, used to prove non-uniqueness in $d \geq 3$. I only comment briefly on the issue of spin-glasses with short-range interactions.

Part III is essentially devoted to mean-field models of spin-glasses. I basically treat two classes of models: Gaussian processes on the hypercube, and models of the Hopfield type. I will go to great lengths to explain in all detail the case of the *random energy model* (REM) and the *generalized random energy model* (GREM), which will give us an idea what a complete solution of such models could look like. Then I will briefly expose the deep ideas of F. Guerra and their reformulation by Aizenman, Sims, and Starr, that show how GREM-like structures can be used to provide bounds, at least for free energies, of general mean-field models in terms of hierarchical structures, essentially explaining the nature of the Parisi solution. Talagrand's proof that this bound is exact will not be given here. Finally, I discuss some of the simpler aspects of the nature of the Gibbs measures in the p-spin SK models. Much more on the SK models can be found in Talagrand's recent book [239].

The last two chapters deal with models that demonstrate the relevance of statistical mechanics beyond the classical 'physics' applications. The Hopfield model of neural networks has played a rather important rôle in the history of the subject, and I try to give a more elementary and to some extent complementary presentation to the one that can be found in Talagrand's book. The final chapter is devoted to the number partitioning problem, where a rather charming connection between a problem from combinatorial optimisation and the REM arises.

My original intention for this book had been to give full proofs of all the main results that are presented, but in the end I found that this was impracticable and that at some places the reader had to be referred to the original literature. So the practice in the book is that full proofs are given when they are reasonably easy, or where I feel they are essential for understanding. In other cases, they are omitted or only outlined.

References are given primarily with the intention to help the reader find a way into the original literature, and not with the ambition of completeness. Overall, the selection of references is due largely to my limited knowledge and memory. More generally, although I do make some comments on the history of the subject, these are by no means to be taken too seriously. This book is intended neither as an encyclopedia nor as an account of the history of the subject.

I have tried to keep the prerequisites for reading as low as reasonable. I assume very little physics background, apart from a rudimentary knowledge of classical mechanics. On the mathematics side, the reader should, however, have a good working knowledge of probability theory, roughly on the level of a graduate course.

There are a great number of very interesting topics that have been left out, either because they are treated elsewhere, or because I know too little about them, or simply for no good reason. One of these are short-range spin-glasses. A good source for this remains Newman's book [185], and at this moment I have nothing serious to add to this. Another topic I skip are Kac models. Kac models make a charming link between mean-field models and finite-range models. There has been a great deal of work concerning them in the context of disordered systems, both in the context of the random field Ising model [73, 74], the Hopfield model [46, 47], and very recently in spin-glasses [38, 104], and it would be nice to cover this. There is a beautiful new book by Errico Presutti dealing with Kac models [209], which does not, however, treat disordered systems.

I want to thank Ole E. Barndorff-Nielsen and Martin Jacobsen for the invitation to teach a course in Copenhagen in the MaPhySto programme, which ultimately

triggered the writing of this book. I am deeply indebted to my collaborators, past and present, on subjects related to this book; notably J. Fröhlich (who initiated me to the subject of disordered systems more than 20 years ago), V. Gayrard, U. Glaus, Ch. Külske, I. Kurkova, M. Löwe, D. Mason, I. Merola, B. Niederhauser, P. Picco, E. Presutti, A. C. D. van Enter, and M. Zahradník, all of whom have contributed greatly to my understanding of the subject. Special thanks are due to Irina Kurkova who kindly provided most of the figures and much of the material of Chapter 10.

I am very grateful to all those who read preliminary versions and pointed out errors, misprints, and omissions, or made other comments, notably J. Černý, V. Gayrard, B. Gentz, M. Jacobsen, I. Kurkova, M. Löwe, E. Orlandi, and T. Schreiber, but in particular Aernout van Enter, who has relentlessly read various versions of the manuscript, and has provided a wealth of suggestions and corrections. I also want to thank my Editor at Cambridge University Press, Diana Gillooly, for suggesting this book project, and for constant help during the writing and production. Finally, I thank Christina van de Sand for her help in the production of the LaTeX version of the typescript.

Nomenclature

$(\Omega, \mathcal{B}, \mathbb{P})$	probability space of disorder
β	inverse temperature
$\mathcal{C}_\Gamma^*$	set of connected clusters
$\delta_x(f)$	local variation of f
$\Delta(f)$	total variation of f
$\mathbb{E}$	expectation
$\mathcal{F}$	sigma-algebra (spin variables)
$\psi_{\beta,N}, \psi_\beta$	overlap distribution
γ	connected contour
$\Gamma(\sigma)$	contour of σ
$\mathcal{G}_\Lambda$	set of polymers
$\mathcal{G}_{\beta,V,N}$	canonical distribution
$\mathcal{K}_T$	genealogical functional
$\mathcal{K}_\beta^\eta$	Aizenman–Wehr metastate
$\mathcal{K}_{\beta,\Lambda}^\eta, \mathcal{K}_\beta^\eta$	metastates
$\mathcal{K}_{\beta,N}, \mathcal{K}_\beta$	empirical distance distribution
$\mathbb{M}_Z$	median of Z
$\mathcal{M}_1(\cdot)$	space of probability measures
$\mathcal{M}_N$	set of values for magnetization
$\mathcal{M}_{\alpha,N}$	point process
μ	chemical potential
μ_β	infinite volume Gibbs measure
$\mu_{\beta,h,N}$	Gibbs measure
$\mu_{\beta,\Lambda}^{(\eta)}$	local Gibbs specification
$\mathbb{P}$	probability
Φ, Φ_A	interaction
$\phi_\beta(m)$	Curie–Weiss function
$\Phi_{\beta,N}$	log of partition function per volume
$\Phi_{\beta,N}(z)$	HS transform rate function
$\mathcal{P}$	Poisson point process
$\mathcal{P}^{(k)}$	Poisson cascade
$\mathbb{Q}_{\beta,N}$	induced measure
$\mathcal{Q}_{\beta,N}$	Hubbard–Stratonovich measure
ρ_Λ	product measure

$\rho_{E,V,N}$	micro-canonical distribution
$\mathcal{R}$	Poisson point process
σ	spin configuration
σ_x	value of spin at x
$\mathcal{S}$	configuation space
$\mathcal{S}_0$	single spin space
$\mathcal{S}_\Lambda$	spin configurations in Λ
$\mathcal{S}_N$	hypercube in dimension N
$\tilde{\mathcal{G}}_{\beta,V,\mu}$	grand canonical distribution
$\tilde{\mathcal{Z}}_{\beta,V,\mu}$	grand canonical partition function
$\mathcal{W}_\alpha$	point process of masses
ξ^μ	pattern (Hopfield model)
$m_\mu(\sigma)$	overlap parameter
$B(\mathcal{S},\mathcal{F})$	bounded measurable functions
B_{loc}	local functions
B_{smql}	quasi-local functions
$C(\mathcal{S})$	continuous functions
C_{loc}	continuous local functions
C_{ql}	continuous quasi-local functions
$d_N(\cdot,\cdot)$	hierarchical overlap
E	energy
F	Helmholtz free energy
$f(\beta,v)$	specific free energy
$F_{\beta,h,V}$	free energy in spin system
G	Gibbs free energy
H	enthalpy
$h(\cdot,\cdot)$	relative entropy
H_Λ	finite volume Hamiltonian
H_N	Hamiltonian function
$I(m)$	Cramèr entropy
$J_N(m)$	correction to Cramèr entropy
$K^\eta_{\beta,\Lambda}$, K^η_β	joint measures
m^*_β	equilibrium magnetization
$m_N(\sigma)$	empirical magnetization
N_x	control fields
p	pressure
$R_N(\cdot,\cdot)$	overlap
S	entropy
S_x, S_C	small fields
T	temperature
$u_N(x)$	scaling function for Gaussian random variable on $\mathcal{S}_N$
V	volume
$w_\Lambda(g)$	activities
X_σ	standardized Gaussian process
$Z_{\beta,V,N}$	canonical partition function
$z_{E,V,N}$	micro-canonical partition function

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Part I

Statistical mechanics

1

Introduction

L'analyse mathématique, n'est elle donc qu'un vain jeu d'esprit? Elle ne peut donner au physicien qu'un langage commode; n'est-ce pas là un médiocre service, dont on aurait pu se passer à la rigueur; et même n'est il pas à craindre que ce langage artificiel ne soit un voile interposé entre la réalité et l'œil du physicien? Loin de là, sans ce langage, la pluspart des analogies intimes des choses nous seraient demeurées à jamais inconnues; et nous aurions toujours ignoré l'harmonie interne du monde, qui est, nous le verrons, la seule véritable réalité objective.[1]

Henri Poincaré, La valeur de la science.

Starting with the Newtonian revolution, the eighteenth and nineteenth centuries saw with the development of analytical mechanics an unprecedented tool for the analysis and prediction of natural phenomena. The power and precision of Hamiltonian perturbation theory allowed even the details of the motion observed in the solar system to be explained quantitatively. In practical terms, analytical mechanics made the construction of highly effective machines possible. Unsurprisingly, these successes led to the widespread belief that, ultimately, mechanics could explain the functioning of the entire universe. On the basis of this confidence, new areas of physics, outside the realm of the immediate applicability of Newtonian mechanics, became the target of the new science of theoretical (analytical) physics. One of the most important of these new fields was the theory of heat, or *thermodynamics*. One of the main principles of Newtonian mechanics was that of the conservation of energy. Now, such a principle could not hold entirely, due to the ubiquitous loss of energy through friction. Thus, all machines on earth require some source of energy. One convenient source of energy is heat, obtainable, e.g., from the burning of wood, coal, or petrol. A central objective of the theory of thermodynamics was to understand how the two types of energy, mechanical and thermal, could be converted into each other. This was originally a completely pragmatic theory, that introduced new concepts related to the phenomenon of heat, *temperature* and *entropy*, and coupled these to mechanical concepts of energy and force. Only towards the end of the nineteenth century, when the success of mechanics reached a peak, was Boltzmann, following earlier work by Bernoulli, Herapath, Joule, Krönig, Claudius,

[1] Approximately: So is mathematical analysis then not just a vain game of the mind? To the physicist it can only give a convenient language; but isn't that a mediocre service, which after all we could have done without; and, is it not even to be feared that this artificial language be a veil, interposed between reality and the physicist's eye? Far from that, without this language most of the intimate analogies of things would forever have remained unknown to us; and we would never have had knowledge of the internal harmony of the world, which is, as we shall see, the only true objective reality.

and Maxwell, able to give a mechanical interpretation of the thermodynamic effects on the basis of the atomistic theory. This *kinetic theory of gases* turned into what we now know as *statistical mechanics* through the work of Gibbs in the early twentieth century. It should be mentioned that this theory, that is now perfectly accepted, met considerable hostility in its early days. The first part of this book will give a short introduction to the theory of statistical mechanics.

It is not a coincidence that at the same time when statistical mechanics was created, another new discipline of physics emerged, that of *quantum mechanics*. Quantum mechanics was concerned with the inadequacies of classical mechanics on the level of microscopic physics, in particular the theory of atoms, and thus concerned the opposite side of what statistical mechanics is about. Interestingly, quantum mechanical effects could explain some deviations of the predictions of statistical mechanics from experimental observation (e.g. the problem of black body radiation that was resolved by Planck's quantum hypothesis). The basic principles of statistical mechanics can be well reconciled with quantum mechanics and give rise the the theory of *quantum statistical mechanics*. However, in many cases, a full quantum mechanical treatment of statistical mechanics turns out to be unnecessary, and much of classical mechanics applies with just some minor changes. In any case, we will here consider only the classical theory. Before approaching our main subject, let us have a very brief look at thermodynamics.

1.1 Thermodynamics

A mechanical system is characterized by essentially geometric quantities, the positions and velocities of its components (which are points of mass). If solid objects are described, the assumption of rigidity allows us to reduce their description to essentially the same kind of coordinates. Such a description does not, however, do complete justice to all the objects we can observe. Even solids are not really rigid, and may change their shape. Moreover, there are liquids, and gases, for which such a description breaks down completely. Finally, there are properties of real objects beyond their positions or velocities that may interfere with their mechanical properties, in particular their *temperature*. In fact, in a dissipative system one may observe that the temperature of a decelerating body often increases. Thermodynamics introduces a description of such new *internal* variables of the system and devises a theory allowing us to control the associated flow of energy.

The standard classical setting of thermodynamics is geared to the behaviour of a gas. A gas is thought to be enclosed in a container of a given (but possibly variable) volume, $V > 0$. This container provides the means of coupling the system to an external mechanical system. Namely, if one can make the gas change the volume of the container, the resulting motion can be used to drive a machine. Conversely, we may change the volume of the container and thus change the properties of the gas inside. Thus, we need a parameter to describe the state of the gas that reacts to the change of volume. This parameter is called the *pressure*, p. The definition of the pressure is given through the amount of mechanical energy needed to change the volume:[2]

$$dE_{\text{mech}} = -p dV \tag{1.1}$$

[2] The minus sign may appear strange (as do many of the signs in thermodynamics). The point, however, is that if the volume increases, work is done by the system (transfered somewhere), so the energy of the system decreases.

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