BOUNDARY ELEMENT METHODS IN TRANSPORT PHENOMENA

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Boundary Element Methods in Transport Phenomena

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Preface

Boundary element methods (BEM) have gained wide acceptance in the areas of engineering analysis related to potential problems and elasticity. These techniques are also being increasingly used for the analysis of heat and mass transfer problems and more recently in flow analysis or momentum transport. The widespread use of the method is due to a number of its advantages, such as reduction of dimensionality for linear differential equations, ease of coding and smaller memory requirements, high accuracy and computational efficiency. Further, the method has undergone rapid developments which have expanded its domain of applications. The descriptive use of the method for the solution of problems encountered in the areas of transport phenomena have been, however, confined to journals and edited monographs, as textbook coverage has not been available. The purpose of this work is to provide a textbook which fills this gap. The book will provide the reader with a complete understanding of the basis of the method and the capability to numerically solve a wide range of transport phenomena problems, especially in heat and mass transfer.

The book can be used as a stand-alone textbook for a one-semester course on this subject, or could be used as a supplement to the traditional transport phenomena texts in a regular course to reinforce the numerical aspects of the subject. The book can also be used as a self study tool for researchers involved in engineering analysis. Thus the novices in BEM will benefit from the clarity of presentation, while the experts in the art of modeling will find the breadth of applications very useful.

The organization of the book comprises of two sets of topics: (i) numerical methods of solution of ordinary and partial differential equations, covered in Chapters 4 to 9; (ii) applications to transport processes, covered in Chapters 10 and 11. Chapters 1 to 3 provide the introductory material. A number of examples are provided throughout the text to illustrate the ideas and the method of solution. The listings of a number of complete programs are also provided in the text and these are also available on a diskette. The programs are valuable learning tools as the reader can test them out on a PC and examine the effect of mesh refinements, influence of the boundary conditions, nature of the solution, etc. The programs are somewhat specific and are not intended to serve as a general library of software, but they are structured in such a way that they can be easily modified for many realistic applications. A number of additional programs for solution of some advanced topics covered in the book are available separately from the author. Exercises are also provided at the end of each chapter for the reader to practice the concepts developed in the text. Many derivations or extensions of the text material are left as exercises in order to economize space. The exercises should, therefore, be viewed as a continuation of the text material. Key references are provided, although a complete set of references is impossible in view of the increasing literature in this field. In this context, for a more detailed literature review, *Boundary Element Communications* (BEC), the abstracts journal produced bimonthly by the current Publisher, provides an excellent starting point.

My interest in the area of boundary elements started in 1985 when, as an accidental tourist to the library, I stumbled upon a book [2.2] by Socrates, Plato and Aristotle on this subject. Due to the comprehensive range of problems covered in their book and due to my research interest in the areas on numerical methods, I started to learn the method by writing my own computer programs for the standard cases. I also began using the method for non-linear one-dimensional (1D) problems using subdomain BEM and was amazed by the robustness of the method. Study of higher dimensional (2D and 3D) Poisson problems was motivated partly by my teaching interests and partly in view of their diverse applications. A number of publications and the associated computer programs grew out of this effort and this book seemed a logical extension. The writing of the textbook gathered momentum in December 1991 when Professor Brebbia visited our university and encouraged me to write a textbook in this field. Thanks Carlos. Little did I realize at that time the efforts needed to complete the work as well as to assimilate the relevant rapidly growing literature in this field. My daughter Nima never believed that the book would ever get completed but was always willing to sacrifice her own time to help me out in the lengthy task. Graduate students, Sriganesh Karur and David Dowell, read through the manuscript and offered several suggestions, some of which were incorporated as the manuscript went through many versions. My thanks are expressed to them as well as to the reviewers for their many helpful hints and to Lance Sucharov for his editorial efforts. Sriganesh also helped in the program development on the dual reciprocity method in Chapter 9, as well as the preparation of the figures for the book. Professor S.C. Dhingra of I.I.T., New Delhi and Dr. Henry Erk of MEMC, St. Peters, also helped with their careful reading of the final version of the text. Henry's comments on document formatting style were very useful. And so it was written and so it shall be.

The book is dedicated to Vishu who taught me the meaning of love and courage. I hope the book will benefit the students in this field. I shall appreciate any praise, criticism, corrections, comments and suggestions for further improvement of this text.

P. A. Ramachandran
Saint Louis, MO
September, 30, 1993.

Computer Programs on Diskette

Diskette Contents

A diskette containing the source listing of the computer programs presented in the text is available separately from Computational Mechanics Publications, Fax 44 (0)703 292853. The programs have been arranged in such a manner that all the subroutines needed to run a particular program are included along with the main program. A sample input data file is also included if needed. The following gives the contents of this diskette.

1. **BEM1D.FOR** This program solves second order differential equations of the boundary value type. The program listing is provided in Section 4.6. The sample input file needed BEM1D.IN is also included.

2. **MULTI.FOR** Solution of multiple second order differential equations of the type described in Section 5.3 is done using the program. The program listing is the same as in Subsection 5.3.2.

3. **BTR1D.FOR** This program solves transient problems in 1D. The listing is provided in Section 6.4. A sample input file BTR1D.IN is also provided.

4. **BE2DC.FOR** Solution of Laplace equation in 2D as outlined in Section 7.3 can be accomplished by this program. A sample input BE2DC.IN is included.

5. **BE2DQ.FOR** This program solves the Laplace equation in 2D using quadratic element and the listing of the program is that in Section 7.6. A sample datafile BE2DQ.IN is included.

6. **DRBEM.FOR** This program solves Poisson type of equations using dual reciprocity method using constant elements. The program is listed in Section 9.3 and the datafile needed DRBEM.IN is included. The datafile has the same structure as BE2DC.IN. The user has to modify the rate function depending on the source term (b function) as indicated in Section 9.3.

Additional Programs

The following additional programs are available with the author for a nominal cost. In some cases, the programs are specific to the examples presented in the book but can be easily modified to other applications since the source code is available and the method of solution is explained in detail in the text. For details on obtaining these programs, contact the author. E-mail address of the author is rama@wuche3.wustl.edu

1. **DISPW.FOR** Solves diffusion-convection-reaction problems in 1D as discussed in Section 4.7 using the 'rate-dependent' weighting functions as discussed in Example 4.9.

2. **THIRD.FOR** Solves third order ordinary differential equations by the method outlined in Section 5.4. The version on the diskette is for the Example 5.6 in the book. The program can be easily modified for other cases by changing the rate function, the node locations and the boundary conditions.

3. **FOURTH.FOR** Solves fourth order ordinary differential equations by the method outlined in Section 5.5. The example problem 5.7 is solved by this program. The program can be suitably modified for other cases.

4. **BEMCUBE.FOR** Solves Laplace equation in 2D using cubic elements discussed in Subsection 7.7.1. The datafile needed is BEMCUBE.IN as provided for Example 7.7.

5. **CYLQ.FOR** Solves the heat conduction equation for axisymmetric cylinders using θ-averaged weighting functions. (See Section 8.3). Datafile for Example 8.3 is included as CYLQ.IN.

6. **DRN2DQ.FOR** Solves the linear Poisson equation in 2D using quadratic elements. (Section 8.8). Datafile needed has the same form as BE2DQ.IN.

7. **PSIOMEGA.FOR** Solves the 2D Stokes equation using the streamfunction-vorticity formulation discussed in Subsection 11.4.2.

Disclaimer of Warranty

The author and the Publisher have made extensive testing of the programs to determine its effectiveness and to verify its accuracy. However, the author and the publisher make no warranty of any kind, expressed or implied, with regards to these programs. The author and the publisher shall not be liable in any event due to any damages resulting from the performance or the use of these programs.

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