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A digital control perspective

Pedro A. Capó-Lugo and Peter M. Bainum



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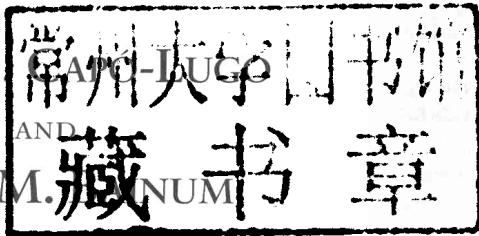
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To my wife, my newborn son, and my family, thanks for their patience
and support.

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To my wife, my son, and his wife for their support and understanding.

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List of symbols

A	State matrix
\hat{A}	Discrete state matrix
A_{CL}	Closed loop matrix
A	Area of the ellipse
A_F	Axial force
A_{VO}	Area of the valve opening
ARE	Adjoint Riccati equation
\bar{a}	Acceleration of the body
$\bar{a}_{0/1}$	Acceleration of the body seen from the reference point
$\bar{a}_{2/0}$	Acceleration of the body seen from the observer 2 near point O
a	Semimajor axis
a_t	Semimajor axis for the transfer orbit
a_N	Coefficient of the polynomial
a	Semi-conjugate axis for the hyperbola
$\bar{\alpha}$	Angle of attack
α	Angle of the incident light
B	Control matrix
\hat{B}	Discrete control matrix
B_2	Constant describing the Earth oblateness
B_C	Ballistic coefficient
\bar{B}	Unit vector applied along the tangential direction
B_0	Earth's magnetic field intensity
\bar{B}_m	Earth's magnetic field vector
\bar{B}_p	Horizontal component of the Earth's magnetic field vector
b	Semiminor axis
\hat{b}_i	Body coordinates
β	Gimbal angle of the motors about the Y axis
β_{max}	Maximum gimbal angle

C	Controllability matrix
\hat{C}	Discrete output matrix
CG	Center of gravity
CM	Center of mass
CP	Center of pressure
c	Number of constraints
$c(t)$	Surface constraint
C_D	Drag coefficient
\bar{c}	Velocity of light in vacuum
DOF	Degrees of freedom
D	Magnetic declination
D_E	Earth's magnetic dipole moment
\bar{D}	Magnetic dipole moment
D_{max}	Maximum magnetic dipole moment
d	Distance from the center of the Earth to the directrix line
d_{RW}	Damping of the reaction wheel
\bar{d}	distance between parallel coordinate systems
$d(\)$	Change or variation of a variable
δ	Gimbal angle for the motors about the Z axis
δf	Difference in the true anomaly angle
Δt	Change in time or sampling time
Δf	Sampling time in the true anomaly angle
$\Delta V_{p,SB}$	Change in velocity at the perigee point for SB
$\Delta V_{p,SA}$	Change in velocity at the perigee point for SA
$\Delta V_{p,SC}$	Change in velocity at the perigee point for SC
$\Delta V_{p,SH}$	Change in velocity at the perigee point for SH
ΔV_a	Change in velocity at the apogee point
∇f	Gradient of a function
$E(z)$	Auxiliary variable in the discrete domain
E	Eccentric anomaly
ECI	Earth centered inertial frame
E_0	Light energy arriving at the infinitesimal surface
E_b	Voltage across the rotor
$E_a(t)$	Applied voltage
E_{max}	Maximum applied voltage
E_r^2	Mean square error
e	Eccentricity of the orbit
ϵ_a	Absorptivity coefficient

ϵ_d	Diffuse reflectance coefficient
ϵ	Reflectivity coefficient
ξ, ζ, η	Coordinate system of the center of mass
e_x	Actual state error
$e_{\dot{x}}$	Time derivative of the actual state error
F	Final state weighting matrix
FS	First stage of Ares V
FL	Fuzzy logic
F_A	Aerodynamic force
\vec{F}	Forces applied to the body
\bar{F}	Solar radiation force
\vec{F}_i^b	Body forces
\vec{F}_i^s	Surface forces
\vec{F}^+	Solar radiation force for absorbing surface
\vec{F}^-	Solar radiation force for reflective surface
$\vec{f}_{J_2, M}$	J2 perturbation force for the maneuvering spacecraft
$\vec{f}_{J_2, R}$	J2 perturbation force for the reference spacecraft
$\vec{f}_{P, M}$	Perturbation force for the maneuvering spacecraft
$\vec{f}_{P, R}$	Perturbation force for the reference spacecraft
f	True anomaly angle
f_L	Final true anomaly angle
f_0	Initial true anomaly angle
f_M	Frequency of the magnetic natural response function
f_v	Damper constant
$G_P(S)$	Transfer function for the process or plant
$G_A(S)$	Transfer function for the actuator
$G_C(S)$	Transfer function for the controller
$G_S(S)$	Transfer function for the sensors
$G_{OL}(S)$	Open loop transfer function
$G_{CL}(S)$	Closed loop transfer function
G	Universal gravitational constant
GA	Genetic algorithms
GP	Gimbal point
\vec{G}	Adjoint Riccati equation
\vec{G}_{∞}	Steady-state adjoint Riccati equation
\vec{G}_{∞}	Discrete steady-state adjoint Riccati equation
g	Gravitational acceleration for the Earth
γ	Flight path angle
γ_L	Learning parameter

$\vec{\gamma}$	Error vector
$\vec{\gamma}_\infty$	Steady-state error vector
H	Density scale height
H_m	Flux density in the rod
\mathcal{H}	Hamiltonian equation
\vec{H}	Angular momentum of the body
\vec{H}_T	Total angular momentum
\vec{H}_{RW}	Reaction wheel angular momentum
h	Angular momentum per unit mass
\bar{h}	Power of electromagnetic wave per unit surface
\bar{h}_0	Solar radiation constant
I	Identity matrix
\hat{I}	Inertia matrix
I	Magnetic inclination angle
I_{max}	Maximum applied current
I_{ij}	Moments of inertia, $i=1,2,3$ and $j=1,2,3$
I_a	Armature current
I_{SP}	Specific impulse for any motor
I_S	Current source
i	Inclination angle
i_S	Inclination angle of the Sun with respect to the Earth
J	Principal moments of inertia
J_{RW}	Reaction wheel moment of inertia
J	Cost function or system performance
K_∞	Steady-state control gain matrix
K	Control gain matrix
K	Momentum of light
K_b	Back electromotive force constant
K_t	Motor torque constant
K_P	Proportional control gain
K_I	Integral control gain
K_D	Derivative control gain
\tilde{K}_P	Proportional adaptive control gain
\tilde{K}_d	Derivative adaptive control gain
K	Spring constant
\vec{K}_∞^P	Steady-state perturbation gain vector
k	Sample taken in the discrete control process

L	Lagrange equation or Lagrange function
$\mathcal{L}\{ \}$	Laplace transform
L	Number of runs in the hierarchical control scheme
L_a	Armature inductance
L_{CP}	Distance from the CG to the CP
L_{GP}	Distance from the CG to the GP
L_{SRB}	Distance from the CG to the center of the nozzle of the SRB
LQR	Linear quadratic regulator
LHS	Left hand side
l	Angular momentum
l_C	Length of the cylinder
$\tilde{\lambda}$	Co-state variable
λ	Longitude
$\lambda_{ecliptic}$	Mean longitude of the ecliptic plane
λ_{sun}	Mean longitude of the Sun
\mathcal{M}	End conditions in the cost function
M	Mean anomaly angle
\bar{M}	Total system mass
M_b	Mass of the body
M_E	Mass of the Earth
M_{sun}	Mean anomaly of the Sun
m	Mass of the body
m_0	Initial mass
m_i	Mass of a particle
\dot{m}	Mass ratio at the exit nozzle of any motor
\dot{m}_{max}	Maximum mass flow rate
μ	Earth gravitational constant which is equal to GM_E
N	Number of particles
N_F	Normal force
N_f	Last sample obtained in the control solution
N_{STALL}	Stall torque
\vec{N}	Unit vector applied along the normal to the orbital plane
\vec{N}	Torques
\vec{N}	End constraints
\vec{N}_A	Torque due to the atmospheric pressure
\vec{N}_{AR}	Torque due to the satellite angular momentum
\vec{N}_E	Gravity-gradient torques
\vec{N}_m	Magnetic torques

\vec{N}_P	Torque due to perturbing forces
\vec{N}_{RW}	Reaction wheel torques
\vec{N}_{SP}	Torque due to solar pressure
\vec{N}_T	Torque due to the thrusters
N_{max}	Maximum torque due to the reaction wheels
n	Mean motion
NN	Neural network
n_T	Number of thrusters for the RoCS
n_t	Number of turns of the coil
\hat{n}	Normal vector to the surface
v	Angular velocity of the reaction wheels
v_-	No load angular velocity
Ω	Anti-symmetric or skew-symmetric matrix
Ω_E	Earth angular velocity about the polar axis
Ω	Right ascension of the ascending node
ω	Argument of perigee
ω_n	Natural frequency
$\vec{\omega}$	Angular velocity of the satellite
$\vec{\omega}_R$	Reference angular velocity of the satellite
$\vec{\omega}_n$	Mean motion of the angular velocity of the satellite
P	P_∞ Steady-state Riccati matrix
\hat{P}_∞	Steady-state discrete Riccati matrix
%OS	Percent of overshoot
p	Semi-latus rectum
ϕ	Latitude
φ	Roll angle
φ_m	Damping function for the magnetic dipole system
φ_n	Natural response function for the magnetic dipole system
ξ	Damping coefficient
ψ	Yaw angle
$\vec{\psi}$	Perturbation column vector
$\vec{\psi}_A$	Perturbation column vector for atmospheric density force
$\vec{\psi}_{SP}$	Perturbation column vector for solar pressure force
$\vec{\psi}_{Unknown}$	Perturbation column vector for unknown forces
Q	State weighting matrix
\tilde{Q}	State weighting matrix in the true anomaly angle
Q	Dynamic pressure
Q_j	Generalized forces

q	Quaternion, and term in the Legendre polynomial expansion of the Earth's potential function
q_i	Generalized coordinates
q_e	Error quaternion
q_c	Commanding quaternion
\mathbf{R}	Rotational matrix
\mathbf{R}	Control weighting matrix
$\bar{\mathbf{R}}$	Control weighting matrix in the true anomaly angle
\mathbf{R}_J	Direction cosines matrix for the principal moments of inertia
$\bar{\mathbf{R}}$	Unit vector applied along the radial direction
R_A	Armature resistance
R_E	Radius of the Earth
R_C	Radius of the cylinder
RoCS	Roll control system
RHS	Right hand side
RE	Riccati equation
R or \bar{R}	Radius that defines the distance from the center of the Earth to the satellite
$\hat{\bar{\mathbf{R}}}$	Unit vector associated with $\bar{\mathbf{R}}$
$\bar{\mathbf{R}}_m$	Distance from the center of mass of the satellite to a point in the body of the satellite
$\bar{\mathbf{R}}_{CM}$	Distance from the center of the Earth to the center of mass of the satellite
r_a	Radius of apogee
r_0, R_{SUN}	Mean distance of the Earth to the Sun
r_p	Radius of perigee
$\bar{\mathbf{r}}$	Position of the maneuvering satellite
$\bar{\mathbf{r}}_i$	Position vector of the body
$\bar{\mathbf{r}}_2$	Distance from point O to the particle
$\bar{\rho}(t)$	Separation distance between a pair of satellites
$\bar{\rho}_{Traj}(t)$	Separation distance between a pair of satellites from the trajectory system
$\bar{\rho}_{NAV}(t)$	Separation distance between a pair of satellites from the navigation system
ρ_{Prop}	Density of the propellant
S	Satellite surface area
\bar{s}	Altitude of the orbit
SRB	Solid rocket booster motors

$sgn(\)$	Signum function
$\hat{\sigma}$	Direction of the light flux
ζ	Function in terms of the true anomaly angle
T	Period of the orbit
T_t	Period of the transfer orbit
T_{JC}	Time in Julian centuries
T_{JD}	Time in Julian dates
T_{J2-X}	Thrust force for the J2-X engines
T_{RoCS}	Thrust force for the RoCS
T_{RS-68b}	Thrust force for the RS-68b motors
T_{SRB}	Thrust force for the SRB motors
T_{max} or T_m	Maximum thrust force
T_K	Kinetic energy
T_p	Time of perigee passage
T_r	Rise time
T_s	Settling time
T_p	Peak time
TPBVP	Two point boundary value problem
tol	Tolerance value
$\vec{\Gamma}(k)$	Solar pressure force disturbance vector
θ	Pitch angle
θ_A	Momentum parameter
ϑ	End conditions co-state variables
U	Potential energy
US	Upper stage flight for Ares V
U_L	Magnetic potential function
$U_e(k)$	Unit value for the error containing the orbital elements
$U_X(k)$	Unit value for the state vector containing the orbital elements
$u^*(t)$	Particular control function
$u_A(t)$	Actuator control function
$u_m(t)$	Maximum input acceleration
$\vec{u}(t)$	Control vector function
$\vec{u}_A(t)$	Atmospheric force per unit mass
$\vec{u}_C(t)$	Control vector function containing adaptive and baseline controller
$\vec{u}_{SP}(t)$	Solar pressure force per unit mass
$\vec{\tilde{u}}(t)$	Adaptive control function
v_f	Relative angular velocity of the satellite

V	Energy equation
V_L	Lyapunov homogeneous function
\mathbb{V}	Volume
V_{Prop}	Velocity of the propellant
$V_{p,SB}$	Velocity at the perigee point for SB
$V_{p,SA}$	Velocity at the perigee point for SA
$V_{p,SC}$	Velocity at the perigee point for SC
$V_{p,SH}$	Velocity at the perigee point for SH
V_A	Velocity at the apogee point
\vec{V}	Velocity of center of mass or velocity vector
\hat{V}	Unit vector of the velocity vector
v	Velocity equation
v_{ESC}	Escape velocity
v_p	Velocity at the perigee point
v_i	Velocity of a particle
W	Work done on a system
W_V	Weight of the vehicle
w	Sum of the true anomaly angle and the argument of perigee
\bar{w}	Light energy per volume
$\vec{\omega}$	Angular velocity
X_B, Y_B, Z_B	Body coordinate frame
X_R, Y_R, Z_R	Reference coordinate frame
X_N, Y_N, Z_N	Nominal separation distance for the satellites in the ECI frame
X_S, Y_S, Z_S	Initial separation distance for the satellites in the ECI frame
\vec{X}	State vector for the orbital elements
\vec{X}_D	State vector for the desired orbital elements
x_{EM}, y_{EM}, z_{EM}	Magnetic field lines
$x_C(s)$	Transfer function for the command function
$x_A(s)$	Transfer function for the actuator function
\vec{x} or x_i	State vector ($i = 1, 2, 3, \dots$)
\vec{x}_0	Initial condition
\vec{x}_1	Final condition
\vec{x}_D	Desired state vector
\bar{y}	Linear transformation in the true anomaly angle of the separation distance

$Z \{ \}$

Z-transform

\bar{z}

State variables with integral terms

$\int_V () dV$

Volumetric integral

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Peter M. Bainum

Preface

This preface attempts to summarize the objective of this text. The authors have more than 54 years of practical real-world experience and attempt to encompass this experience in the chapters and suggested problems. Many of these problems have been adapted from these actual space projects and cannot be found in many contemporary text books.

This book will be used by practicing engineers and other texts may provide a deeper theoretical background. The reader can refer to more than 170 historical references, beginning with Arthur C. Clarke's 1945 reference describing his ideas for formation flying of three communication stations in a geosynchronous orbit in order to provide, through relay technology, instantaneous communication to/from any location on the Earth's surface, and also to/from various orbiting spacecraft.

Although the text begins with a review of orbital mechanics, the emphasis here is placed on digital control techniques and specific application for formation flying, deployment, station keeping, and reconfiguration. Part of the formation flying is based on research contracts with the Applied Physics Laboratory, and DARPA, focusing on on-track (string of pearls) and 3-D tetrahedron configurations in highly elliptical orbits. The tetrahedron was suggested as part of a design challenged by NASA Goddard.

Advanced modern control techniques, including intelligent control applications such as fuzzy logic, hierarchical control, and adaptive control, are proposed for the first time for formation flying applications. Some of these methods have been previously implemented for industrial applications, such as in the use of stepper motors.

Where possible, problems with analytical or closed-form solutions are suggested; in some cases a background in MATLAB and ability to solve nonlinear differential equations and in discrete form is assumed, as well as the ability to produce results in graphical format. In some cases answers and/or hints are provided. The philosophy here is that most of us learn by practicing, which means by working problems of various degrees of difficulty. Suggested problems attempt to follow the material covered in the preceding chapter.