

INTERNATIONAL STUDENT VERSION

Calculus

Early Transcendentals

Howard Anton

Irl C. Bivens

Stephen Davis

COMBINED
NINTH EDITION



9th
EDITION

CALCULUS

EARLY TRANSCENDENTALS

International Student Version

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with contributions by

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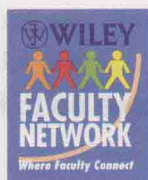
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About HOWARD ANTON

Howard Anton wrote the original version of this text and was the author of the first six editions. He obtained his B.A. from Lehigh University, his M.A. from the University of Illinois, and his Ph.D. from the Polytechnic University of Brooklyn, all in mathematics. In the early 1960s he worked for Burroughs Corporation and Avco Corporation at Cape Canaveral, Florida, where he was involved with the manned space program. In 1968 he joined the Mathematics Department at Drexel University, where he taught full time until 1983. Since that time he has been an adjunct professor at Drexel and has devoted the majority of his time to textbook writing and activities for mathematical associations. Dr. Anton was president of the EPADEL Section of the Mathematical Association of America (MAA), served on the Board of Governors of that organization, and guided the creation of the Student Chapters of the MAA. He has published numerous research papers in functional analysis, approximation theory, and topology, as well as pedagogical papers. He is best known for his textbooks in mathematics, which are among the most widely used in the world. There are currently more than one hundred versions of his books, including translations into Spanish, Arabic, Portuguese, Italian, Indonesian, French, Japanese, Chinese, Hebrew, and German. For relaxation, Dr. Anton enjoys traveling and photography.

About IRL BIVENS

Irl C. Bivens, recipient of the George Polya Award and the Merten M. Hasse Prize for Expository Writing in Mathematics, received his A.B. from Pfeiffer College and his Ph.D. from the University of North Carolina at Chapel Hill, both in mathematics. Since 1982, he has taught at Davidson College, where he currently holds the position of professor of mathematics. A typical academic year sees him teaching courses in calculus, topology, and geometry. Dr. Bivens also enjoys mathematical history, and his annual History of Mathematics seminar is a perennial favorite with Davidson mathematics majors. He has published numerous articles on undergraduate mathematics, as well as research papers in his specialty, differential geometry. He has served on the editorial boards of the MAA Problem Book series and *The College Mathematics Journal* and is a reviewer for *Mathematical Reviews*. When he is not pursuing mathematics, Professor Bivens enjoys juggling, swimming, walking, and spending time with his son Robert.

About STEPHEN DAVIS

Stephen L. Davis received his B.A. from Lindenwood College and his Ph.D. from Rutgers University in mathematics. Having previously taught at Rutgers University and Ohio State University, Dr. Davis came to Davidson College in 1981, where he is currently a professor of mathematics. He regularly teaches calculus, linear algebra, abstract algebra, and computer science. A sabbatical in 1995–1996 took him to Swarthmore College as a visiting associate professor. Professor Davis has published numerous articles on calculus reform and testing, as well as research papers on finite group theory, his specialty. Professor Davis has held several offices in the Southeastern section of the MAA, including chair and secretary-treasurer. He is currently a faculty consultant for the Educational Testing Service Advanced Placement Calculus Test, a board member of the North Carolina Association of Advanced Placement Mathematics Teachers, and is actively involved in nurturing mathematically talented high school students through leadership in the Charlotte Mathematics Club. He was formerly North Carolina state director for the MAA. For relaxation, he plays basketball, juggles, and travels. Professor Davis and his wife Elisabeth have three children, Laura, Anne, and James, all former calculus students.

About THOMAS POLASKI, contributor to the ninth edition

Thomas W. Polaski received his B.S. from Furman University and his Ph.D. in mathematics from Duke University. He is currently a professor at Winthrop University, where he has taught since 1991. He was named Outstanding Junior Professor at Winthrop in 1996. He has published articles on mathematics pedagogy and stochastic processes and has authored a chapter in a forthcoming linear algebra textbook. Professor Polaski is a frequent presenter at mathematics meetings, giving talks on topics ranging from mathematical biology to mathematical models for baseball. He has been an MAA Visiting Lecturer and is a reviewer for *Mathematical Reviews*. Professor Polaski has been a reader for the Advanced Placement Calculus Tests for many years. In addition to calculus, he enjoys travel and hiking. Professor Polaski and his wife, LeDayne, have a daughter, Kate, and live in Charlotte, North Carolina.

To
my wife Pat and my children: Brian, David, and Lauren

In Memory of
my mother Shirley
my father Benjamin
my thesis advisor and inspiration, George Bachman
my benefactor in my time of need, Stephen Girard (1750–1831)
—HA

To
my son Robert
—IB

To
my wife Elisabeth
my children: Laura, Anne, and James
—SD



PREFACE

This ninth edition of *Calculus* maintains those aspects of previous editions that have led to the series' success—we continue to strive for student comprehension without sacrificing mathematical accuracy, and the exercise sets are carefully constructed to avoid unhappy surprises that can derail a calculus class. However, this edition also has many new features that we hope will attract new users and also motivate past users to take a fresh look at our work. We had two main goals for this edition:

- To make those adjustments to the order and content that would align the text more precisely with the most widely followed calculus outlines.
- To add new elements to the text that would provide a wider range of teaching and learning tools.

All of the changes were carefully reviewed by an advisory committee of outstanding teachers comprised of both users and nonusers of the previous edition. The charge of this committee was to ensure that all changes did not alter those aspects of the text that attracted users of the eighth edition and at the same time provide freshness to the new edition that would attract new users. Some of the more substantive changes are described below.



NEW FEATURES IN THIS EDITION

New Elements in the Exercises We added new true/false exercises, new writing exercises, and new exercise types that were requested by reviewers of the eighth edition.

Making Connections We added this new element to the end of each chapter. A Making Connections exercise synthesizes concepts drawn across multiple sections of its chapter rather than using ideas from a single section as is expected of a regular or review exercise.

Reorganization of Review Material The precalculus review material that was in Chapter 1 of the eighth edition forms Chapter 0 of the ninth edition. The body of material in Chapter 1 of the eighth edition that is not generally regarded as precalculus review was moved to appropriate sections of the text in this edition. Thus, Chapter 0 focuses exclusively on those preliminary topics that students need to start the calculus course.

Parametric Equations Reorganized In the eighth edition, parametric equations were introduced in the first chapter and picked up again later in the text. Many instructors asked that we return to the traditional organization, and we have done so; the material on parametric equations is now first introduced and then discussed in detail in Section 10.1 (*Parametric Curves*). However, to support those instructors who want to continue the eighth edition path of giving an early exposure to parametric curves, we have provided Web materials (Web Appendix I) as well as self-contained exercise sets on the topic in Section 6.4 (*Length of a Plane Curve*) and Section 6.5 (*Area of a Surface of Revolution*).

Also, Section 14.4 (*Surface Area; Parametric Surfaces*) has been reorganized so surfaces of the form $z = f(x, y)$ are discussed before surfaces defined parametrically.

Differential Equations Reorganized We reordered and revised the chapter on differential equations so that instructors who cover only separable equations can do so without a forced diversion into general first-order equations and other unrelated topics. This chapter can be skipped entirely by those who do not cover differential equations at all in calculus.

New 2D Discussion of Centroids and Center of Gravity In the eighth edition and earlier, centroids and center of gravity were covered only in three dimensions. In this edition we added a new section on that topic in Chapter 6 (*Applications of the Definite Integral*), so centroids and center of gravity can now be studied in two dimensions, as is common in many calculus courses.

Related Rates and Local Linearity Reorganized The sections on related rates and local linearity were moved to follow the sections on implicit differentiation and logarithmic, exponential, and inverse trigonometric functions, thereby making a richer variety of techniques and functions available to study related rates and local linearity.

Rectilinear Motion Reorganized The more technical aspects of rectilinear motion that were discussed in the introductory discussion of derivatives in the eighth edition have been deferred so as not to distract from the primary task of developing the notion of the derivative. This also provides a less fragmented development of rectilinear motion.

Other Reorganization The section *Graphing Functions Using Calculators and Computer Algebra Systems*, which appeared in the text body of the eighth edition, is now a text appendix (Appendix A), and the sections *Mathematical Models* and *Second-Order Linear Homogeneous Differential Equations* are now posted on the Web site that supports the text.

OTHER FEATURES

Flexibility This edition has a built-in flexibility that is designed to serve a broad spectrum of calculus philosophies—from traditional to “reform.” Technology can be emphasized or not, and the order of many topics can be permuted freely to accommodate each instructor’s specific needs.

Rigor The challenge of writing a good calculus book is to strike the right balance between rigor and clarity. Our goal is to present precise mathematics to the fullest extent possible in an introductory treatment. Where clarity and rigor conflict, we choose clarity; however, we believe it to be important that the student understand the difference between a careful proof and an informal argument, so we have informed the reader when the arguments being presented are informal or motivational. Theory involving ϵ - δ arguments appears in a separate section so that it can be covered or not, as preferred by the instructor.

Rule of Four The “rule of four” refers to presenting concepts from the verbal, algebraic, visual, and numerical points of view. In keeping with current pedagogical philosophy, we used this approach whenever appropriate.

Visualization This edition makes extensive use of modern computer graphics to clarify concepts and to develop the student’s ability to visualize mathematical objects, particularly

those in 3-space. For those students who are working with graphing technology, there are many exercises that are designed to develop the student's ability to generate and analyze mathematical curves and surfaces.

Quick Check Exercises Each exercise set begins with approximately five exercises (answers included) that are designed to provide students with an immediate assessment of whether they have mastered key ideas from the section. They require a minimum of computation and are answered by filling in the blanks.

Focus on Concepts Exercises Each exercise set contains a clearly identified group of problems that focus on the main ideas of the section.

Technology Exercises Most sections include exercises that are designed to be solved using either a graphing calculator or a computer algebra system such as *Mathematica*, *Maple*, or the open source program *Sage*. These exercises are marked with an icon for easy identification.

Applicability of Calculus One of the primary goals of this text is to link calculus to the real world and the student's own experience. This theme is carried through in the examples and exercises.

Career Preparation This text is written at a mathematical level that will prepare students for a wide variety of careers that require a sound mathematics background, including engineering, the various sciences, and business.

Trigonometry Review Deficiencies in trigonometry plague many students, so we have included a substantial trigonometry review in Appendix B.

Appendix on Polynomial Equations Because many calculus students are weak in solving polynomial equations, we have included an appendix (Appendix C) that reviews the Factor Theorem, the Remainder Theorem, and procedures for finding rational roots.

Principles of Integral Evaluation The traditional Techniques of Integration is entitled "Principles of Integral Evaluation" to reflect its more modern approach to the material. The chapter emphasizes general methods and the role of technology rather than specific tricks for evaluating complicated or obscure integrals.

Historical Notes The biographies and historical notes have been a hallmark of this text from its first edition and have been maintained. All of the biographical materials have been distilled from standard sources with the goal of capturing and bringing to life for the student the personalities of history's greatest mathematicians.

Margin Notes and Warnings These appear in the margins throughout the text to clarify or expand on the text exposition or to alert the reader to some pitfall.

SUPPLEMENTS

SUPPLEMENTS FOR THE STUDENT

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The Student Companion Site provides access to the following student supplements:

- Web Quizzes, which are short, fill-in-the-blank quizzes that are arranged by chapter and section.
- Additional textbook content, including answers to odd-numbered exercises and appendices.

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WileyPLUS, Wiley's digital-learning environment, is loaded with all of the supplements above, and also features the following:

- The E-book, which is an exact version of the print text, but also features hyperlinks to questions, definitions, and supplements for quicker and easier support.
- The Student Study Guide provides concise summaries for quick review, checklists, common mistakes/pitfalls, and sample tests for each section and chapter of the text.
- The Graphing Calculator Manual helps students to get the most out of their graphing calculator and shows how they can apply the numerical and graphing functions of their calculators to their study of calculus.
- Guided Online (GO) Exercises prompt students to build solutions step by step. Rather than simply grading an exercise answer as wrong, GO problems show students precisely where they are making a mistake.
- Are You Ready? quizzes gauge student mastery of chapter concepts and techniques and provide feedback on areas that require further attention.
- Algebra and Trigonometry Refresher quizzes provide students with an opportunity to brush up on material necessary to master calculus, as well as to determine areas that require further review.

SUPPLEMENTS FOR THE INSTRUCTOR

Instructor Companion Site

The Instructor Companion Site provides detailed information on the textbook's features, contents, and coverage and provides access to the following instructor supplements:

- PowerPoint slides cover the major concepts and themes of each section in a chapter.
- Personal-Response System questions ("Clicker Questions") appear at the end of each PowerPoint presentation and provide an easy way to gauge classroom understanding.

- Additional textbook content, such as Calculus Horizons and Explorations, back-of-the-book appendices, and selected biographies.

WileyPLUS

WileyPLUS, Wiley's digital-learning environment, is loaded with all of the supplements above, and also features the following:

- Homework management tools, which easily allow you to assign and grade questions, as well as gauge student comprehension.
- QuickStart features predesigned reading and homework assignments. Use them as-is or customize them to fit the needs of your classroom.
- The E-book, which is an exact version of the print text but also features hyperlinks to questions, definitions, and supplements for quicker and easier support.
- Animated applets, which can be used in class to present and explore key ideas graphically and dynamically—especially useful for display of three-dimensional graphs in multivariable calculus.

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THE ROOTS OF CALCULUS

Today's exciting applications of calculus have roots that can be traced to the work of the Greek mathematician Archimedes, but the actual discovery of the fundamental principles of calculus was made independently by Isaac Newton (English) and Gottfried Leibniz (German) in the late seventeenth century. The work of Newton and Leibniz was motivated by four major classes of scientific and mathematical problems of the time:

- Find the tangent line to a general curve at a given point.
- Find the area of a general region, the length of a general curve, and the volume of a general solid.
- Find the maximum or minimum value of a quantity—for example, the maximum and minimum distances of a planet from the Sun, or the maximum range attainable for a projectile by varying its angle of fire.
- Given a formula for the distance traveled by a body in any specified amount of time, find the velocity and acceleration of the body at any instant. Conversely, given a formula that

specifies the acceleration of velocity at any instant, find the distance traveled by the body in a specified period of time.

Newton and Leibniz found a fundamental relationship between the problem of finding a tangent line to a curve and the problem of determining the area of a region. Their realization of this connection is considered to be the “discovery of calculus.” Though Newton saw how these two problems are related ten years before Leibniz did, Leibniz published his work twenty years before Newton. This situation led to a stormy debate over who was the rightful discoverer of calculus. The debate engulfed Europe for half a century, with the scientists of the European continent supporting Leibniz and those from England supporting Newton. The conflict was extremely unfortunate because Newton's inferior notation badly hampered scientific development in England, and the Continent in turn lost the benefit of Newton's discoveries in astronomy and physics for nearly fifty years. In spite of it all, Newton and Leibniz were sincere admirers of each other's work.



ISAAC NEWTON (1642–1727)

Newton was born in the village of Woolsthorpe, England. His father died before he was born and his mother raised him on the family farm. As a youth he showed little evidence of his later brilliance, except for an unusual talent with mechanical devices—he apparently built a working water clock and a toy flour mill powered by a mouse. In 1661 he entered Trinity College in Cambridge with a deficiency in geometry. Fortunately, Newton caught the eye of Isaac Barrow, a gifted mathematician and teacher. Under Barrow's guidance Newton immersed himself in mathematics and science, but he graduated without any special distinction. Because the bubonic plague was spreading rapidly through London, Newton returned to his home in Woolsthorpe and stayed there during the years of 1665 and 1666. In those two momentous years the entire framework of modern science was miraculously created in Newton's mind. He discovered calculus, recognized the underlying principles of planetary motion and gravity, and determined that “white” sunlight was composed of all colors, red to violet. For whatever reasons he kept his discoveries to himself. In 1667 he returned to Cambridge to obtain his Master's degree and upon graduation became a teacher at Trinity. Then in 1669 Newton succeeded his teacher, Isaac Barrow, to the

Lucasian chair of mathematics at Trinity, one of the most honored chairs of mathematics in the world.

Thereafter, brilliant discoveries flowed from Newton steadily. He formulated the law of gravitation and used it to explain the motion of the moon, the planets, and the tides; he formulated basic theories of light, thermodynamics, and hydrodynamics; and he devised and constructed the first modern reflecting telescope. Throughout his life Newton was hesitant to publish his major discoveries, revealing them only to a select circle of friends,

perhaps because of a fear of criticism or controversy. In 1687, only after intense coaxing by the astronomer, Edmond Halley (discoverer of Halley's comet), did Newton publish his masterpiece, *Philosophiae Naturalis Principia Mathematica* (The Mathematical Principles of Natural Philosophy). This work is generally considered to be the most important and influential scientific book ever written. In it Newton explained the workings of the solar system and formulated the basic laws of motion, which to this day are fundamental in engineering and physics. However, not even the pleas of his friends could convince Newton to publish his discovery of calculus. Only after Leibniz published his results did Newton relent and publish his own work on calculus.

After twenty-five years as a professor, Newton suffered depression and a nervous breakdown. He gave up research in 1695 to accept a position as warden and later master of the London mint. During the twenty-five years that he worked at the mint, he did virtually no scientific or mathematical work. He was knighted in 1705 and on his death was buried in Westminster Abbey with all the honors his country could bestow. It is interesting to note that Newton was a learned theologian who viewed the primary value of his work to be its support of the existence of God. Throughout his life he worked passionately to date biblical events by relating them to astronomical phenomena. He was so consumed with this passion that he spent years searching the Book of Daniel for clues to the end of the world and the geography of hell.

Newton described his brilliant accomplishments as follows: "I seem to have been only like a boy playing on the seashore and diverting myself in now and then finding a smoother pebble or prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me."



GOTTFRIED WILHELM LEIBNIZ (1646–1716)

This gifted genius was one of the last people to have mastered most major fields of knowledge—an impossible accomplishment in our own era of specialization. He was an expert in law, religion, philosophy, literature, politics, geology, metaphysics, alchemy, history, and mathematics.

Leibniz was born in Leipzig, Germany. His father, a professor of moral philosophy at the University of Leipzig, died when Leibniz was six years old. The precocious boy then gained access to his father's library and began reading voraciously on a wide range of subjects, a habit that he maintained throughout his life. At age fifteen he entered the University of Leipzig as a law student and by the age of twenty received a doctorate from the University of Altdorf. Subsequently, Leibniz followed a career in law and international politics, serving as counsel to kings and princes. During his numerous foreign missions, Leibniz came in contact with outstanding mathematicians and scientists who stimulated his interest in mathematics—most notably, the physicist Christian Huygens. In mathematics Leibniz was self-taught, learning the subject by reading papers and journals. As a result of this fragmented mathematical education, Leibniz often rediscovered the results of others, and this helped to fuel the debate over the discovery of calculus.

Leibniz never married. He was moderate in his habits, quick-tempered but easily appeased, and charitable in his judgment of other people's work. In spite of his great achievements, Leibniz never received the honors showered on Newton, and he spent his final years as a lonely embittered man. At his funeral there was one mourner, his secretary. An eyewitness stated, "He was buried more like a robber than what he really was—an ornament of his country."



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