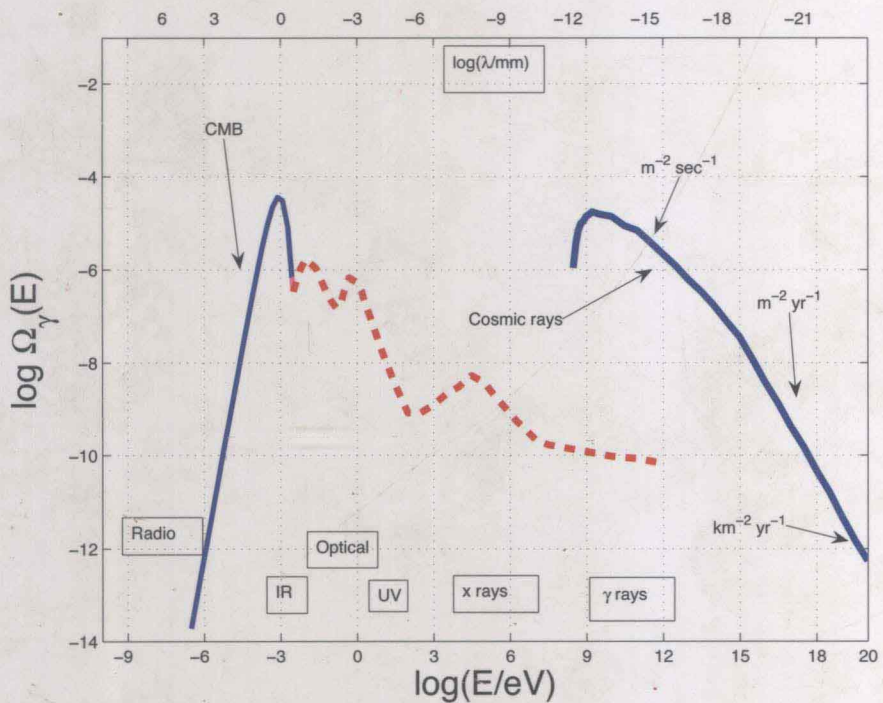


A Primer on the Physics of the Cosmic Microwave Background



Massimo Giovannini

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To Sergio Fubini

Preface

More than three score years ago, high-energy physicists were driven to scrutinize the properties of the cosmic radiation then available (i.e. cosmic rays). Today the same situation is realized not only with cosmic rays but also with different cosmological data: most notably, with the Cosmic Microwave Background (CMB in what follows). While I am writing this preface, European science is at the forefront of the developments in high-energy physics and cosmology thanks to the Large Hadron Collider program and thanks also to the Planck explorer mission. Today laboratory physics and celestial physics give us contradictory indications: it seems that all the matter accessible to terrestrial laboratory experiments contributes only 5% to the total energy budget of the Universe.

Cosmologists and astrophysicists today cannot ignore the knowledge of the micro-world provided by high-energy physics. In similar terms, high-energy physicists cannot avoid being exposed to some of the key concepts in modern gravitation and cosmology. While grand unifications of all fundamental forces are one of the intriguing hopes suggested by current theoretical speculations, the opportunity of a small unification lies already before us in the years to come: the construction of a common language which will allow, in the near future, a more effective exchange of information and ideas between contiguous branches of the physics community. The present book seeks to be a modest contribution to this mighty endeavor.

This book grew through the last decade because of various series of lectures that were either directly or indirectly connected to CMB physics and, more generally, to gravitation. In the last couple of years I came to the conclusion that an effective way of presenting a cosmology course (either for last year undergraduate or for PhD students) is to use CMB as a guiding theme. While lecturing to PhD students I have been confronted with the

problem of giving a sufficiently accurate and updated information to an audience that was, very often, rather composite. Not all PhD students were exposed to General Relativity or field theory in their undergraduate courses. Similarly, not all PhD students have a preliminary knowledge of astrophysics. I have tried, therefore, to present the material in a reasonably self-contained manner also in view of the time limitations imposed by a PhD course.

My warm acknowledgment goes to the Enrico Fermi center for a senior research grant entitled *From the Planck Scale to the Hubble Radius*. Without this support my efforts would have been forlorn. In commencing this script I wish also to express my very special and sincere gratitude to Prof. G. Cocconi and Prof. E. Picasso. I am indebted to G. Cocconi for his advice in the preparation of the first section. I am indebted to E. Picasso for delightful discussions which have been extremely relevant both for the selection of topics and for the overall quality of the manuscript.

Massimo Giovannini

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Part I

Cosmic Microwave Background Physics and the Formulation of the Standard Cosmological Model

Chapter 1

Why CMB Physics?

When approaching a new subject of study, especially within the realm of empirical sciences, the relevant question to ask is always the same: why should we learn about this? So, why should we learn about CMB physics? To answer this kind of questions one might be tempted to invoke either historical or subjective arguments. For instance one could say that, historically, blackbody emission is rather interesting in itself since it represented, at the dawn of the century, one of the fragile bridges that allowed us to pass from a classical description of macroscopic phenomena to the quantum mechanical language which is today the most appropriate for the discussion of microscopic physics. On a more aesthetic (and hence subjective) level, one could also affirm that blackbody emission is beautiful since it depends only upon one crucial parameter, i.e. the temperature. Subjectivity in science is very important since it drives the enthusiasm of researchers towards new and exciting fields of investigation. At the same time any subjective self-excitation should be gauged by more objective elements of judgment. Objectivity, for natural scientists, rhymes with testability. The quest for objectivity does not imply the lack of fantasy but, on the contrary, it just focuses our theoretical endeavor.

In this introductory chapter the theme will be to stress that there are objective elements that make CMB physics one of the most attractive and promising frameworks for gathering indirect informations on the early stages of the life of our own Universe. After a general introduction to blackbody emission, the motivations of this script will be spelled out. The bottom line will be that, indeed, the CMB is cosmological and represents the dominant component of the detected extra-galactic emission.

The whole observable Universe will therefore be approached, in the first approximation, as a system emitting electromagnetic radiation. The topics

to be treated in the present chapter are therefore the following:

- electromagnetic emission of the Universe;
- the blackbody spectrum;
- a bit of history of the CMB observations;
- the entropy of the CMB and its implications;
- the time evolution of the CMB temperature;
- a quick glance at the Sunyaev-Zeldovich effect.

All along this script the natural system of units will be adopted. In this system

$$\hbar = c = k_B = 1, \quad (1.1)$$

where $\hbar = h/2\pi$, c is the speed of light and k_B is the Boltzmann constant. In order to pass from one system of units to the other it is useful to recall that

- $\hbar c = 197.327 \text{ MeV fm}$;
- $K = 8.617 \times 10^{-5} \text{ eV}$;
- $(\hbar c)^2 = 0.389 \text{ GeV}^2 \text{ mbarn}$;
- $c = 2.99792 \times 10^{10} \text{ cm/sec}$.

In Fig. 1.1 a rather intriguing plot summarizes the electromagnetic emission of our own Universe. Only the extra-galactic emissions are reported.^a On the horizontal axis we have the logarithm of the energy of the photons (expressed in eV). On the vertical axis we reported the logarithm (to base 10) of $\Omega_\gamma(E)$ which is defined as

$$\Omega_\gamma(E) = \frac{1}{\rho_{\text{crit}}} \frac{d\rho_\gamma}{d \ln E}. \quad (1.2)$$

The specific form of $\Omega_\gamma(E)$ in the case of the CMB branch of the spectrum will be discussed in the following section (see, for instance, Eq. (1.12)). For the moment it suffices to note that $\Omega_\gamma(E)$ measures the energy density of the emitted radiation in critical units. The critical energy density ρ_{crit} can be understood, grossly speaking, as the mean energy density of the Universe, i.e. for the current values of the cosmological parameters, the energy density equivalent to about six proton masses per cubic meter (see, for instance, Eq. (1.11)).

^aBy extra-galactic emissions we mean radiation coming from the outside of our galaxy. Of course, as stressed later on, it must be borne in mind that our own galaxy is also an efficient emitter of electromagnetic radiation.