

THE FRANK J. FABOZZI SERIES

The background of the top half of the cover features a dark, blurred image of financial market data. It includes a candlestick chart with white and black bars, a line graph with several white lines, and a table of numerical data. The data table has columns with values like 10590, 10591, 10595, 10598, -60, -70, 1.1, -0.322%, -0.375%, 0.598%, -62, -77, and -202.

INTRODUCTION *to* FIXED INCOME ANALYTICS

SECOND
EDITION

*Relative Value Analysis,
Risk Measures, and Valuation*

FRANK J. FABOZZI • STEVEN V. MANN

Introduction to Fixed Income Analytics Second Edition

Relative Value Analysis, Risk Measures, and Valuation

FRANK J. FABOZZI
STEVEN V. MANNI



WILEY

John Wiley & Sons, Inc.

Copyright © 2010 by John Wiley & Sons, Inc. All rights reserved.

Published by John Wiley & Sons, Inc., Hoboken, New Jersey.
Published simultaneously in Canada.

No part of this publication may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, electronic, mechanical, photocopying, recording, scanning, or otherwise, except as permitted under Section 107 or 108 of the 1976 United States Copyright Act, without either the prior written permission of the Publisher, or authorization through payment of the appropriate per-copy fee to the Copyright Clearance Center, Inc., 222 Rosewood Drive, Danvers, MA 01923, (978) 750-8400, fax (978) 646-8600, or on the web at www.copyright.com. Requests to the Publisher for permission should be addressed to the Permissions Department, John Wiley & Sons, Inc., 111 River Street, Hoboken, NJ 07030, (201) 748-6011, fax (201) 748-6008, or online at <http://www.wiley.com/go/permissions>.

Limit of Liability/Disclaimer of Warranty: While the publisher and author have used their best efforts in preparing this book, they make no representations or warranties with respect to the accuracy or completeness of the contents of this book and specifically disclaim any implied warranties of merchantability or fitness for a particular purpose. No warranty may be created or extended by sales representatives or written sales materials. The advice and strategies contained herein may not be suitable for your situation. You should consult with a professional where appropriate. Neither the publisher nor author shall be liable for any loss of profit or any other commercial damages, including but not limited to special, incidental, consequential, or other damages.

For general information on our other products and services or for technical support, please contact our Customer Care Department within the United States at (800) 762-2974, outside the United States at (317) 572-3993, or fax (317) 572-4002.

Wiley also publishes its books in a variety of electronic formats. Some content that appears in print may not be available in electronic books. For more information about Wiley products, visit our web site at www.wiley.com.

Library of Congress Cataloging-in-Publication Data:

Fabozzi, Frank J.

Introduction to fixed income analytics : relative value analysis, risk measures, and valuation / Frank J. Fabozzi, Steven V. Mann.—2nd ed.

p. cm.—(Frank J. Fabozzi series ; 191)

Includes index.

ISBN 978-0-470-57213-9 (cloth); 978-0-470-92209-5 (ebk); 978-0-470-92210-1 (ebk)

1. Fixed-income securities. 2. Fixed income securities—Mathematics. 3. Rate of return. 4. Risk management. I. Mann, Steven V. II. Title.

HG4650.F335 2010

332.63'2—dc22

2010026721

Printed in the United States of America.

10 9 8 7 6 5 4 3 2 1

**Introduction to
Fixed Income
Analytics
Second Edition**

The Frank J. Fabozzi Series

- Fixed Income Securities, Second Edition* by Frank J. Fabozzi
- Focus on Value: A Corporate and Investor Guide to Wealth Creation* by James L. Grant and James A. Abate
- Handbook of Global Fixed Income Calculations* by Dragomir Krgin
- Managing a Corporate Bond Portfolio* by Leland E. Crabbe and Frank J. Fabozzi
- Real Options and Option-Embedded Securities* by William T. Moore
- Capital Budgeting: Theory and Practice* by Pamela P. Peterson and Frank J. Fabozzi
- The Exchange-Traded Funds Manual* by Gary L. Gastineau
- Professional Perspectives on Fixed Income Portfolio Management, Volume 3* edited by Frank J. Fabozzi
- Investing in Emerging Fixed Income Markets* edited by Frank J. Fabozzi and Efstathia Pilarinu
- Handbook of Alternative Assets* by Mark J. P. Anson
- The Global Money Markets* by Frank J. Fabozzi, Steven V. Mann, and Moorad Choudhry
- The Handbook of Financial Instruments* edited by Frank J. Fabozzi
- Collateralized Debt Obligations: Structures and Analysis* by Laurie S. Goodman and Frank J. Fabozzi
- Interest Rate, Term Structure, and Valuation Modeling* edited by Frank J. Fabozzi
- Investment Performance Measurement* by Bruce J. Feibel
- The Handbook of Equity Style Management* edited by T. Daniel Coggin and Frank J. Fabozzi
- The Theory and Practice of Investment Management* edited by Frank J. Fabozzi and Harry M. Markowitz
- Foundations of Economic Value Added, Second Edition* by James L. Grant
- Financial Management and Analysis, Second Edition* by Frank J. Fabozzi and Pamela P. Peterson
- Measuring and Controlling Interest Rate and Credit Risk, Second Edition* by Frank J. Fabozzi, Steven V. Mann, and Moorad Choudhry
- Professional Perspectives on Fixed Income Portfolio Management, Volume 4* edited by Frank J. Fabozzi
- The Handbook of European Fixed Income Securities* edited by Frank J. Fabozzi and Moorad Choudhry
- The Handbook of European Structured Financial Products* edited by Frank J. Fabozzi and Moorad Choudhry
- The Mathematics of Financial Modeling and Investment Management* by Sergio M. Focardi and Frank J. Fabozzi
- Short Selling: Strategies, Risks, and Rewards* edited by Frank J. Fabozzi
- The Real Estate Investment Handbook* by G. Timothy Haight and Daniel Singer
- Market Neutral Strategies* edited by Bruce I. Jacobs and Kenneth N. Levy
- Securities Finance: Securities Lending and Repurchase Agreements* edited by Frank J. Fabozzi and Steven V. Mann
- Fat-Tailed and Skewed Asset Return Distributions* by Svetlozar T. Rachev, Christian Menn, and Frank J. Fabozzi
- Financial Modeling of the Equity Market: From CAPM to Cointegration* by Frank J. Fabozzi, Sergio M. Focardi, and Petter N. Kolm
- Advanced Bond Portfolio Management: Best Practices in Modeling and Strategies* edited by Frank J. Fabozzi, Lionel Martellini, and Philippe Priaulet
- Analysis of Financial Statements, Second Edition* by Pamela P. Peterson and Frank J. Fabozzi
- Collateralized Debt Obligations: Structures and Analysis, Second Edition* by Douglas J. Lucas, Laurie S. Goodman, and Frank J. Fabozzi
- Handbook of Alternative Assets, Second Edition* by Mark J. P. Anson
- Introduction to Structured Finance* by Frank J. Fabozzi, Henry A. Davis, and Moorad Choudhry
- Financial Econometrics* by Svetlozar T. Rachev, Stefan Mittnik, Frank J. Fabozzi, Sergio M. Focardi, and Teo Jasic
- Developments in Collateralized Debt Obligations: New Products and Insights* by Douglas J. Lucas, Laurie S. Goodman, Frank J. Fabozzi, and Rebecca J. Manning
- Robust Portfolio Optimization and Management* by Frank J. Fabozzi, Peter N. Kolm, Dessislava A. Pachamanova, and Sergio M. Focardi
- Advanced Stochastic Models, Risk Assessment, and Portfolio Optimizations* by Svetlozar T. Rachev, Stogan V. Stoyanov, and Frank J. Fabozzi
- How to Select Investment Managers and Evaluate Performance* by G. Timothy Haight, Stephen O. Morrell, and Glenn E. Ross
- Bayesian Methods in Finance* by Svetlozar T. Rachev, John S. J. Hsu, Biliana S. Bagasheva, and Frank J. Fabozzi
- Structured Products and Related Credit Derivatives* by Brian P. Lancaster, Glenn M. Schultz, and Frank J. Fabozzi
- Quantitative Equity Investing: Techniques and Strategies* by Frank J. Fabozzi, CFA, Sergio M. Focardi, Petter N. Kolm

FJF

To my wife Donna
and my children Patricia, Karly, and Francesco

SVM

To my wife, Mary – TDA

Preface

Participants in the fixed income market are inundated with terms and concepts in both the popular press and, more typically, in research reports and professional journal articles. Making life more difficult for professionals in this market sector is the fact that for some important analytical concepts, the same concept is referred to in different ways by different dealer firms and asset management firms. The purpose of this book is to describe the key analytical concepts used in the fixed income market and illustrate how they are computed. The book is not only intended for professionals but also newcomers to the field. It is for this reason that we provide end of chapter questions.

Although market professionals often want a walk through demonstration of how a metric is computed, once they are comfortable with the concept and its computation, professionals then rely on vendors of analytical systems. Probably the most popular system relied upon by fixed income professionals is the Bloomberg System. For this reason, every chapter ties in the analytical concepts that are available on Bloomberg and walks the reader through the relevant Bloomberg screens. We want to thank Bloomberg Financial for granting us permission to reproduce the screens that we used in our exhibits.

We begin the book with an explanation of the most basic concept in finance: the time value of money. In Chapter 2, we describe yield curve analysis, discussing the importance of spot rates and forward rates. The fixed income market has adopted various conventions for determining the number of days when computing accrued interest when trades are settled. These market conventions are the subject of Chapter 3.

The basics of bond valuation are covered in Chapter 4. Our focus in this chapter is on option-free bonds (i.e., bonds that are not callable, puttable or convertible) and that have a fixed coupon rate. Yield measures for bonds are covered in Chapter 5.

The analysis of floating rate securities and bonds whose coupon interest is linked to some inflation measure are the subjects of Chapters 6 and 14, respectively. Bonds with embedded options are the subjects of Chapters 7, 9, and 10. Chapter 7 explains how to analyze callable and puttable agency and corporate bonds. All residential mortgage-backed securities and certain

asset-backed securities grant borrowers a prepayment option and, therefore, these securities have an embedded call option. Chapter 9 explains how these bonds are valued. For those readers unfamiliar with mortgage-backed and asset-backed securities, Chapter 8 explains them and how their cash flows are estimated. Convertible bond valuation is the subject of Chapter 10.

While one often hears about yield measures, portfolio managers are assessed based on their performance, which is measured in terms of total return. Chapter 11 demonstrates the calculation of this measure for individual bonds and portfolios.

A key analytical concept for quantifying and controlling the interest risk of a portfolio or trading position is duration and convexity. These measures of interest rate risk are explained in Chapter 12. One of the limitations of these two measures for use in portfolio risk management is that they assume that if interest rates change, the interest rate for all maturities change by the same amount. This is known as the parallel yield curve shift assumption. An analytical framework for assessing how a portfolio's value might change if this assumption is relaxed is the calculation of a portfolio's key rate durations, which is also explained in Chapter 12.

There are other measures used frequently for quantifying a portfolio's risk exposure. The most popular one is the value-at-risk (VaR) metric. In Chapter 13 we explain not only the reason for the popularity of this metric and alternative methodologies for calculating it, but the severe limitations of this measure. We explain a superior metric for quantifying risk, conditional VaR.

The approach to bond valuation described in the earlier chapters of the book are based on the discounted cash flow framework. Another approach to valuing bonds for inclusion in a portfolio or positioning for a trade is relative valuation. When properly interpreted, the tools of relative value analysis offer investors some clues about how similar bonds are currently priced in the market on a relative basis. Relative value analysis is the subject of Chapter 15.

An important derivative instrument in the fixed income market for controlling risk is an interest rate swap and is the subject of Chapter 16. After describing a swap's counterparties, risk-return profile, and economic interpretation, we illustrate how to value it.

As explained in several chapters, a key input into a valuation model is the expected interest rate volatility or expected yield volatility. How this measure is estimated is covered in Chapter 17.

We would like to thank Kimberly Bradshaw for her editorial assistance and Megan Orem for her patience in typesetting this book.

Frank J. Fabozzi
Steven V. Mann

About the Authors

Frank J. Fabozzi, Ph.D., CFA, CPA is Professor in the Practice of Finance in the Yale School of Management. Prior to joining the Yale faculty, he was a Visiting Professor of Finance in the Sloan School at MIT. Professor Fabozzi is a Fellow of the International Center for Finance at Yale University and on the Advisory Council for the Department of Operations Research and Financial Engineering at Princeton University. He is the editor of the *Journal of Portfolio Management* and an associate editor of the *Journal of Fixed Income* and the *Journal of Structured Finance*. He is a trustee for the BlackRock family of closed-end funds. In 2002, Professor Fabozzi was inducted into the Fixed Income Analysts Society's Hall of Fame and is the 2007 recipient of the C. Stewart Sheppard Award given by the CFA Institute. He has authored numerous books in investment management and structured finance. Professor Fabozzi earned a doctorate in economics from the City University of New York in 1972 and earned the designation of Chartered Financial Analyst and Certified Public Accountant.

Steven V. Mann, Ph.D., is Professor of Finance and Chair at the Moore School of Business, University of South Carolina. He has co-authored four books, edited two books, and written numerous articles in the area of investments, primarily fixed income securities and derivatives. Professor Mann is an accomplished teacher winning over 20 awards for excellence in teaching. He also works as a consultant to investment banks and commercial banks, and has conducted training programs for financial institutions throughout the United States. Professor Mann also serves as an expert witness in legal proceedings on matters involving fixed income securities.

Contents

Preface	xiii
About the Authors	xv
CHAPTER 1	
Time Value of Money	1
Future Value of a Single Cash Flow	1
Present Value of a Single Cash Flow	4
Compounding/Discounting When Interest Is Paid More Than Annually	8
Future and Present Values of an Ordinary Annuity	10
Yield (Internal Rate of Return)	20
Concepts Presented in this Chapter	26
Appendix: Compounding and Discounting in Continuous Time	27
Questions	31
CHAPTER 2	
Yield Curve Analysis: Spot Rates and Forward Rates	33
A Bond Is a Package of Zero-Coupon Instruments	33
Theoretical Spot Rates	34
Forward Rates	44
Dynamics of the Yield Curve	57
Concepts Presented in this Chapter	60
Questions	60
CHAPTER 3	
Day Count Conventions and Accrued Interest	63
Day Count Conventions	63
Computing the Accrued Interest	74
Concepts Presented in this Chapter	76
Questions	76

CHAPTER 4	
Valuation of Option-Free Bonds	77
General Principles of Valuation	77
Determining a Bond's Value	80
The Price/Discount Rate Relationship	84
Time Path of Bond	86
Valuing a Zero-Coupon Bond	90
Valuing a Bond Between Coupon Payments	90
Traditional Approach to Valuation	94
The Arbitrage-Free Valuation Approach	96
Concepts Presented in this Chapter	107
Questions	108
CHAPTER 5	
Yield Measures	109
Sources of Return	109
Traditional Yield Measures	113
Yield to Call	121
Yield to Put	123
Yield to Worst	123
Cash Flow Yield	124
Portfolio Yield Measures	125
Yield Measures for U.S. Treasury Bills	128
Yield Spread Measures Relative to a Spot Rate Curve	134
Concepts Presented in this Chapter	137
Appendix: Mathematics of the Internal Rate of Return	138
Questions	139
CHAPTER 6	
Analysis of Floating Rate Securities	141
General Features of Floaters	141
Valuing a Risky Floater	150
Valuation of Floaters with Embedded Options	157
Margin Measures	157
Concepts Presented in this Chapter	166
Questions	167
CHAPTER 7	
Valuation of Bonds with Embedded Options	169
Overview of the Valuation of Bonds with Embedded Options	169
Option-Adjusted Spread and Option Cost	170

Lattice Model	172
Binomial Model	175
Illustration	196
Concepts Presented in this Chapter	198
Questions	198
CHAPTER 8	
Cash Flow for Mortgage-Backed Securities and Amortizing Asset-Backed Securities	199
Cash Flow of Mortgage-Backed Securities	199
Amortizing Asset-Backed Securities	238
Concepts Presented in this Chapter	242
Questions	244
CHAPTER 9	
Valuation of Mortgage-Backed and Asset-Backed Securities	247
Static Cash Flow Yield Analysis	247
Monte Carlo Simulation/OAS	249
Concepts Presented in this Chapter	270
Questions	270
CHAPTER 10	
Analysis of Convertible Bonds	273
General Characteristics of Convertible Bonds	273
Tools for Analyzing Convertibles	276
Call and Put Features	278
Convertible Bond Arbitrage	279
Other Types of Convertibles	283
Concepts Presented in this Chapter	285
Questions	285
CHAPTER 11	
Total Return	287
Computing the Total Return	287
OAS-Total Return	290
Total Return to Maturity	291
Total Return for a Mortgage-Backed Security	299
Portfolio Total Return	301
Total Return Analysis for Multiple Scenarios	301
Concepts Presented in this Chapter	314
Questions	314

CHAPTER 12	
Measuring Interest Rate Risk	317
The Full Valuation Approach	317
Price Volatility Characteristics of Bonds	324
Duration	334
Other Duration Measures	350
Convexity	360
Price Value of a Basis Point	365
The Importance of Yield Volatility	367
Concepts Presented in this Chapter	369
Questions	370
CHAPTER 13	
Value-at-Risk Measure and Extensions	373
Value-at-Risk	373
Conditional Value-at-Risk	384
Concepts Presented in this Chapter	385
Questions	386
CHAPTER 14	
Analysis of Inflation-Protected Bonds	387
Breakeven Inflation rate	388
Valuation of TIPS	389
Measuring Interest Rate Risk	394
Concepts Presented in this Chapter	397
Questions	397
CHAPTER 15	
The Tools of Relative Value Analysis	399
How Portfolio Managers Add Value	399
Yield Spreads over Swap and Treasury Curves	400
Asset Swaps	403
Credit Default Swaps	410
Concepts Presented in this Chapter	413
Questions	414
CHAPTER 16	
Analysis of Interest Rate Swaps	417
Description of an Interest Rate Swap	417
Interpreting a Swap Position	419
Terminology, Conventions, and Market Quotes	421

Valuing Interest Rate Swaps	424
Primary Determinants of Swap Spreads	440
Dollar Duration of a Swap	445
Concepts Presented in this Chapter	447
Questions	447
CHAPTER 17	
Estimating Yield Volatility	451
Historical Volatility	451
Implied Volatility	455
Forecasting Yield Volatility	459
Concepts Presented in this Chapter	463
Questions	463
INDEX	485

Time Value of Money

A security is a package of cash flows. The cash flows are delivered across time with varying degrees of uncertainty. To value a security, we must determine how much this package of cash flows is worth today. This process employs a fundamental finance principle—the *time value of money*. Simply stated, one dollar today is worth more than one dollar to be received in the future. The reason is that the money has a time value. One dollar today can be invested, start earning interest immediately, and grow to a larger amount in the future. Conversely, one dollar to be received one year from today is worth less than one dollar delivered today. This is true because an individual can invest an amount of money less than one dollar today and at some interest rate it will grow to one dollar in a year's time.

The purpose of this chapter is to introduce the fundamental principles of future value (i.e., compounding cash flows) and present value (i.e., discounting cash flows). These principles will be employed in every chapter in the remainder of the book. To be sure, no matter how complicated the security's cash flows become (e.g., bonds with embedded options, interest rate swaps, etc.), determining how much they are worth today involves taking present values. In addition, we introduce the concept of yield, which is a measure of potential return and explain how to compute the yield on any investment.

FUTURE VALUE OF A SINGLE CASH FLOW

Suppose an individual invests \$100 at 5% compounded annually for three years. We call the \$100 invested the *original principal* and denote it as P . In this example, the annual interest rate is 5% and is the compensation the investor receives for giving up the use of his or her money for one year's time. Intuitively, the interest rate is a bribe offered to induce an individual to postpone their consumption of one dollar until some time in the future. If interest is compounded annually, this means that interest is paid for use of the money only once per year.

We denote the interest rate as i and put it in decimal form. In addition, N is the number of years the individual gives up use of his or her funds and FV_N is the future value or what the original principal will grow to after N years. In our example,

$$P = \$100$$

$$i = 0.05$$

$$N = 3 \text{ years}$$

So the question at hand is how much \$100 will be worth at the end of three years if it earns interest at 5% compounded annually?

To answer this question, let's first determine what the \$100 will grow to after one year if it earns 5% interest annually. This amount is determined with the following expression

$$FV_1 = P(1 + i)$$

Using the numbers in our example

$$FV_1 = \$100(1.05) = \$105$$

In words, if an individual invests \$100 that earns 5% compounded annually, at the end of one year the amount invested will grow to \$105 (i.e., the original principal of \$100 plus \$5 interest).

To find out how much the \$100 will be worth at the end of two years, we repeat the process one more time

$$FV_2 = FV_1(1 + i)$$

From the expression above, we know that

$$FV_1 = P(1 + i)$$

Substituting this in the expression and then simplifying, we obtain

$$FV_2 = P(1 + i)(1 + i) = P(1 + i)^2$$

Using the numbers in our example, we find that

$$FV_2 = \$100(1.05)^2 = \$110.25$$

Note that during the second year, we earn \$5.25 in interest rather than \$5 because we are earning interest on our interest from the first year. This

example illustrates an important point about how securities' returns work; returns reproduce multiplicatively rather than additively.

To find out how much the original principal will be worth at the end of three years, we repeat the process one last time

$$FV_3 = FV_2(1 + i)$$

Like before, we have already determined FV_2 , so making this substitution and simplifying gives us

$$FV_3 = P(1 + i)^2(1 + i)$$

$$FV_3 = P(1 + i)^3$$

Using the numbers in our example, we find that

$$FV_3 = \$100(1.05)^3 = \$115.7625$$

The future value of \$100 invested for three years earning 5% interest compounded annually is \$115.7625.

The general formula for the future value of a single cash flow N years in the future given an interest rate i is

$$FV_N = P(1 + i)^N \quad (1.1)$$

From this expression, it is easy to see that for a given original principal P the future value will depend on the interest rate (i) and the number of years (N) that the cash flow is allowed to grow at that rate. For example, suppose we take the same \$100 and invest it at 5% interest for 10 years rather than five years, what is the future value? Using the expression presented above, we find that the future value is

$$FV_N = \$100(1.05)^{10} = \$162.8894$$

Now let us leave everything unchanged except the interest rate. What is the future value of \$100 invested for 10 years at 6%? The future value is now

$$FV_N = \$100(1.06)^{10} = \$179.0848$$

As we will see in due course, the longer the investment, the more dramatic the impact of even relatively small changes in interest rates on future values.