

THE FRANK J. FABOZZI SERIES



INTRODUCTION *to* FIXED INCOME ANALYTICS

SECOND
EDITION

*Relative Value Analysis,
Risk Measures, and Valuation*

FRANK J. FABOZZI • STEVEN V. MANN

Introduction to Fixed Income Analytics Second Edition

Relative Value Analysis, Risk Measures, and Valuation

FRANK J. FABOZZI
STEVEN V. MANNE



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FJF

To my wife Donna
and my children Patricia, Karly, and Francesco

SVM

To my wife, Mary – TDA

Preface

Participants in the fixed income market are inundated with terms and concepts in both the popular press and, more typically, in research reports and professional journal articles. Making life more difficult for professionals in this market sector is the fact that for some important analytical concepts, the same concept is referred to in different ways by different dealer firms and asset management firms. The purpose of this book is to describe the key analytical concepts used in the fixed income market and illustrate how they are computed. The book is not only intended for professionals but also newcomers to the field. It is for this reason that we provide end of chapter questions.

Although market professionals often want a walk through demonstration of how a metric is computed, once they are comfortable with the concept and its computation, professionals then rely on vendors of analytical systems. Probably the most popular system relied upon by fixed income professionals is the Bloomberg System. For this reason, every chapter ties in the analytical concepts that are available on Bloomberg and walks the reader through the relevant Bloomberg screens. We want to thank Bloomberg Financial for granting us permission to reproduce the screens that we used in our exhibits.

We begin the book with an explanation of the most basic concept in finance: the time value of money. In Chapter 2, we describe yield curve analysis, discussing the importance of spot rates and forward rates. The fixed income market has adopted various conventions for determining the number of days when computing accrued interest when trades are settled. These market conventions are the subject of Chapter 3.

The basics of bond valuation are covered in Chapter 4. Our focus in this chapter is on option-free bonds (i.e., bonds that are not callable, putable or convertible) and that have a fixed coupon rate. Yield measures for bonds are covered in Chapter 5.

The analysis of floating rate securities and bonds whose coupon interest is linked to some inflation measure are the subjects of Chapters 6 and 14, respectively. Bonds with embedded options are the subjects of Chapters 7, 9, and 10. Chapter 7 explains how to analyze callable and putable agency and corporate bonds. All residential mortgage-backed securities and certain

asset-backed securities grant borrowers a prepayment option and, therefore, these securities have an embedded call option. Chapter 9 explains how these bonds are valued. For those readers unfamiliar with mortgage-backed and asset-backed securities, Chapter 8 explains them and how their cash flows are estimated. Convertible bond valuation is the subject of Chapter 10.

While one often hears about yield measures, portfolio managers are assessed based on their performance, which is measured in terms of total return. Chapter 11 demonstrates the calculation of this measure for individual bonds and portfolios.

A key analytical concept for quantifying and controlling the interest risk of a portfolio or trading position is duration and convexity. These measures of interest rate risk are explained in Chapter 12. One of the limitations of these two measures for use in portfolio risk management is that they assume that if interest rates change, the interest rate for all maturities change by the same amount. This is known as the parallel yield curve shift assumption. An analytical framework for assessing how a portfolio's value might change if this assumption is relaxed is the calculation of a portfolio's key rate durations, which is also explained in Chapter 12.

There are other measures used frequently for quantifying a portfolio's risk exposure. The most popular one is the value-at-risk (VaR) metric. In Chapter 13 we explain not only the reason for the popularity of this metric and alternatives methodologies for calculating it, but the severe limitations of this measure. We explain a superior metric for quantifying risk, conditional VaR.

The approach to bond valuation described in the earlier chapters of the book are based on the discounted cash flow framework. Another approach to valuing bonds for inclusion in a portfolio or positioning for a trade is relative valuation. When properly interpreted, the tools of relative value analysis offer investors some clues about how similar bonds are currently priced in the market on a relative basis. Relative value analysis is the subject of Chapter 15.

An important derivative instrument in the fixed income market for controlling risk is an interest rate swap and is the subject of Chapter 16. After describing a swap's counterparties, risk-return profile, and economic interpretation, we illustrate how to value it.

As explained in several chapters, a key input into a valuation model is the expected interest rate volatility or expected yield volatility. How this measure is estimated is covered in Chapter 17.

We would like to thank Kimberly Bradshaw for her editorial assistance and Megan Orem for her patience in typesetting this book.

Frank J. Fabozzi
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Time Value of Money

A security is a package of cash flows. The cash flows are delivered across time with varying degrees of uncertainty. To value a security, we must determine how much this package of cash flows is worth today. This process employs a fundamental finance principle—the *time value of money*. Simply stated, one dollar today is worth more than one dollar to be received in the future. The reason is that the money has a time value. One dollar today can be invested, start earning interest immediately, and grow to a larger amount in the future. Conversely, one dollar to be received one year from today is worth less than one dollar delivered today. This is true because an individual can invest an amount of money less than one dollar today and at some interest rate it will grow to one dollar in a year's time.

The purpose of this chapter is to introduce the fundamental principles of future value (i.e., compounding cash flows) and present value (i.e., discounting cash flows). These principles will be employed in every chapter in the remainder of the book. To be sure, no matter how complicated the security's cash flows become (e.g., bonds with embedded options, interest rate swaps, etc.), determining how much they are worth today involves taking present values. In addition, we introduce the concept of yield, which is a measure of potential return and explain how to compute the yield on any investment.

FUTURE VALUE OF A SINGLE CASH FLOW

Suppose an individual invests \$100 at 5% compounded annually for three years. We call the \$100 invested the *original principal* and denote it as P . In this example, the annual interest rate is 5% and is the compensation the investor receives for giving up the use of his or her money for one year's time. Intuitively, the interest rate is a bribe offered to induce an individual to postpone their consumption of one dollar until some time in the future. If interest is compounded annually, this means that interest is paid for use of the money only once per year.

We denote the interest rate as i and put it in decimal form. In addition, N is the number of years the individual gives up use of his or her funds and FV_N is the future value or what the original principal will grow to after N years. In our example,

$$P = \$100$$

$$i = 0.05$$

$$N = 3 \text{ years}$$

So the question at hand is how much \$100 will be worth at the end of three years if it earns interest at 5% compounded annually?

To answer this question, let's first determine what the \$100 will grow to after one year if it earns 5% interest annually. This amount is determined with the following expression

$$FV_1 = P(1 + i)$$

Using the numbers in our example

$$FV_1 = \$100(1.05) = \$105$$

In words, if an individual invests \$100 that earns 5% compounded annually, at the end of one year the amount invested will grow to \$105 (i.e., the original principal of \$100 plus \$5 interest).

To find out how much the \$100 will be worth at the end of two years, we repeat the process one more time

$$FV_2 = FV_1(1 + i)$$

From the expression above, we know that

$$FV_1 = P(1 + i)$$

Substituting this in the expression and then simplifying, we obtain

$$FV_2 = P(1 + i)(1 + i) = P(1 + i)^2$$

Using the numbers in our example, we find that

$$FV_2 = \$100(1.05)^2 = \$110.25$$

Note that during the second year, we earn \$5.25 in interest rather than \$5 because we are earning interest on our interest from the first year. This

example illustrates an important point about how securities' returns work; returns reproduce multiplicatively rather than additively.

To find out how much the original principal will be worth at the end of three years, we repeat the process one last time

$$FV_3 = FV_2(1 + i)$$

Like before, we have already determined FV_2 , so making this substitution and simplifying gives us

$$FV_3 = P(1 + i)^2(1 + i)$$

$$FV_3 = P(1 + i)^3$$

Using the numbers in our example, we find that

$$FV_3 = \$100(1.05)^3 = \$115.7625$$

The future value of \$100 invested for three years earning 5% interest compounded annually is \$115.7625.

The general formula for the future value of a single cash flow N years in the future given an interest rate i is

$$FV_N = P(1 + i)^N \quad (1.1)$$

From this expression, it is easy to see that for a given original principal P the future value will depend on the interest rate (i) and the number of years (N) that the cash flow is allowed to grow at that rate. For example, suppose we take the same \$100 and invest it at 5% interest for 10 years rather than five years, what is the future value? Using the expression presented above, we find that the future value is

$$FV_N = \$100(1.05)^{10} = \$162.8894$$

Now let us leave everything unchanged except the interest rate. What is the future value of \$100 invested for 10 years at 6%? The future value is now

$$FV_N = \$100(1.06)^{10} = \$179.0848$$

As we will see in due course, the longer the investment, the more dramatic the impact of even relatively small changes in interest rates on future values.