Applied
CALCULUS
for Business, Economics, Life Sciences, and Social Sciences
Sixth Edition
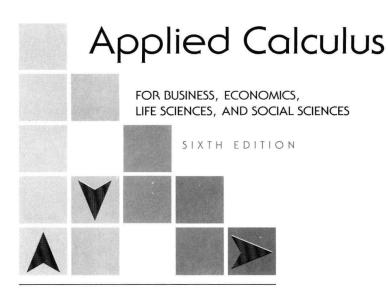

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Applied Calculus

FOR BUSINESS, ECONOMICS,
LIFE SCIENCES, AND SOCIAL SCIENCES

SIXTH EDITION

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CHAPTER DEPENDENCIES

PART ONE   A LIBRARY OF ELEMENTARY FUNCTIONS*

1
A Beginning Library of
Elementary Functions

2
Additional Elementary
Functions

PART TWO   CALCULUS

3
The Derivative

4
Graphing and Optimization

5
Additional Derivative Topics

6
Integration

8
Multivariable
Calculus

12
Trigonometric
Functions

9
Differential
Equations

7
Additional
Integration Topics

10
Taylor Polynomials
and Infinite Series

11
Probability
and Calculus

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* Selected topics from Part One may be referred to as needed in Part Two or reviewed systematically
before starting Part Two.

PREFACE

The sixth edition of *Applied Calculus for Business, Economics, Life Sciences, and Social Sciences* is designed for a one- or two-term course in calculus and for students who have had $1\frac{1}{2}$–2 years of high school algebra or the equivalent. The choice and independence of topics make the text readily adaptable to a variety of courses (see the chapter dependency chart on the preceding page). It is one of six books in the authors' college mathematics series.

Improvements in this edition evolved out of the generous response from a large number of users of the last and previous editions as well as survey results from instructors, mathematics departments, course outlines, and college catalogs. Fundamental to a book's growth and effectiveness is classroom use and feedback. Now in its sixth edition, *Applied Calculus for Business, Economics, Life Sciences, and Social Sciences* has had the benefit of having a substantial amount of both.

■ EMPHASIS AND STYLE

The text is **written for student comprehension.** Great care has been taken to write a book that is mathematically correct and accessible to students. Emphasis is on computational skills, ideas, and problem solving rather than mathematical theory. Most derivations and proofs are omitted except where their inclusion adds significant insight into a particular concept. General concepts and results are usually presented only after particular cases have been discussed.

■ EXAMPLES AND MATCHED PROBLEMS

Over 270 completely worked examples are used to introduce concepts and to demonstrate problem-solving techniques. Many examples have multiple parts, significantly increasing the total number of worked examples. Each example is followed by a similar **matched problem for the student to work** while reading the material. This actively involves the student in the learning process. The answers to these matched problems are included at the end of each section for easy reference.

■ EXPLORATION AND DISCUSSION

Every section contains **Explore–Discuss** boxes interspersed at appropriate places to encourage the student to think about a relationship or process before a result is stated, or to investigate additional consequences of a development in the text. **Verbalization** of mathematical concepts, results, and processes is encouraged in these Explore–Discuss boxes, as well as in some matched problems, and in some problems in almost every exercise set. The Explore–Discuss material also can be used as

in-class or out-of-class **group activities.** In addition, at the end of every chapter (before the chapter review), we have included a special **chapter group activity** that involves several of the concepts discussed in the chapter. To emphasize the importance of these special activities, Explore–Discuss and group activity materials are shaded in color, and similar problems in the exercise sets are identified by placing a color screen over the problem number—for example, 25.

■ EXERCISE SETS

The book contains over 3,700 problems. Many problems have multiple parts, significantly increasing the total number of problems. Each exercise set is designed so that an average or below-average student will experience success and a very capable student will be challenged. Exercise sets are mostly divided into A (routine, easy mechanics), B (more difficult mechanics), and C (difficult mechanics and some theory) levels.

■ APPLICATIONS

A major objective of this book is to give the student substantial experience in **modeling and solving real-world problems.** Enough applications are included to convince even the most skeptical student that mathematics is really useful (see the Applications Index inside the back cover). Worked examples involving applications are identified by ▶. **Almost every exercise set contains application problems,** usually divided into business and economics, life science, and social science groupings. An instructor with students from all three disciplines can let them choose applications from their own field of interest; if most students are from one of the three areas, then special emphasis can be placed there. Most of the applications are simplified versions of actual real-world problems taken from professional journals and books. No specialized experience is required to solve any of the applications.

■ TECHNOLOGY

The generic term **graphing utility** is used to refer to any of the various graphing calculators or computer software packages that might be available to a student using this book. (See the description of the software accompanying this book later in this Preface.) Although **access to a graphing utility is not assumed,** it is likely that many students will want to make use of one of these devices. To assist these students, **optional graphing utility activities** are included in appropriate places in the book. These include brief discussions in the text, examples or portions of examples solved on a graphing utility, and problems for the student to solve. All the optional graphing utility material is clearly identified by either ▭ or C and can be omitted without loss of continuity, if desired.

■ GRAPHS

All graphs are new and are computer-generated to ensure mathematical accuracy. Graphing utility screens displayed in the text are actual output from a graphing calculator.

■ STUDENT AIDS

Annotation of examples and developments, in small color type, is found throughout the text to help students through critical stages (see Sections 1-1 and 3-2). **Think boxes** (dashed boxes) are used to enclose steps that are usually performed mentally (see Sections 1-1 and 3-4). **Boxes** are used to highlight important definitions, theorems, results, and step-by-step processes (see Sections 1-1 and 3-2). **Caution** statements appear throughout the text where student errors often occur (see Sections 3-2 and 4-1). **Functional use of color** improves the clarity of many illustrations, graphs, and developments, and guides students through certain critical steps (see Sections 1-1 and 3-2). **Boldface type** is used to introduce new terms and highlight important comments. **Chapter review** sections include a review of all important terms and symbols and a comprehensive review exercise. **Answers to most review exercises,** keyed to appropriate sections, are included in the back of the book. Answers to all other odd-numbered problems are also in the back of the book.

■ CONTENT

The text begins with the development of a library of elementary functions in Chapters 1 and 2, including their properties and uses. We encourage students to investigate mathematical ideas and processes **graphically** and **numerically,** as well as **algebraically.** This development lays a firm foundation for studying mathematics both in this book and in future endeavors. Depending on the syllabus for the course and the background of the students, some or all of this material can be covered at the beginning of a course, or selected portions can be referred to as needed later in the course.

The material in Part Two consists of **differential calculus** (Chapters 3–5), **integral calculus** (Chapters 6 and 7), and a variety of **additional calculus topics** (Chapters 8–12). In general, Chapters 3–7 must be covered in sequence; however, certain sections can be omitted or given brief treatments, as pointed out in the discussion that follows. Chapters 8–12 are independent and can be covered in any order after completing Chapters 3–7 (see the chapter dependency chart on page ix).

Chapter 3 introduces the **derivative,** covers the **limit properties** essential to understanding the definition of the derivative, develops the **rules of differentiation** (including the chain rule for power forms), and introduces **applications** of derivatives in business and economics. The interplay between graphical, numerical, and algebraic concepts is emphasized here and throughout the text.

Chapter 4 focuses on **graphing** and **optimization.** The first three sections cover continuity and first-derivative and second-derivative graph properties, while emphasizing **polynomial graphing. Rational function** graphing is covered in Section 4-4.

In a course that does not include graphing rational functions, this section can be omitted or given a brief treatment. Optimization is covered in Section 4-5, including examples and problems involving end-point solutions.

The first three sections of Chapter 5 extend the derivative concepts discussed in Chapters 3 and 4 to **exponential and logarithmic functions** (including the general form of the chain rule). This material is required for all the remaining chapters. **Implicit differentiation** is introduced in Section 5-4 and applied to **related rate problems** in Section 5-5. These topics are not referred to elsewhere in the text and can be omitted.

Chapter 6 introduces **integration.** The first two sections cover **antidifferentiation** techniques essential to the remainder of the text. Section 6-3 discusses some applications involving **differential equations** that can be omitted. Sections 6-4 and 6-5 discuss the **definite integral** in terms of **Riemann sums,** including **approximations** with various types of sums and some **simple error estimation.** As before, the interplay between the graphical, numeric, and algebraic properties is emphasized. These two sections also are required for the remaining chapters in the text.

Chapter 7 covers **additional integration topics** and is organized to provide maximum flexibility for the instructor. The first section extends the **area** concepts introduced in Chapter 6 to the area between two curves and related applications. Section 7-2 covers three more **applications** of integration, and Sections 7-3 and 7-4 deal with additional **techniques of integration.**

The first five sections of Chapter 8 deal with **differential multivariable calculus** and can be covered any time after Section 5-3 has been completed. Section 8-6 requires the **integration** concepts discussed in Chapter 6.

After introducing the basic concepts and terminology used in the study of **differential equations** in Section 9-1, **separable differential equations** (Section 9-2) and **first-order linear differential equations** (Section 9-3) and related applications are covered thoroughly. All the growth laws introduced in Section 6-3 are covered again—this time with more emphasis on student recognition of the relevant growth law.

Chapter 10 begins by discussing the approximation of functions by **Taylor polynomials. Infinite series** follow naturally from the presentation, eliminating the need for a lengthy treatment of series of constants and all the associated convergence tests. The operations that can be performed on Taylor series and the effects of these operations on the interval of convergence are carefully discussed, since this is the way Taylor series are used in most real-world applications.

After discussing **improper integrals** in Section 11-1, the remainder of the chapter discusses properties of continuous probability density functions, including the uniform, exponential, and normal probability distributions. The presentation, though comprehensive, assumes no previous experience with probability.

Chapter 12 provides brief coverage of **trigonometric functions** that can be incorporated into the course, if desired. Section 12-1 provides a review of basic trigonometric concepts. Section 12-2 can be covered any time after Section 5-3 has been completed. Section 12-3 requires the material in Chapter 6.

■ PRINCIPAL CHANGES FROM THE FIFTH EDITION

As mentioned earlier, exploration and discussion activities have been distributed uniformly throughout the book. These new elements include Explore–Discuss questions in the text and exercise sets, and chapter group activities. The optional material on graphing utilities is also more uniformly distributed, but the major emphasis of the book is still on solving problems without the aid of technology.

Part One has been reorganized and mostly rewritten in order to present the student with a library of elementary functions and to encourage viewing mathematical ideas and processes graphically, numerically, and algebraically.

The material on limits has been reorganized. Limit concepts are introduced as they occur naturally in the development of calculus and now appear in three different sections. Section 3-2 develops the limit properties necessary to find derivatives by the definition, Section 4-1 discusses continuity and graphs, and Section 4-4 discusses limits at infinity and infinite limits.

The development of the definite integral has been extensively revised. Riemann sums, areas of rectangles, and simple error estimations are used to introduce the definite integral and motivate the fundamental theorem of calculus. This provides a traditional approach to this subject that emphasizes concept understanding and the relationship between graphs, numerical estimation, and antidifferentiation.

In Chapter 9, the sections on second-order differential equations and systems of differential equations have been omitted to provide a more concise treatment. Chapter 11 now focuses exclusively on applying calculus techniques to continuous probability density functions and no longer discusses any discrete probability distributions. The chapter on numerical techniques has been dropped. Some of these techniques are now discussed at appropriate places in the text, and others are presented as graphing utility activities. Appendix A, dealing with special topics from algebra, has been omitted. This material will be made available to interested schools in other forms.

■ SUPPLEMENTS FOR THE STUDENT

1. A **Student's Solutions Manual** by Garret J. Etgen is available through a book store. The manual includes detailed solutions to all odd-numbered problems and all review exercises.
2. *Visual Calculus* **software** for IBM-compatible computers is available at a nominal cost through a book store. *Visual Calculus* by David Schneider contains over twenty routines that provide additional insight into the topics discussed in the text. Although this software has much of the computing power of standard calculus software packages, it is primarily a teaching tool that focuses on understanding mathematical concepts, rather than on computing. These routines incorporate graphics whenever possible to illustrate topics such as secant lines; tangent lines; velocity; optimization; the relationship between the graphs of f, f', f''; and the various approaches to approximating definite integrals. All the routines in this

software package are menu-driven and very easy to use. The software is accompanied by a manual with instructions and additional exercises for the student. Hardware requirements are an IBM-compatible computer with at least 384K of memory and a graphics adapter: CGA, EGA, VGA, or Hercules.

3. A **Graphics Calculator Manual** by Carolyn L. Meitler contains examples illustrating the use of a graphics calculator to solve problems similar to those discussed in the text. The manual follows the chapter organization of the text, making it easy to find examples in the manual illustrating appropriate calculator solution methods for problems in the text. The manual includes keystrokes for the TI-81, TI-82, TI-83, and TI-85 calculators. However, the examples and techniques can be used with any graphing utility.

4. A **Supplemental Applications and Topics** manual by Jon E. Baum is available from the publisher. Part I of the manual expands the application exercises in the text and reinforces the important role of the mathematics presented. These exercises provide the student with a richer and more varied experience in solving real-world problems. Part II of the manual presents some applications that are not covered in the text, including transportation problems, assignment problems, sensitivity analysis, and a variety of finance topics. After completing the prerequisite material in the text, students interested in these more specialized topics will realize substantial benefits by studying this portion of the manual.

■ SUPPLEMENTS FOR THE INSTRUCTOR

For a summary of all available supplementary materials and detailed information regarding examination copy requests and orders, see page xx.

1. **PH Custom Test, a menu-driven random test system** for either IBM-compatible or Macintosh computers, is available to instructors without cost. The test system has been greatly expanded and now offers **on-line testing.** Carefully constructed algorithms use random-number generators to produce different, yet equivalent, versions of each of these problems. In addition, the system incorporates a unique **editing function** that allows the instructor to create additional problems, or alter any of the existing problems in the test, using a full set of mathematical notation. The test system offers **free-response, multiple-choice, and mixed exams.** An almost unlimited number of quizzes, review exercises, chapter tests, midterms, and final examinations, each different from the other, can be generated quickly and easily. At the same time, the system will produce answer keys, student worksheets, and a gradebook for the instructor, if desired.

2. An **Instructor's Resource Manual** provides over 120 transparency masters and all the answers not included in the text, as well as hard copy of test items available in PH Custom Test. This manual is available to instructors without charge.

3. A **Student's Solutions Manual** by Garret J. Etgen (see Student Aids) is available to instructors without charge from the publisher.

4. *Visual Calculus* **software** and accompanying **manual** by David Schneider (see Student Aids) are available to instructors without charge. The manual

contains complete instructions for using the software, eliminating the need to spend class time discussing these details, and examples and exercises for the student. In addition to providing students with the opportunity to use the computer as an effective tool in the learning process, instructors will find the software very useful for activities such as preparing examples for class, constructing test questions, and classroom demonstrations.

5. A **Graphics Calculator Manual** by Carolyn L. Meitler (see Student Aids) is available to instructors without charge from the publisher. The manual contains all the necessary information for a student with no previous experience with a graphic calculator, eliminating the need for the instructor to prepare materials related to calculator usage. In particular, separate appendixes for the TI-81, TI-82, TI-83, and TI-85 graphic calculators contain detailed instructions, including calculator-specific keystrokes, for performing the various operations required to effectively use each of these calculators to solve problems in the text. Furthermore, the methods illustrated for these calculators are easily adapted to other graphing utilities. The manual is very effective both for a class where all students purchase the same calculator and in a setting where students are using a variety of different calculators—an important consideration as more and more students arrive at college having already purchased a graphic calculator.

6. A **Supplemental Applications and Topics** manual by Jon E. Baum (see Student Aids) is available to instructors without charge from the publisher. Instructors can use Part I of this manual to supplement the exercise sets in the text, providing students with additional experience in solving applications utilizing the mathematics presented in the text. Part II of the manual can be used to provide coverage of applications not covered in the text, such as transportation problems, assignment problems, sensitivity analysis, and a variety of finance topics, either as part of the syllabus for a course or as subjects for independent study.

7. A **Basic Algebra Review** supplement, covering material previously included in Appendixes A and B, is available from the publisher.

■ ERROR CHECK

Because of the careful checking and proofing by a number of mathematics instructors (acting independently), the authors and publisher believe this book to be substantially error-free. For any errors remaining, the authors would be grateful if they were sent to: Michael R. Ziegler, 509 W. Dean Court, Fox Point, WI 53217; or, by e-mail, to: michael@mscs.mu.edu

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Producing this new edition with the help of all these extremely competent people has been a most satisfying experience.

R. A. Barnett
M. R. Ziegler

■ Ordering Information

When requesting examination copies or placing orders for this text or any of the related supplementary material listed below, please refer to the corresponding ISBN numbers.

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Applied Calculus for Business, Economics, Life Sciences, and Social Sciences, Sixth Edition	0-13-573213-1
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Supplemental Applications and Topics to accompany the Barnett and Ziegler College Mathematics Series	
Basic Algebra Review	

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