CALCULUS of a SINGLE VARIABLE

Thomas P. Dick
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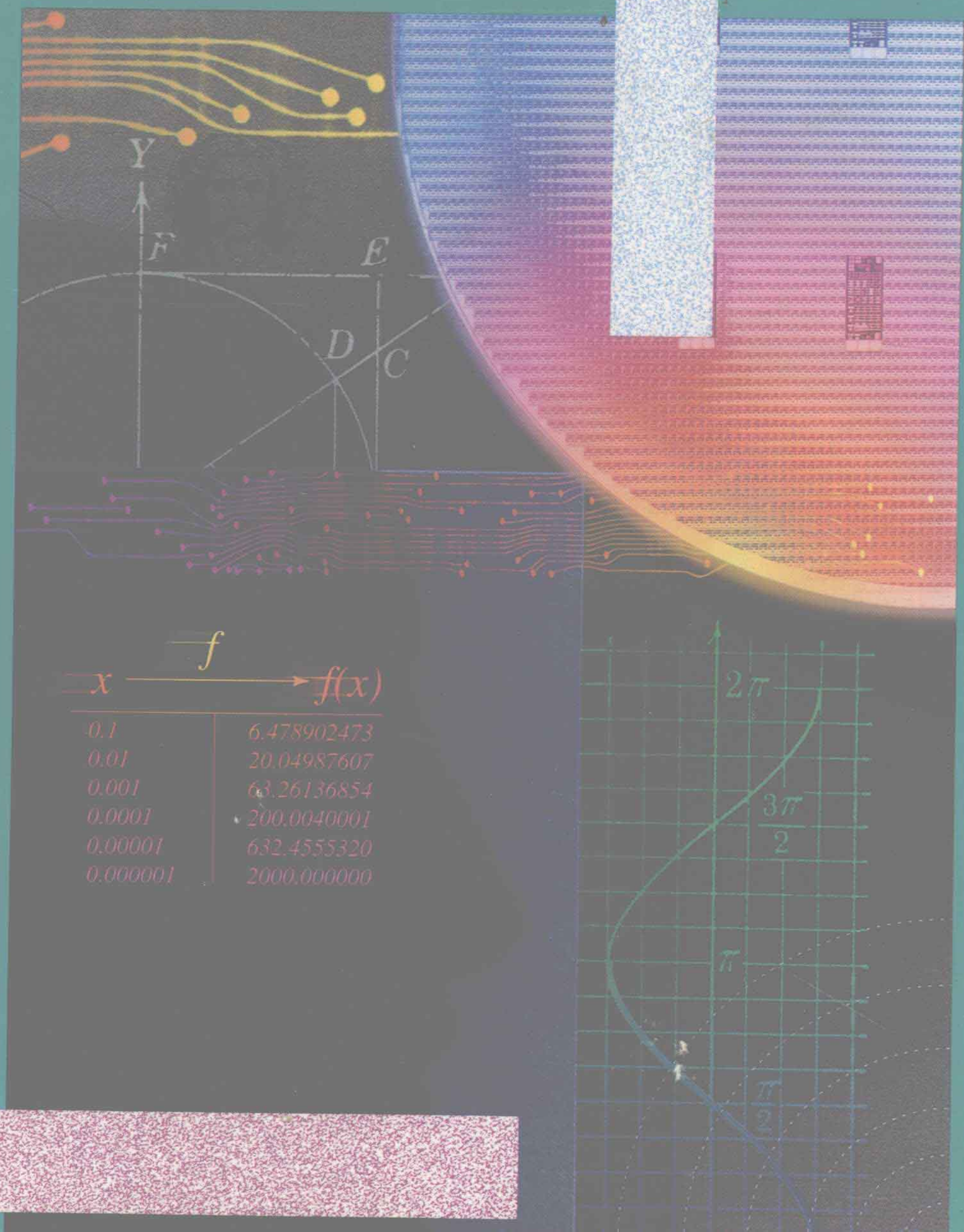

THE OREGON STATE UNIVERSITY
CALCULUS CONNECTIONS PROJECT

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Preface to the Instructor

Curriculum revision is generally a process of gradual evolution of scope and sequence, occasionally punctuated by calls for more fundamental changes in content or delivery. The launching of Sputnik precipitated such a call for reform in mathematics education in the 1950s. We now find ourselves in the midst of a new period of widespread revitalization efforts in mathematics curriculum and instruction. A forward-looking vision of the entire K-12 mathematics curriculum is outlined in the *Curriculum and Evaluation Standards* of the National Council of Teachers of Mathematics. The Mathematical Sciences Education Board has made an eloquent and urgent case for revitalizing mathematics instruction at all levels in preparation for our country's future workforce needs in *Everybody Counts*. Both of these influential documents recognize the emergence of sophisticated computer and calculator technology as redefining the tools of mathematics education.

Calculus occupies a particularly critical position in mathematics education as the gateway to advanced training in most scientific and technical fields. It is fitting that calculus should receive particular attention as we prepare for the needs of the twenty-first century. The Sloan Conference (Tulane, 1986) and the Calculus for a New Century Conference (Washington, 1987) sounded the call for reform in the calculus curriculum. Now the entire introductory course in calculus is being reexamined under the closest scrutiny that it has received in several years. Through a special funding initiative, the National Science Foundation has made resources available for a variety of calculus curriculum revision efforts to be tried and implemented. The Calculus Connections Project is one of these NSF-funded efforts, and this book is a major result of the project.

MAJOR THEMES

The text does not differ radically from a traditional calculus text in terms of major topics. This is as it should be - calculus reform will not change the importance and vitality of the major ideas of calculus, and any wholesale departure from those ideas should be viewed with great skepticism. What *is* possible is a fresh approach to these important ideas in light of the availability of modern technology. In particular, the technology can invite us to change or adopt new emphases in instruction.

Making intelligent use of technology

Computer algebra systems, spreadsheets, and graphing calculators are just a few of the readily available technological tools providing students with new windows of understanding and new opportunities for applying calculus. However, technology should not be viewed as a panacea for calculus instruction. This book seeks to take advantage of these new tools, while at the same time alerting the student to their inherent limitations and the care that must be taken to use technology wisely.

While being technology-aware, the text itself does not assume the availability of any particular machine or software. To do so would invite immediate obsolescence and ignore how quickly technology advances. Rather, the text adopts a language appropriate for the kinds of numerical, graphical, and symbolic capabilities that are found (and will continue to be found) on a wide variety of computer software packages and sophisticated calculators. For example, the language of "zooming in" on the graph of a function is powerfully suggestive without the need for listing specific keystrokes or syntax.

Technology can provide students new opportunities for understanding calculus, but it must be used with care. Numerical computations performed by a machine are subject to magnitude and precision limitations. For example, the calculation of difference quotients is naturally prone to cancellation errors. Machine-generated graphs can also provide misleading information, since graphs consist of discrete collections of pixels whose locations are computed numerically. Symbolic algebra results need to be interpreted in context. Helping students understand the limitations of technology is a major goal of this text. Students are reminded of the care that must be taken to make intelligent use of technology without becoming a victim of its pitfalls.

Since no specific hardware or software is assumed, an instructor will need to judge the appropriateness of any particular activity in light of the technology available. However, the exercises are designed to be compatible with a very wide variety of available software and hardware. A graphing calculator will be adequate for most of the activities.

Multiple representation approach to functions

The most important concept in all of mathematics is that of *function*, and the function concept is central in calculus. The idea of a function as a process accepting inputs and returning outputs can be captured in a variety of representations - numerically as a table of input-output pairs, graphically as a plot of outputs vs. inputs, and symbolically as a formula describing or modeling the input-output process. The interpretations of the core calculus topics of limits and continuity, differentiation, and integration all have different flavors when approached through different representations. The connections we forge among them enrich our personal concept of function.

All too often, students leave the calculus course with an impoverished mental image of function formed in a context dominated by symbolic forms. This book seeks to take a more balanced three-fold approach to functions. With each new topic or result, an explicit effort is made to interpret the meaning and consequences in a numerical, graphical, and symbolic context. Such an approach does not require technology, but the availability of an appropriate device allows us greater access to numerical and graphical representations, while at the same time reducing the need for heavy emphasis of rote "by hand" symbol manipulation skills.

Visualization and approximation

Two themes that become increasingly important with the availability of technology are visualization and approximation. The ability to obtain a machine-generated graph as a first step instead of a last one can completely turn around our approach to a variety of calculus topics. Graphical interpretation skills become primary. In particular, graphing can be used as a powerful problem-solving aid, both in estimating and in monitoring the reasonableness of results obtained numerically or symbolically. Whenever possible, explicit mention is made of the visual interpretation of definitions, theorems, and example solutions, often with direct reference to machine-generated graphs.

Much of calculus grew out of problems of approximation, and many of the key concepts of calculus are best understood as limits of approximations. Numerical tools make once exorbitantly tedious calculations into viable computational estimation strategies.

Accordingly, approximation and estimation techniques are given a high priority throughout the text.

Overview of the materials

Chapter 0 is a brief introduction to the major ideas of calculus, including a discussion of how the notions of infinite processes and approximation arise naturally in the study of real number measurement. Review material on absolute value and interval notation is included. Important issues in the use of technology in calculus are addressed, particularly the limitations of computers and calculators, such as numerical precision and the discrete nature of machine graphics. Functions are introduced as input-output processes, with an emphasis on multiple representations that recurs throughout the text. For students who do not have previous experience with the use of graphing calculators or software, the section on using graphing as a tool will provide a good introduction. Depending on the backgrounds of your students, the amount of time spent on Chapter 0 will vary widely from instructor to instructor.

Chapter 1 presents a library of functions. From the outset, transcendental functions are treated, including trigonometric, exponential, logarithmic, and inverse trigonometric functions. With respect to terminology and notation, the distinctions between variables, constants, and parameters are highlighted.

Chapter 2 discusses limits and continuity. Numerical and graphical approaches receive equal if not greater emphasis than symbolic techniques. The rigorous epsilon-delta definitions are included, but these are explained with reference to their numerical and graphical consequences, rather than a heavy emphasis on proofs. For example, the definition of continuity of a function has a dynamic interpretation in terms of the scaling of a graphing window. The "delta-hunt" for a particular epsilon becomes a search for a certain horizontal scaling, given a vertical scaling. Numerically, epsilons and deltas can be interpreted as output and input tolerances.

Chapter 3 starts with a review of linear functions, and then uses piece-wise linear functions to discuss the notions of *local slope* and *local linearity*. Differentiable functions can then be considered as *approximately* locally linear functions, an idea visually reinforced by zooming in on the graphs of functions. The physical interpretation of derivative as a rate of change is motivated by the problem of estimating a car's speedometer reading using its odometer and a stopwatch.

Difference quotients are used repeatedly to approximate derivative values, and are not considered just an artifact of the formal definition of derivative. Derivative properties and rules are developed and a dictionary of derivative formulas for all the basic algebraic and transcendental functions is included at the end of the chapter. Extra exercises on the mechanics of computing derivatives can be found in the appendices.

Chapter 4 emphasizes the use of the derivative as a measurement tool. The chapter begins by discussing the physical interpretation of derivative as a rate of change. A tangent line is considered as the graph of the best linear approximation of a differentiable function at a point (first-order Taylor forms). The use of the derivative to analyze function behavior, critical points, and extrema, and the consequences of the Mean Value Theorem are interpreted from both physical and graphical perspectives.

Chapter 5 treats applications and extensions of the derivative. The use of calculus to solve optimization problems is illustrated with examples and exercises drawn from the context of the day-to-day operations of a manufacturing facility. Higher-order derivatives are introduced using the physical example of a car's acceleration, and then used in a discussion of concavity and inflection points. This chapter also treats implicit differentiation and its application to related rates, and parametric equations and their application to particle motion.

Chapter 6 motivates the idea of definite integral through both the geometrical interpretation of area and the physical interpretation of accumulated change. By using piece-wise linear functions as examples, many of the properties of definite integrals are explored without the necessity of special summation formulas. A Riemann sum is then motivated as a reasonable approximation technique for more general functions. Antiderivatives are introduced by reversing the problem of determining a car's speedometer reading from its odometer and clock readings to one of determining distance covered from speed and time readings. *Slope fields* (direction fields) are used as a graphical means of approximating the graph of an antiderivative. Noting that $d = rt$ represents the area under the graph of a car's *constant* speed r over time t leads to the more general conjecture that definite integrals can be used to generate antiderivatives. The two fundamental theorems of calculus tie together differentiation and integration. Chapter 6 closes with a discussion of numerical and symbolic techniques of integration. The method of substitution is discussed as the integral counterpart to the chain rule. Additional material on techniques of integration can be found in the appendices.

Chapter 7 emphasizes applications of the definite integral as a measurement tool. The role of Riemann sums in modeling measurements involving continuously varying quantities is highlighted over and over again. Geometric examples include the measurement of area, volume, and arc length. Many of the definite integrals encountered in this chapter require the use of machine numerical integration. There is a discussion of applications of integration to measuring various averages, including moving averages. In turn, the notion of average value of a function suggests a Monte Carlo technique of numerical integration. The chapter then turns to physical applications such as velocity, force, and work, and ends with an introduction to improper integrals.

Chapter 8 discusses differential equations from a variety of viewpoints. Integration by parts is discussed as the counterpart to the product rule in searching for antiderivatives. Slope fields are used to visualize solutions to differential equations. The first fundamental theorem of calculus is reviewed for its use in creating antiderivatives. Exponential and logarithmic functions are both re-examined as the solutions of special differential equations. Applications of exponential functions to problems of growth and decay are included. Chapter 8 ends by treating Euler's method as a graphical application of the fundamental theorem of calculus.

Chapter 9 deals with function approximations. Error bounds for numerical integration are discussed from the point of view of interpolating polynomials. After Taylor polynomials are further developed as function approximations, techniques for using graphics to compare functions are discussed. This leads naturally to a revisiting of limits and indeterminate forms (L'Hopital's Rule). Chapter 9 concludes by examining cubic splines, which combine some of the attributes of both interpolating polynomials and Taylor polynomials (this section can be readily omitted without loss of continuity in the material).

Chapter 10 opens with a discussion of long division as a familiar example of an infinite process that can produce an infinite sequence of approximations. Several examples of sequences, including recursive and iterative sequences, are examined. Root-finding methods, including the bisection method and Newton's method, are also included as examples of iterative techniques yielding sequences of approximations. The Archimedean property of real numbers and Zeno's paradox are used to motivate the idea of a series. A series is then defined as the limit of a sequence of partial sums. Tests of convergence include the Nth term test, comparison and limit comparison tests, the integral test, the alternating series test, and the root and ratio tests. Absolute and conditional convergence are contrasted. After a discussion of power series, including interval and radius of convergence, this chapter concludes with a closer look at iterative methods in general.

The appendices provide review material on trigonometry as well as additional material on techniques of integration (including the method of partial fractions), polar coordinates, complex numbers, and Taylor's formula. The appendices conclude with additional practice exercises for differentiation and integration. Besides short answers to almost all the odd-numbered exercises, several useful formulas and tables can be found in the end pages of the book.

Corrections, comments, and criticisms of the materials are welcomed, and can be directed to the authors.

ANCILLARIES

The ***Instructor's Resource Manual*** provides answers to all the exercises, a pool of test items for each chapter (with answers), and additional commentary on goals and philosophy of the book, suggestions for pacing, and section-by-section notes to aid instructors using the materials.

The ***Student's Resource Manual*** is a supplement that provides detailed answers to selected exercises, as well as programs for graphing calculators that should prove useful in the sections on numerical integration, slope fields, Newton's method, and Euler's method.

If your school makes use of a computer algebra system in teaching calculus, or if students have access to such software - Mathematica, Maple, Derive, or Theorist - other resources are available from PWS Publishing Company. ***Notebooks*** have been prepared to accompany this book for each of these computer algebra systems. Each includes examples of step-by-step worked exercises from the text. Contact your bookstore for more information.

Preface to the Student

Books are written to be read. Yes, that is true even of mathematics books! We strongly encourage you to read the chapter introductions and the explanations in each section carefully, and to follow closely the discussion of examples. Perhaps you are accustomed to skipping to the exercises of a mathematics textbook first, and then searching back for an example that is a "clone" of the problem at hand that you can mimic. Certainly, this book has many examples and exercises to illustrate and help you practice your calculus skills. But there are also many problems in this text that ask you to reflect on and explain in your own words some of the important ideas of calculus. Other problems are designed to force you to think about these ideas in new ways. You may feel frustrated at times, but keep in mind that the effort you make to really understand the ideas in calculus will give you an ownership of them that will last long after you forget some of the specific technical details.

USING TECHNOLOGY TO STUDY CALCULUS

Some of the technology made possible by calculus includes devices such as graphing calculators and computers. We live in an exciting age where these powerful computational tools enable us to perform complex numerical and symbolic computations and provide tremendous graphics capabilities at our fingertips. In turn, we now have both new ways to understand the ideas of calculus and new opportunities to apply calculus.

However, even the most powerful technology is of little use if we do not know how it can and cannot be applied. We recognize that new technological tools are available and this book was written with the *intelligent* use of those tools in mind. The use of technology to study functions is not without its pitfalls. To use calculators and computers intelligently requires a knowledge of their limitations. Solving important mathematical problems will always require the inspiration, recognition, and application of the right idea at the right time.

If you have access to one of the computer algebra systems now being used to teach calculus at many schools - Mathematica, Maple, Derive, or Theorist - you may be interested in another problem-solving aid provided by the publisher. The *Notebooks* prepared to accompany this book are data disks comprised of worked examples and exercises from the text. The examples on each disk show, step-by-step, how to use a particular computer algebra system to solve selected problems from the text. (Contact your bookstore for more information.)

In calculus and other branches of mathematics, you will often encounter problems for which there is no specific recipe to solve them. Even in these instances, there are a variety of strategies you can use to make progress toward a solution. The next section gives you some hints from a master problem solver. We hope you find them useful.

GENERAL HINTS FOR SOLVING PROBLEMS: POLYA'S FOUR STEPS

George Polya (1887-1985) was considered by many as the greatest teacher of mathematical problem solving. In his work *How To Solve It*, Polya discusses in detail many aspects of the problem-solving process. He provides several useful general strategies (or heuristics) for mathematical problem solving. Here are the four basic steps Polya outlined in the problem-solving process.

POLYA'S FOUR STEPS IN PROBLEM SOLVING

1. UNDERSTAND THE PROBLEM
2. DEVISE A PLAN
3. CARRY OUT THE PLAN
4. LOOK BACK

Let's elaborate on these problem-solving steps.

1. UNDERSTAND THE PROBLEM

This means *understand what the problem is asking for*. While that may seem obvious, there are many times when we dive into a problem and waste a lot of time and effort that could have been saved by a few extra moments of reflection at the beginning. Ask yourself these questions: Do I understand all the terminology? What is given? What is the goal? Am I required to find something or to prove something? Is there enough information? Is there extraneous information? Have I seen a similar problem before? Rewriting the problem in your own words, drawing a figure, trying some examples are all ways to clarify a problem statement.

The full power of algebra and calculus can be unleashed if we can model a problem situation as a function or as an equation or inequality. We may be able to introduce a coordinate system for the purposes of graphing. The act of identifying and labeling variable quantities in and of itself may clarify aspects of the problem to us.

2. DEVISE A PLAN

Devise a plan of action for the problem. If you don't know where to begin, then try a general problem-solving heuristic or strategy. Three very useful heuristics include:

Trial and Error. At worst, you may get a better feel for the constraints of the problem situation. At best, you may stumble on the answer directly. Trial and error doesn't necessarily mean blind guesswork; our early guesses can help guide us in making better guesses. Making a list of the results of our trials may reveal a pattern or relationship. Mathematics is sometimes called the science or art of finding patterns.

Try a Simpler Problem. If the original problem seems too complex or confusing, try simplifying it first and solving that version. The solution to the simpler problem may give insights on how to solve the original problem. Exactly how do you make a problem simpler? Some of the ways include: substituting a smaller number in place of a larger one given in the problem; substituting a specific numerical value for an unknown constant or parameter (0 or 1 are often good substitution choices); making up a related problem that involves fewer dimensions or unknowns; and adding or dropping some of the problem constraints.

Try Extreme or Special Cases. We may get a special understanding from examining the problem situation in extreme or special cases. For example, if a problem involved the *elliptical* orbits of planets, we might benefit by considering the special case of a *circular* orbit. Substituting extreme values for an unknown variable can also give us useful information. For example, a question involving lines in the plane can be examined for the special cases of horizontal (zero slope) and vertical (undefined slope) lines. Making a list of special cases may also reveal a pattern or relationship.

3. CARRY OUT THE PLAN

Carry out your plan of action. Implement the strategy you've chosen until the problem is solved or until a new course of action is suggested. Give yourself a reasonable period of time to solve the problem. Monitor yourself. If you feel that you've embarked on a dead-end road then consider a change of strategy. Don't be afraid of starting all over. Many times a fresh start and a new strategy lead to success. You can have a flash of insight when you least expect it!

4. LOOK BACK

Check your answer to see if it really satisfies the requirements of your problem. Looking back means more than just checking your answer, though. Also look at your method of solution. Can you see another way of coming up with the answer? Can you see how your method could be used on other problems? Look forward to how you might generalize or extend your solution.

Calculus arose in response to the need to solve certain problems, and to understand the what and why of calculus requires understanding how, when, and where calculus can be used to solve problems. Calculus provides some very powerful tools for calculating quantities related to change. The derivative provides a means of measuring rates of change, and the integral provides a means of measuring accumulated change. This book is devoted to helping you understand these fundamental ideas so that you can successfully apply calculus to solving problems.

Your study of calculus can be an exciting intellectual adventure. Good luck on your journey!

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