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Nussbaum



**Electromagnetic
and Quantum Properties
of Materials**

SERIES IN SOLID STATE PHYSICAL ELECTRONICS

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**Electromagnetic
and Quantum Properties
of Materials**

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This book is for my
parents-in-law
Nellie and Tom Sydee

PREFACE

In this book, our study of the properties of materials rests principally on electromagnetic field theory and quantum theory, each of which is treated at both a beginning and an advanced level.

Chapter 1 surveys the behavior of electric and magnetic fields in free space, for the benefit of the reader who has either not had a fields course or who needs a review of the topics covered in introductory physics. It is assumed that the user is familiar with vector algebra (addition and the scalar and vector products) and elementary calculus. The vector operators are explained as part of the field concepts, and emphasis is placed on physical interpretation.

Chapter 2 covers quantum theory in an elementary way, plus the Boltzmann and Fermi distributions. The material here is sufficient for a complete understanding of semiconductors, dielectrics, and magnetic materials, and for most of the chapter on quantum electronics. Hence, the treatment of the Schroedinger equation and quantum statistics given in Chapter 3 may be by-passed completely at the discretion of the instructor. Other topics of an advanced nature, such as tensors, may also be omitted; these are denoted by an asterisk.

Chapter 4 is devoted to semiconductor theory and devices, with emphasis on the concepts of the Fermi level and the electrostatic, chemical, and electrochemical potentials. Chapter 5, on dielectrics, starts with a review of

fields in a material medium; since the discussion of microscopic properties leans so heavily on this idea, it was felt worthwhile to include it. A similar approach is taken in Chapter 6, on magnetic materials. The final chapter, on quantum electronics, also starts with a review, covering the vector model of the atom. One of the main subjects treated in this chapter is a qualitative description of masers and lasers. I have deliberately avoided the quantitative approach, and refer the interested reader to the specialized books.

In writing this book, I have received help from so many sources that it is difficult to enumerate them all. In the first place, this is a sequel to *Electromagnetic Theory for Engineers and Scientists*, Prentice-Hall, Inc. (1965), and I wish to reacknowledge those who gave assistance there. Of the many colleagues and friends who allowed me to utilize their ideas and original thoughts, I would particularly like to thank the following:

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Finally, I would like to express my appreciation to Nicholas C. Romanelli of Prentice-Hall, Inc., with whom it has been a pleasure to work, and to Mrs. Georgine B. Enger, for her extraordinary effort and skill in typing the manuscript.

A.N.

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1

VECTORS AND FIELDS

1-1 INTRODUCTION

The primary purpose of this chapter is to review two fundamental concepts of electromagnetic theory. One of them is the idea of a *vector* and the other is the notion of a *field*. These quantities have both mathematical and physical characteristics, and we shall briefly consider each of these aspects. First, however, let us introduce the material which forms a basis for our study of both electromagnetic fields and materials.

1-2 PHYSICAL QUANTITIES AND UNITS

All quantities used in physics and engineering can be expressed in terms of the following fundamental ones:

<i>mass</i>	<i>m</i>
<i>length</i>	<i>l</i>
<i>time</i>	<i>t</i>
<i>charge</i>	<i>q</i>
<i>temperature</i>	<i>T</i>

These five are considered to be undefinable and we think of them in an

intuitive fashion. For example, we associate with temperature the concepts of "hotness" or "coldness" without trying to give a precise definition of these terms.

The units of measurement which we shall adopt (and which are almost universally used in electrical engineering) are the *meter* (m) for length, the *kilogram* (kg) for mass, and the *second* (sec) for time. This choice is called the *MKS system*. If we then add the *coulomb* (coul) for charge, we have the fundamental quantities necessary to define all other electromagnetic parameters. It should be kept in mind that the choice of fundamental quantities made here is highly arbitrary. For example, mechanical engineers commonly employ a system in which *force* is used in place of mass in the above list and the unit of force is called the *pound*. The MKS system has the advantage of being compatible with electrical quantities such as the watt and the volt which have been in common use for many years.

1-3 SOME SCALAR AND VECTOR QUANTITIES

As stated in the preface, we are assuming that the reader has a background in vector algebra, and we shall use procedures such as vector addition and the scalar and vector products without definition or explanation. We shall also use the common convention of denoting vectors in printed text by bold-faced type (**A**) and in diagrams (or written notes) by placing an arrow over the symbol (\vec{A}).

We can define many familiar physical concepts in terms of the fundamental ones of the previous section. For example, *velocity* **v** is defined as the time-rate of change of length or *displacement* **l**, or

$$\mathbf{v} = \frac{d\mathbf{l}}{dt} \quad (1-1)$$

The magnitude v of the velocity **v** is called the *speed*, which is a scalar quantity, and both speed and velocity have dimensions of m/sec.

Next, we define *acceleration* **a** by

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{l}}{dt^2} \quad (1-2)$$

with units of m/sec², and *momentum* **p** by

$$\mathbf{p} = m\mathbf{v} \text{ kg-m/sec} \quad (1-3)$$

In terms of momentum, the quantitative definition of *force* **F** is

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \text{ newtons} \quad (1-4)$$

and the substitution of (1-3) into (1-4) gives the more familiar

$$\mathbf{F} = m\mathbf{a} \quad (1-5)$$

provided m is a constant.

Work W is defined as the application of a force over a distance, and the quantity of work is the product of the displacement and the amount of force exerted along the direction of motion. Since the force may be variable in both magnitude and direction, we express the amount of work as

$$W = \int F \cos \theta \, dl \text{ joules} \quad (1-6)$$

where θ is the angle between the force \mathbf{F} and the displacement $d\mathbf{l}$. Using the definition of the scalar product, Eq. (1-6) can be written

$$W = \int \mathbf{F} \cdot d\mathbf{l} \quad (1-7)$$

in vector notation.

A quantity related to work is *power* P , defined by

$$P = \frac{dW}{dt} \text{ watts} \quad (1-8)$$

where a watt is a joule/sec. Note that work and power are scalar quantities.

1-4 COULOMB'S LAW

Although it was stated that electric charge is a fundamental, undefinable quantity, we can give a precise meaning to the *magnitude* q of a charge in terms of the concept of force for which we do have a definition. Let us consider two charged particles in a vacuum (or air) and separated by a fixed distance r . The quantity or magnitude q of charge is that property of either particle to which the mutual force between them is directly proportional, or

$$F \propto q_1 q_2 \quad (1-9)$$

where q_1 and q_2 are the respective charges on each particle. Now Coulomb showed experimentally that for two given charges with a separation r , the force obeys the inverse-square law

$$F \propto \frac{1}{r^2} \quad (1-10)$$

Combining (1-9) and (1-10) and introducing a proportionality constant C gives

$$F = C \frac{q_1 q_2}{r^2} \quad (1-11)$$

and this relation is known as *Coulomb's law*.

A convenient natural standard of electric charge is furnished by the