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获奖论文集

区国强　吴恭孚　丘成桐　主编

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數學獎
HANG LUNG
MATHEMATICS AWARDS

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Hang Lung Mathematics Awards
Collection of Winning Papers 2006

恒隆
數學獎
HANG LUNG
MATHEMATICS AWARDS

數學科學研究所
The Institute of Mathematical Sciences
香港中文大學數學系
Department of Mathematics,
The Chinese University of Hong Kong
恒隆地產
HANG LUNG PROPERTIES

Preface

2006 Hang Lung Mathematics Awards (HLMA) marked the second occasion that the Department of Mathematics at The Chinese University of Hong Kong and Hang Lung Properties Ltd. collaborated and jointly held the biennial mathematics research competition for secondary school students in Hong Kong. We were pleased not only by the growing participation and interest among students and teachers, but also by the creativity and ingenuity of the students in carrying out their research. We were particularly delighted to hear that some schools have even started interest groups in mathematics research.

This book has collected the research papers of the winners of the 2006 HLMA which have met a high level of academic standards comparable to that of the top-tier university level thesis. It was gratifying to see that many gifted young students have risen to the challenge and have delivered excellent results under the quality guidance of their teachers.

We believe that mathematics is the foundation upon which scientific discoveries are built. By encouraging secondary students to engage in intellectual research, we hope to promote and cultivate a sense of serious scholarship among the younger generations.

We wish to take this opportunity to thank the editors for their tireless efforts, dedication and patience throughout the editing process. We hope the publishing of these excellent papers would serve to inspire more students to engage in academic discovery, and to enlist the support of teachers, schools, government, and the community in this process.

Professor Shing-Tung Yau

Director, The Institute of Mathematical Sciences
The Chinese University of Hong Kong

Mr. Ronnie C. Chan

Chairman
Hang Lung Properties Ltd.

October 2012

Acknowledgement

The publishing of this book has been made possible by donations from Hang Lung Properties Ltd.

The contribution to mathematics education from the initiators of Hang Lung Mathematics Awards, Mr. Ronnie C. Chan and Professor Shing-Tung Yau, will be marked in history. Professor Yau provides the academic leadership and vision for the Awards and he is also one of the editors of this volume. Hang Lung Properties Ltd. has been providing the financial support for the Awards and additional funding for the editorial and publishing cost of this volume.

The reputation of an academic award is always established upon its rigorous process of assessment and review. The Scientific Committee is the cornerstone of this intellectual excellence. The Screening Panel would like to thank Denix Auroux, Jianguo Huang, Jiaxin Hu, Charles Li, Qinghui Liu, Hui Rao, Yuguang Shi, Baorui Song, Ping-Kwan Tam, Yiming Xiao, Guangyuan Zhang, Guobiao Zhou for their professional judgment.

The Steering Committee has contributed greatly to the popularity of the Awards in society. Many students, teachers, and schools have participated in the competition. The Executive Committee, chaired by Thomas Au, is indebted to the secretariat led by Serena Yip and the assistance of Vris Cheung. The cooperation of Susan Wong, Joyce Kwong, and Joyce Leung from Hang Lung Properties has been invaluable.

This volume will never be perfect without the commitment of Kung-Fu Ng who have led the editorial work. He would like to express his sincere gratitude to Yuen-Ying Wong and Wei Zhou for their careful reading and comment on the winning papers. The technical work of Yat-Ming Cheung and Chun-Ngai Cheung and the coordination of Mavis Chan had also played a crucial role in the editorial process.

Hang Lung Mathematics Awards

Introduction The Hang Lung Mathematics Awards (HLMA), co-organized by Hang Lung Properties Ltd. and the Department of Mathematics at The Chinese University of Hong Kong, is a biennial research-based mathematics competition for secondary school students in Hong Kong. Founded in 2004 by Mr. Ronnie C. Chan, Chairman of Hang Lung Properties Ltd., and world-renowned mathematician Professor Shing-Tung Yau, a 1982 Fields Medalist and 2010 Wolf Prize recipient, the competition aims to stimulate creativity and to encourage intellectual discovery in mathematics and science among secondary school students in Hong Kong.

Schools are invited to form teams and, under the supervision of a lead teacher, the teams design and carry out a mathematics research project. Each team submits a project report summarizing the findings, which is evaluated by the Scientific Committee in a multi-step process similar to that for the selection of publication in a scientific journal. Short-listed teams are invited to participate in an oral defense of their project before members of the Scientific Committee. This final stage is modeled after a doctoral degree defense and comprises two parts: a public presentation of the research project followed by a closed-door inquiry. The winners of the HLMA will be decided after the oral defense.

Hang Lung Properties Ltd. donated over HK$2 million to each competition. The Department of Mathematics at The Chinese University of Hong Kong provides tuition scholarships to the teachers of the winning schools. The Department of Mathematics handles all administrative, operational, and educational aspects of the competition.

Participation Up to five students of the same secondary school may form a team to participate in the competition. The team shall be led by a teacher of the school. After a simple registration process which started in early 2005, each team performed their study and research on a topic selected by the team. Research reports were submitted in August 2006. There were 96 teams of 294 students from 51 schools participated and 55 of those teams submitted research reports.

Assessment To decide the winners, there are two stages of assessment: research report review and oral defense. The assessment in each stage is the jurisdiction of the Scientific Committee, with the support of the Screening Panel and external experts from the international mathematics community. In the reviewing stage, each research report has to pass an initial screening. Then it is sent to at least two referees of external experts. A shortlist will be selected by the Scientific Committee to proceed to the second stage of assessment, which is the oral defense. In this stage, each team will make a brief presentation of their research in front of the Scientific Committee. The presentation is open to the public. It is then followed by a closed door inquiry. The winners will be decided afterwards.

Awards Up to eight awards are presented to the mathematics projects that meet the highest academic standard in terms of research methodology and originality: Gold, Silver, Bronze, and up to five Honorable Mentions. Each award consists of four components: a "Student Education Award" to be shared equally among team members and applied towards their university studies; a "Teacher Leadership Award" for the supervising teacher; a "School Development Award" to promote mathematics education at the school; and a "Tuition Scholarship" for any teacher at the winning schools to earn a Master of Science (MSc) in Mathematics from The Chinese University of Hong Kong. The winning students and teachers also received a crystal trophy and a certificate while the school was presented with a crystal trophy.

Organization

The two principal committees of the Hang Lung Mathematics Awards are the Scientific Committee and the Steering Committee.

Scientific Committee, 2006

The Scientific Committee comprises world renowned mathematicians and is the academic and adjudicating body of the Hang Lung Mathematics Awards. The Scientific Committee oversees the whole process of assessment, including the review of the research project reports and conducting the oral defense. The committee upholds the academic integrity and high standard of the mathematics research competition as well as the awards.

List of members (affiliations at the time of the event)
Chair: Professor Tony F. Chan, University of California, Los Angeles
Professor Shiu Yuen Cheng, The Hong Kong University of Science and Technology
Professor John H. Coates, Cambridge University
Professor Jean-Pierre Kahane, Universite Paris-Sud Orsay
Professor Ka-Sing Lau, The Chinese University of Hong Kong
Professor Peter D. Lax, Courant Institute, New York University
Professor Christopher J. Lennard, University of Pittsburgh
Professor Kenneth C. Millett, University of California, Santa Barbara
Professor Ngaiming Mok, University of Hong Kong
Professor Cathleen S. Morawetz, New York University
Professor Gilbert Strang, Massachusetts Institute of Technology
Professor Robert S. Strichartz, Cornell University
Professor Tom Yau-Heng Wan, Chinese University of Hong Kong
Professor Lo Yang, Chinese Academy of Sciences
Professor Andrew Chi-Chih Yao, Tsing Hua University

Screening Panel is a subcommittee of the Scientific Committee. It handles the initial screening of each report, supervises external review process, and serves as a bridge between all referees and members of Scientific Committee.

Chair: Professor Tom Yau-Heng Wan, The Chinese University of Hong Kong
Professor Wing Sum Cheung, The University of Hong Kong
Professor Conan Nai Chung Leung, The Chinese University of Hong Kong

Steering Committee, 2006

The Steering Committee comprises mathematicians and representatives from different sectors of society and serves as the advisory body. The committee also includes mathematics department heads of major Hong Kong universities. Some members from the Scientific Committee and Executive Committee also serve in the Steering Committee so that it has an overall perspective of all the aspects.

List of members (affiliations at the time of the event)

Chair: Professor Sir James A. Mirrlees, 1996 Nobel Laureate in Economics

Professor Thomas Kwok-Keung Au, Mathematics, The Chinese University of Hong Kong

Professor Wing Sum Cheung, Chairman of Mathematics Department, The University of Hong Kong

Professor Ka-Sing Lau, Chairman of Mathematics Department, The Chinese University of Hong Kong

Professor Jian-Shu Li, Chairman of Mathematics Department, The Hong Kong University of Science and Technology

Mr. Siu Leung Ma, CEO, Fung Kai Public Schools

Mr. Chun Kau Poon, Principal, The Hong Kong Federation of Youth Group Lee Shau Kee College

Ms. Susan Wong, Chairman's Office, Hang Lung Properties Ltd.

Professor Lo Yang, Deputy Director, Morningside Center of Mathematics, Chinese Academy of Sciences

Mr. Chee Tim Yip, Principal, Pui Ching Middle School

Executive Committee is a subcommittee of the Steering Committee. It is in charge of the administration, promotion, team registration, communication, and organization of events including the oral defense.

Chair: Professor Thomas Kwok-Keung Au, The Chinese University of Hong Kong

Dr. Ka-Luen Cheung, The Chinese University of Hong Kong

Dr. Leung-Fu Cheung, The Chinese University of Hong Kong

Dr. Chi-Hin Lau, The Chinese University of Hong Kong

Secretariat: Ms. Serena Wing-Hang Yip, The Chinese University of Hong Kong

Gold, Silver, and Bronze

GOLD

Team member(s): Cheuk-Hin Cheng
School: S.K.H. Lam Woo Memorial Secondary School
Teacher: Mr. Kwok-Tai Wong
Topic: *How to Keep Water Cold –*
A Study about the Wet Contact Surface Area in Cylinder

SILVER

Team member(s): Yun-Pui Tsoi
School: Sha Tin Government Secondary School
Teacher: Mr. Wai-Man Ko
Topic: *On the Prime Number Theorem*

BRONZE

Team member(s): Chung-Yam Li, Fai Li, King-Ching Li,
Daniel Chung-Sing Poon
School: Wong Shiu Chi Secondary School
Teacher: Mr. Chun-Yu Kwong
Topic: *Construction of Tangents to Circles in Poincaré Model*

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HOW TO KEEP WATER COLD
A STUDY ABOUT THE WET CONTACT SURFACE AREA IN CYLINDER

TEAM MEMBERS
CHEUK-HIN CHENG[1]

SCHOOL
S.K.H. LAM WOO MEMORIAL SECONDARY SCHOOL

ABSTRACT. The question investigated in this essay is: Given the dimensions of a cylindrical container and the volume of the water contained in it, which position would give the minimum wet contact area? In part 1, we will discuss about which position of the container, horizontal or vertical, will give a smaller wet contact surface area when the volume of water varies. In part 2, we will still discuss the wet contact area in the cylinder, but considering the volume of water as a constant and allowing the cylinder to incline with a variable angle α. We will try to find out the value of α such that the total wet surface area is minimum.

1. Introduction

Actually, doing this research was not initially for Hang Lung Mathematics Award, but for a winter holiday assignment from the Mathematics Enrichment Class in my secondary school two years before writing this paper:

"Think of an interesting mathematics question, then try to solve it."

This reminded me of a problem which was long in my mind.

One day, I looked at a bottle of water on my desk. I laid it down and put it back to upright, repeatedly. Then I observed that the contact area between the bottle and water might be different when the bottle is placed horizontally and vertically. I would like to find out which position of the bottle, placed horizontally or vertically, can give the minimum wet contact

[1]This work is done under the supervision of the author's teacher, Mr. Kwok-Tai Cathay Wong.

area for different volume of water. I found this problem interesting because it may have some significance in the storage of some viscous liquids such as crude oil or chemicals. Or, at least I would know how to reduce the wet surface area and keep my water cold for a longer time.

After I described the problem to my teacher in the Enrichment Class, he told me that I have to study differentiation to solve this kind of problem. Later when I studied differentiation and solved the problem, I felt the happiness of using textbook knowledge to solve daily-life problems. I discovered an interesting result which was out of my expectation. I expected that, as the volume of water decreases, there will be only one switching point to change the position with smaller wet surface area, but finally I found that there can be either one or three switching points.

After I have solved the above problem (which is part 1 in this report), my mathematics teacher suggested me to investigate some generalisation of the original problem, such as a different shape of the container. Finally I decided to generalise the inclined angle of the container, so that the container is not restricted to only two positions. I found the problem in part two is more difficult, as there are more dependent variables which are interrelated, but they cannot be expressed explicitly in terms of other variables. Eventually I got another unexpected result: When the water level is lower than half the height of the cylinder, you should incline the cylinder for some angle to minimize the wet contact area!

In this report, when an equation apparently cannot be solved analytically, the Solver programme in Microsoft Excel will be used. However, I tried to use mathematical analysis to tackle the question whenever possible.

2. Definitions

As there are many variables and functions introduced throughout the report, a list of their definitions and notations is given here. The page number in the bracket indicates the first page the notation appears.

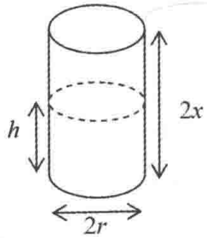
r	the radius of the cylinder (p.4)
$2x$	the height of the cylinder (p.4)
h	the water level when the cylinder is placed vertically (p.4)
V	the volume of the water (p.4)
A_1	the wet contact area when the cylinder is placed vertically (p.4)

A_2	the wet contact area when the cylinder is placed horizontally (p.4)
θ	the angle at center formed by the water segment on the base when the cylinder is placed horizontally, which is a variable (p.5)
$K(\theta)$	the wet contact area on the base when the cylinder is placed horizontally (p.5)
$f(\theta)$	$A_1 - A_2$ (p.6)
a	the first θ-intercept of the curve $y = f(\theta)$ (p.10)
b	the third θ-intercept of the curve $y = f(\theta)$ (p.10)
α	the inclined angle made with the ground when the cylinder is placed obliquely (p.13)
A_3	the wet contact area when the water covers the base of the cylinder totally and does not touch the lid (p.14)
A_4	the wet contact area when the water does not touch the lid and does not totally cover the base (p.14)
A_5	the wet contact area when the water touches both the lid and the base (p.14)
l	the shortest height of the water when the cylinder is placed obliquely (p.14)
p	the longest height minus the shortest height of the water when the cylinder is placed obliquely (p.14)
β	the angle at center formed by the water segment on the base when the cylinder is placed obliquely (p.15)
h'	the height of the cone formed by the water when the cylinder is placed obliquely (p.15)
γ	the angle at centre of the base formed by water when the water just touches the lid (p.15)
d	the depth of water on the base when the cylinder is placed obliquely (p.16)
$g(\beta)$	$\beta - \sin\beta$ (p.17)
ϕ	the angle at centre formed by the water segment when the cylinder is placed horizontally, which is a constant (p.17)
λ	the angle at centre formed by the water segment on the lid when the cylinder is placed obliquely (p.18)

3. Part 1 - Vertical or Horizontal?

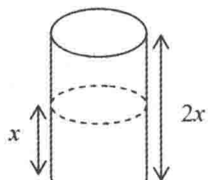
The problem we will investigate in this part is: Given the dimensions of a cylinder and the volume of water in the cylinder, which position, horizontal or vertical, would give a smaller total wet contact surface area? How will the result change if the volume of water varies?

Let the radius of the cylinder be r ($r > 0$), the height of the cylinder be $2x$ ($x > 0$), and the water level when the cylinder is in vertical position be h ($h \geqslant 0$). Then the volume of water V, which equals to $\pi r^2 h$, directly varies as h.



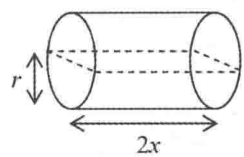
We can easily observe that if the cylinder is half full of water, the wet contact areas in the two positions are equal. They should be both equal to half of the curved surface area plus the area of one base of the cylinder.

The following is the proof.



When the cylinder is placed vertically,

$$\text{contact area}(A_1) = r^2 + 2r\pi x.$$



When the cylinder is placed horizontally,

$$\begin{aligned}\text{contact area}(A_2) &= 2 \times \frac{1}{2} r^2 \pi + \frac{1}{2} \times 2r\pi \times 2x \\ &= r^2\pi + 2r\pi x.\end{aligned}$$

Therefore, $A_1 = A_2$.

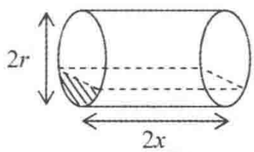
Now we consider the general case of $0 \leqslant h \leqslant 2x$.

It is obvious that when h is just slightly greater than 0, the horizontal position will give a smaller wet contact area, because the water touches the whole base in the vertical position, but just touch a very small area in the horizontal position. However, when h is just slightly less than $2x$, the vertical position will give a smaller wet contact area, because the water almost touches all the surface in the horizontal position, but the water will not touch the top surface of the cylinder in the vertical position. Therefore, we would like to investigate in what range of the value of h, the vertical (or the horizontal) position would give a smaller wet contact area.

When the cylinder is placed vertically,

$$\text{contact area}(A_1) = \begin{cases} \pi r^2 + 2r\pi h & \text{when } 0 < h < 2x, \\ 0 & \text{when } h = 0, \\ 2\pi r^2 + 2r\pi h & \text{when } h = 2x, \end{cases}$$

$$\text{volume of water}(V) = r^2\pi h.$$



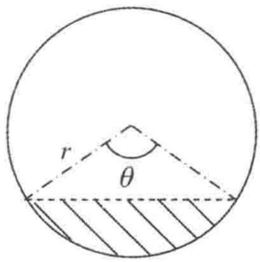


Figure of the base

When the cylinder is placed horizontally, let the shaded area, which is the wet contact area on the base, be $K(\theta)$. Then $K(\theta) = \frac{r^2\theta}{2} - \frac{r^2 \sin\theta}{2}$, where θ is the angle at center and $0 \leqslant \theta \leqslant 2\pi$.

$$\begin{aligned} \text{Contact area}(A_2) &= 2K(\theta) + r\theta \times 2x \\ &= 2K(\theta) + 2rx\theta \\ &= 2\left[\frac{r^2\theta}{2} - \frac{r^2\sin\theta}{2}\right] + 2rx\theta \\ &= r^2\theta - r^2\sin\theta + 2rx\theta. \end{aligned}$$

$$\begin{aligned} \text{Volume of water}(V) &= K(\theta) \times 2x \\ &= \left[\frac{r^2\theta}{2} - \frac{r^2\sin\theta}{2}\right] \times 2x \\ &= (r^2\theta - r^2\sin\theta)x. \end{aligned}$$