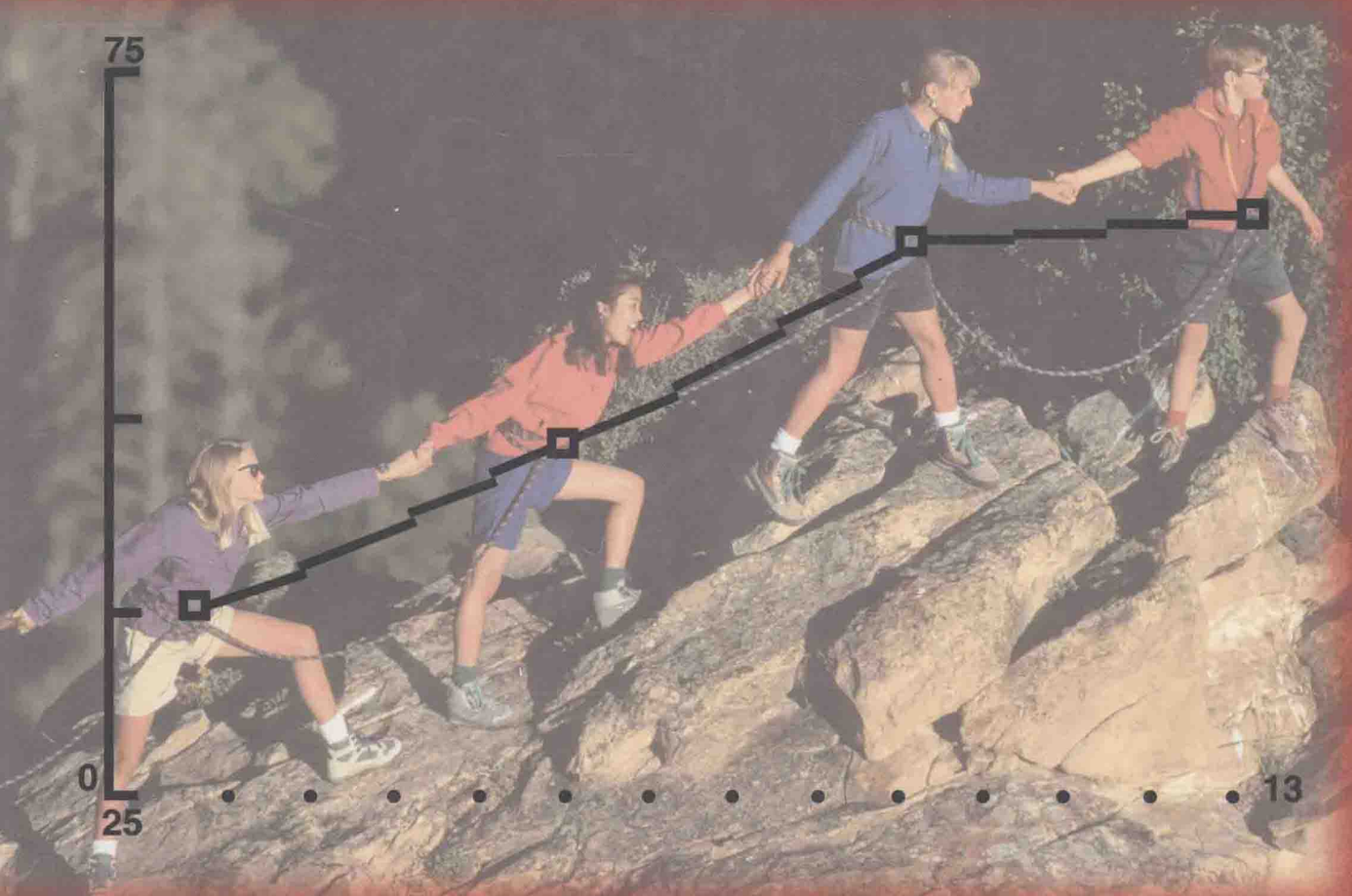


SECOND EDITION

# College Algebra

ENHANCED WITH GRAPHING UTILITIES



Michael Sullivan ♦

Michael Sullivan, III

SECOND  
EDITION

# College Algebra

*Enhanced with Graphing Utilities*

Michael Sullivan

*Chicago State University*

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*Joliet Junior College*

Prentice Hall

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# PREFACE TO THE INSTRUCTOR

As professors, one at an urban public university and the other at a community college, Michael Sullivan and Michael Sullivan III are aware of the varied needs of College Algebra students, ranging from those having little mathematical background and fear of mathematics courses to those who have had a strong mathematical education and are highly motivated. For some of these students, this will be their last course in mathematics, while others might decide to further their mathematical education. This text is written for both groups. As the author of precalculus, engineering calculus, finite math, and business calculus texts, and as a teacher, Michael understands what students must know if they are to be focused and successful in upper level math courses. However, as a father of four, including the co-author, he also understands the realities of college life. Michael Sullivan III believes passionately in the value of technology as a tool for learning that enhances understanding without sacrificing important skills. Both authors have taken great pains to insure that the text contains solid, student-friendly examples and problems, as well as a clear, seamless writing style.

## In the Second Edition

The second edition builds upon a strong foundation by integrating new features and techniques that further enhance student interest and involvement. The elements of the previous edition that have proved successful remain, while many changes, some obvious, others subtle, have been made. The text has been streamlined to increase accessibility. A huge benefit of authoring a successful series is the broad-based feedback upon which improvements and additions are ultimately based. Virtually every change to this edition is the result of thoughtful comments and suggestions made by colleagues and students who have used the previous edition. This feedback has proved invaluable and has been used to make changes that improve the flow and usability of the text. For example, some topics have been moved to better reflect the way teachers approach the course. In other places, problems have been added where more practice was needed. The supplements package has been enhanced through upgrading traditional supplements and adding innovative media components.

## Changes to the Second Edition

### Specific Organizational Changes

- Chapters 1, 3, and certain Appendix topics have been rewritten and reorganized to now appear in Chapter 1. This format will allow a more efficient use of the text and will improve the flow of the course. Once Chapter 1 is completed, the development of functions proceeds uninterrupted.
- From Chapter 3 to Chapter 1: Solving Equations; Setting up Equations; Applications; Solving Inequalities

- From the Appendix to Chapter 1: Solving Equations; Completing the Square
- From Chapter 1 to Chapter 2: Linear Curve Fitting; Direct Variation (These topics are now treated in the context of a linear function).
- From Chapter 3 to Chapter 4: Complex Numbers; Quadratic Equations with a Negative Discriminate (This section, which may be done after the quadratic equation in Chapter 1, now appears as motivation for the Fundamental Theorem of Algebra). Polynomial and Rational Inequalities (This topic is now handled after the discussion on polynomial and rational functions).
- From Chapter 2 to Chapter 5: One-to-One Functions; Inverse Functions (This topic now appears where it is needed at the start of the Exponential and Logarithmic Function chapter).
- Chapter 2 now appears as Chapters 2 and 3. Included now in Chapter 2 are Quadratic Functions and Models (formerly in Chapter 4).
- The chapter on Systems of Equations and Inequalities appears earlier to accommodate the syllabi of many schools. The section on Nonlinear Equations was moved to the Conics chapter, to make use of the graphs of conics in the solution.
- The chapter on Sequences; Induction; Sets; Probability now consists of two chapters. The last three chapters of the book may be covered in any order now.

### Specific Content Changes

- In this edition emphasis is placed on the role of modeling in college algebra. To this end, dedicated sections appear on Linear Functions and Models, Quadratic Functions and Models, Power Functions and Models, Polynomial Functions and Models, and Exponential and Logarithmic Functions and Models. The focus of many of these applications is in the area of business, finance and economics.
- A section on univariate data and obtaining probability from data is new to this edition.
- A more extensive and more useable Appendix is also new. Now reference is made in the text to the Appendix for students who may require more information.
- The use of technology has been updated to include the more powerful features of ZERO(ROOT) VALUE and INTERSECT. The use of TRACE is minimal.

As a result of these changes, this edition will be an improved teaching device for professors and a better learning tool for students.

### Acknowledgments

Textbooks are written by authors, but evolve from an idea into final form through the efforts of many people. Special thanks to Don Dellen, who first suggested this book and the other books in this series. Don's extensive contributions to publishing and mathematics are well known; we all miss him dearly.

There are many people we would like to thank for their input, encouragement, patience, and support. They have our deepest thanks and appreciation. We apologize for any omissions . . .

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# PREFACE TO THE STUDENT

As you begin your study of College Algebra, you may feel overwhelmed by the numbers of theorems, definitions, procedures, and equations that confront you. You may even wonder whether or not you can learn all of this material in the time allotted. These concerns are normal. Do not become frustrated and say “This is impossible.” It is possible! Work hard, concentrate, use all the resources available to you, and, above all, don’t give up.

For many of you, this may be your last math course; for others, it is just the first in a series of many. Either way, this text was written to help you, the student, master the terminology and basic concepts of College Algebra. These aims have helped to shape every aspect of the book. Many learning aids are built into the format of the text to make your study of the material easier and more rewarding. This book is meant to be a “machine for learning,” that can help you focus your efforts and get the most from the time and energy you invest.

## About the authors

Michael Sullivan has taught College Algebra courses for over thirty years. He is also the father of four college graduates, including this text’s co-author, all of whom called home, from time to time, frustrated and with questions. He knows what you’re going through. Michael Sullivan, III, as a fairly recent graduate, experienced, first-hand, learning math using graphing calculators and computer algebra systems. He recognizes that, while this technology has many benefits, it can only be used effectively with an understanding of the underlying mathematics. Having earned advanced degrees in both economics and mathematics, he sees the importance of mathematics in applications.

So we, father and son, have written a text that doesn’t overwhelm, or unnecessarily complicate, College Algebra, but at the same time provides you with the skills and practice you need to be successful. Please do not hesitate to contact us through Prentice Hall with any suggestions or comments that would improve this text.

Best Wishes!

Michael Sullivan

Michael Sullivan, III



# OVERVIEW

## CHAPTER OPENERS AND CHAPTER PROJECTS

**Chapter openers** use current articles to set up **Chapter Projects**. Many of the concepts that you encounter in this course relate directly to today's headlines and issues. These chapter projects are designed to give you a chance to use math to better understand the world. The list of topics for review will help you in two major ways, ... First, it allows you to review basic concepts immediately before using them in context. Second, it emphasizes the natural building of mathematical concepts throughout the course.

### CHAPTER

# 3

## Functions and Their Graphs



### Wednesday February 10, 1999 The Oregonian "Ship awaits salvage effort"

COOS BAY—Cleanup crews combed the oil-scarred south coast Tuesday as authorities raced to finish plans to refloat a 639-foot cargo ship mired for six days 150 yards off one of Oregon's most biologically rich beaches.

All day Tuesday, streaks of oil oozed from the cracked hull of the bulk cargo carrier *New Carissa* and spread over six miles of beach. Despite the breached hull, authorities believed they had a better chance of pulling the stricken ship out of beach sands than pumping nearly 400,000 gallons of oil off the ship in the winter surf.

See Chapter Project 1.

### Preparing for This Chapter

Before getting started on this chapter, review the following concepts:

- Slope of a Line (pp. 67–68)
- Intervals (pp. 54–55)
- Tests for Symmetry of an Equation (p. 23)
- Graphs of Certain Equations (Example 2, p. 16; Example 10, p. 23; Example 12, p. 25)
- Domain of a Function (pp. 99 and 104)

### Outline

- 3.1 Characteristics of Functions; Library of Functions
- 3.2 Graphing Techniques: Transformations
- 3.3 Operations on Functions; Composite Functions
- 3.4 Mathematical Models: Constructing Functions
- Chapter Review
- Chapter Projects

### CHAPTER PROJECTS



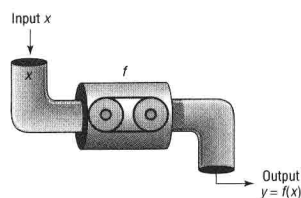
1. **Oil Spill** An oil tanker strikes a sand bar that rips a hole in the hull of the ship. Oil begins leaking out of the tanker with the spilled oil forming a circle around the tanker. The radius of the circle is increasing at the rate of 2.2 feet per hour.
  - (a) Write the area of the circle as a function of the radius,  $r$ .
  - (b) Write the radius of the circle as a function of time,  $t$ .
  - (c) What is the radius of the circle after 2 hours? What is the radius of the circle after 2.5 hours?
  - (d) Use the result of part (c) to determine the area of the circle after 2 hours and 2.5 hours.
  - (e) Determine a function that represents area as a function of time,  $t$ .
  - (f) Use the result of part (e) to determine the area of the circle after 2 hours and 2.5 hours.
  - (g) Compute the average rate of change of the area of the circle from 2 hours to 2.5 hours.
  - (h) Compute the average rate of change of the area of the circle from 3 hours to 3.5 hours.
  - (i) Based on the results obtained in parts (g) and (h), what is happening to the average rate of change of the area of the circle as time passes?
  - (j) If the oil tanker is 150 yards from shore, when will the oil spill first reach the shoreline? (1 yard = 3 feet).
  - (k) How long will it be until 6 miles of shoreline is contaminated with oil? (1 mile = 5280 feet)
2. **Economics** For a manufacturer of computer chips, it has been determined that the number  $Q$  of chips produced as a function of labor  $L$ , in hours, is given by  $Q = 8L^{1/2}$ . In addition, the cost of production as a function of the number of chips produced per month is  $C(Q) = 0.002Q^3 + 0.5Q^2 - Q + 1000$ .
  - (a) Determine cost as a function of labor.
  - (b) If the demand function for the chips is  $Q(P) = 5000 - 20P$ , where  $P$  is the

## CLEAR WRITING STYLE

Sullivan's **accessible writing style** is apparent throughout, often utilizing various approaches to the same concept. An author who writes clearly makes potentially difficult concepts intuitive, making class time more productive.

Sometimes it is helpful to think of a function  $f$  as a machine that receives as input a number from the domain, manipulates it, and outputs the value. See Figure 3.

Figure 3



The restrictions on this input/output machine are

1. It only accepts numbers from the domain of the function.
2. For each input, there is exactly one output (which may be repeated for different inputs).

## 2.1 FUNCTIONS

- Determine Whether a Relation Represents a Function
- Find the Value of a Function
- Find the Domain of a Function
- Identify the Graph of a Function
- Obtain Information from or about the Graph of a Function

In Section 1.1, we said that a **relation** is a correspondence between two variables, say  $x$  and  $y$ . When relations are written as ordered pairs  $(x, y)$ , we say that  $x$  is related to  $y$ . Often, we are interested in specifying the type of relation (such as an equation) that might exist between the two variables.

For example, the relation between the revenue  $R$  resulting from the sale of  $x$  items selling for \$10 each may be expressed by the equation  $R = 10x$ . If we know how many items have been sold, then we can calculate the revenue by using the equation  $R = 10x$ . This equation is an example of a **function**.

As another example, suppose that an icicle falls off a building from a height of 64 feet above the ground. According to a law of physics, the distance

## LEARNING OBJECTIVES

Begin each section by reading the **Learning Objectives**. These objectives will help you organize your studies and prepare for class. The Learning Objectives also tie the text to MathPro Explorer's Section Objectives.

## STEP-BY-STEP EXAMPLES

**Step-by-step examples** insure that you follow the entire solution process and give you an opportunity to check your understanding of each step.

### 136 Chapter 2 Linear and Quadratic Functions

#### SUMMARY Steps for Graphing a Quadratic Function $f(x) = ax^2 + bx + c$ , $a \neq 0$ by hand

- STEP 1. Determine the vertex,  $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ .
- STEP 2. Determine the axis of symmetry,  $x = \frac{-b}{2a}$ .
- STEP 3. Determine the y-intercept,  $f(0)$ .
- STEP 4. (a) If  $b^2 - 4ac > 0$ , then the graph of the quadratic function has two  $x$ -intercepts, which are found by solving  $ax^2 + bx + c = 0$ .  
(b) If  $b^2 - 4ac = 0$ , the vertex is the  $x$ -intercept.  
(c) If  $b^2 - 4ac < 0$ , there are no  $x$ -intercepts.
- STEP 5. Determine an additional point if  $b^2 - 4ac \leq 0$  by using the y-intercept and the axis of symmetry.
- STEP 6. Plot the points and draw the graph.

#### Applications

Real-world problems may lead to mathematical problems that involve quadratic functions. Two such problems follow.

## REAL-WORLD DATA

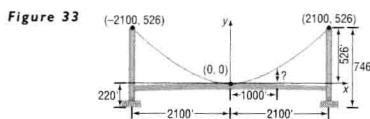
Real-world data is incorporated into examples and exercise sets to emphasize that mathematics is a tool used to understand the world around us. As you use these problems and examples, you will see the relevance and utility of the skills being covered.

**27. Federal Income Tax** Two 1998 Tax Rate Schedules are given in the accompanying table. If  $x$  equals the amount on Form 1040, line 37, and  $y$  equals the tax due, construct a function  $f$  for each schedule.

### 1998 TAX RATE SCHEDULES

SCHEDULE X — IF YOUR FILING STATUS IS SINGLE				SCHEDULE Y-1 — USE IF YOUR FILING STATUS IS MARRIED FILING JOINTLY OR QUALIFYING WIDOW(ER)			
If the amount on Form 1040, line 37, is: Over—	But not over—	Enter on Form 1040, line 38	of the amount over—	If the amount on Form 1040, line 37, is: Over—	But not over—	Enter on Form 1040, line 38	of the amount over—
\$0	\$25,750	15%	\$0	\$0	\$43,050	15%	\$0
25,750	62,450	\$3,862.50 + 28%	25,750	43,050	104,050	\$6,457.50 + 28%	43,050
62,450	130,250	14,138.50 + 31%	62,450	104,050	158,550	23,537.50 + 31%	104,050
130,250	283,150	35,156.50 + 36%	130,250	158,550	283,150	40,432.50 + 36%	158,550
283,150	—	90,200.50 + 39.6%	283,150	283,150	—	85,288.50 + 39.6%	283,150

**Solution** We begin by choosing the placement of the coordinate axes so that the  $x$ -axis coincides with the road surface and the origin coincides with the center of the bridge. As a result, the twin towers will be vertical (height  $746 - 220 = 526$  feet above the road) and located 2100 feet from the center. Also, the cable, which has the shape of a parabola, will extend from the towers, open up, and have its vertex at  $(0, 0)$ . As illustrated in Figure 33, the choice of placement of the axes enables us to identify the equation of the parabola as  $y = ax^2$ ,  $a > 0$ . We can also see that the points  $(-2100, 526)$  and  $(2100, 526)$  are on the graph.



Based on these facts, we can find the value of  $a$  in  $y = ax^2$ .

$$y = ax^2$$

$$526 = a(2100)^2$$

$$a = \frac{526}{(2100)^2}$$

The equation of the parabola is therefore

$$y = \frac{526}{(2100)^2}x^2$$

The height of the cable when  $x = 1000$  is

$$y = \frac{526}{(2100)^2}(1000)^2 \approx 119.3$$

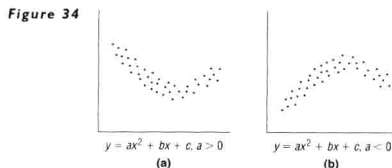
Thus, the cable is 119.3 feet high at a distance of 1000 feet from the center of the bridge.

**NOW WORK PROBLEM 31.**

### Fitting a Quadratic Function to Data

**3**

In Section 2.2, we found the line of best fit for data that appeared to be linearly related. It was noted that data may also follow a nonlinear relation. Figures 34(a) and (b) show scatter diagrams of data that follow a quadratic relation.



## "NOW WORK" PROBLEMS

Many examples end with "Now Work Problems." The problems suggested here are similar to the corresponding examples and provide a great way to check your understanding as you work through the chapter. The solutions to all "Now Work" problems can be found in the back of the text as well as the *Student Solutions Manual*.

- 31. Suspension Bridge** A suspension bridge with weight uniformly distributed along its length has twin towers that extend 75 meters above the road surface and are 400 meters apart. The cables are parabolic in shape and are suspended from the tops of the towers. The cables touch the road surface at the center of the bridge. Find the height of the cables at a point 100 meters from the center. (Assume that the road is level.)

## PROCEDURES

Procedures, both algebraic and technical, are clearly expressed throughout the text.

### EXAMPLE 3 Solving a Quadratic Equation by Graphing and by Using the Quadratic Formula

Find the real solutions, if any, of the equation  $3x^2 - 5x + 1 = 0$ .

**Solution** The equation is in standard form, so we compare it to  $ax^2 + bx + c = 0$  to find  $a$ ,  $b$ , and  $c$ .

$$\begin{aligned} 3x^2 - 5x + 1 &= 0 \\ ax^2 + bx + c &= 0 \end{aligned}$$

With  $a = 3$ ,  $b = -5$ , and  $c = 1$ , we evaluate the discriminant  $b^2 - 4ac$ .

$$b^2 - 4ac = (-5)^2 - 4(3)(1) = 25 - 12 = 13$$

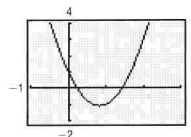
Since  $b^2 - 4ac > 0$ , there are two unequal real solutions.

**Graphing Solution** Figure 18 shows the graph of equation

$$Y_1 = 3x^2 - 5x + 1$$

As expected, we see that there are two  $x$ -intercepts: one between 0 and 1, the other between 1 and 2. Using Zero (or Root), twice, the solutions to the equation are 0.23 and 1.43 rounded to two decimal places.

Figure 18



**Algebraic Solution** We use the quadratic formula with  $a = 3$ ,  $b = -5$ ,  $c = 1$ , and  $b^2 - 4ac = 13$ .

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{5 \pm \sqrt{13}}{6}$$

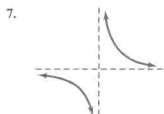
The solution set is  $\left\{ \frac{5 - \sqrt{13}}{6}, \frac{5 + \sqrt{13}}{6} \right\}$ . These solutions are exact.

**NOW WORK PROBLEM 13.**

## 3.1 EXERCISES

In Problems 1–8, match each graph to the function listed whose graph most resembles the one given.

- |                            |                              |
|----------------------------|------------------------------|
| A. Constant function       | B. Linear function           |
| C. Square function         | D. Cube function             |
| E. Square root function    | F. Reciprocal function       |
| G. Absolute value function | H. Greatest-integer function |



## END-OF-SECTION EXERCISES

Sullivan's exercises are unparalleled in terms of thorough coverage and accuracy. Each **end-of-section exercise set** begins with visual and concept based problems, starting you out with the basics of the section. Well-thought-out exercises better prepare you for exams.

28. **Employment and the Labor Force** The following data represent the civilian labor force (people aged 16 years and older, excluding those serving in the military) and the number of employed people in the United States for the years 1981–1991. Treat the size of the labor force as the independent variable and the number employed as the dependent variable. Both the size of the labor force and the number employed are measured in thousands of people.



Year	Civilian Labor Force	Number Employed
1981	108,670	100,397
1982	110,204	99,526
1983	111,550	100,834
1984	113,544	105,005
1985	115,461	107,150
1986	117,834	109,597
1987	119,865	112,440
1988	121,669	114,968
1989	123,869	117,342
1990	124,797	117,914
1991	125,303	116,877

Source: Business Statistics, 1993–1991 U.S. Department of Commerce, Economics and Statistics Administration, Bureau of Economic Analysis, June 1992.

- Use a graphing utility to draw a scatter diagram.
- Use a graphing utility to find the line of best fit to the data. Express the solution using function notation.
- Interpret the slope.
- Predict the number of employed people if the civilian labor force is 122,340 thousand people.

## MODELING

Many examples and exercises connect real-world situations to mathematical concepts. Learning to work with models is a skill that transfers to many disciplines.

## GRAPHING UTILITIES AND TECHNIQUES

Increase your understanding, visualize, discover, explore, and solve problems using a **graphing utility**. Sullivan uses the graphing utility to further your understanding of concepts not to circumvent essential math skills.

Table 8

X	Y <sub>1</sub>	Y <sub>2</sub>
-3	ERROR	1.7321
-2	ERROR	1.4142
-1	ERROR	1
0	ERROR	0
1	ERROR	1
2	ERROR	1.4142
3	ERROR	1.7321

(a)

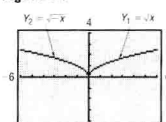
X	Y <sub>1</sub>	Y <sub>2</sub>
-3	-2	4
-2	-1	1
-1	0	0
0	1	1
1	2	4
2	3	9
3	4	16

(b)

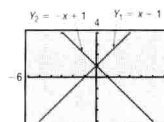
X	Y <sub>1</sub>	Y <sub>2</sub>
-3	8	8
-2	4	4
-1	1	1
0	0	0
1	1	1
2	4	4
3	8	8

(c)

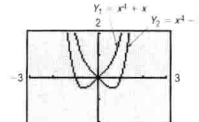
Figure 34



(a)



(b)



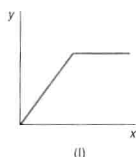
(c)

When the graph of the function  $y = f(x)$  is known, the graph of the new function  $y = f(-x)$  is the **reflection about the y-axis** of the graph of the function  $y = f(x)$ .

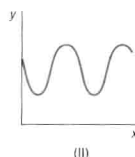


63. Match each of the following functions with the graphs that best describe the situation.

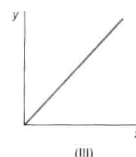
- The cost of building a house as a function of its square footage.
- The height of an egg dropped from a 300-foot building as a function of time.
- The height of a human as a function of time.
- The demand for Big Macs as a function of price.
- The height of a child on a swing as a function of time.



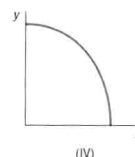
(I)



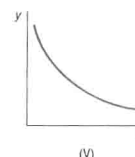
(II)



(III)



(IV)



(V)

## DISCUSSION WRITING AND READING PROBLEMS

These **problems** are designed to get you to "think outside the box," therefore fostering an intuitive understanding of key mathematical concepts. In this example, matching the graph to the functions insures that you understand functions at a fundamental level.

## CHAPTER REVIEW

### LIBRARY OF FUNCTIONS

#### Linear function

$$f(x) = mx + b$$

Graph is a line with slope  $m$  and  $y$ -intercept  $b$ .

#### Constant function

$$f(x) = b$$

Graph is a horizontal line with  $y$ -intercept  $b$  (see Figure 14).

#### Identity function

$$f(x) = x$$

Graph is a line with slope 1 and  $y$ -intercept 0 (see Figure 15).

#### Square function

$$f(x) = x^2$$

Graph is a parabola with vertex at  $(0, 0)$  (see Figure 16).

#### Cube function

$$f(x) = x^3$$

See Figure 17.

#### Square root function

$$f(x) = \sqrt{x}$$

See Figure 18.

#### Reciprocal function

$$f(x) = 1/x$$

See Figure 19.

#### Absolute value function

$$f(x) = |x|$$

See Figure 20.

### THINGS TO KNOW

#### Average rate of change of a function

The average rate of change of  $f$  from  $c$  to  $x$  is

$$\frac{\Delta y}{\Delta x} = \frac{f(x) - f(c)}{x - c}, \quad x \neq c$$

#### Increasing function

A function  $f$  is increasing on an open interval  $I$  if, for any choice of  $x_1$  and  $x_2$  in  $I$ , with  $x_1 < x_2$ , we have  $f(x_1) < f(x_2)$ .

#### Decreasing function

A function  $f$  is decreasing on an open interval  $I$  if, for any choice of  $x_1$  and  $x_2$  in  $I$ , with  $x_1 < x_2$ , we have  $f(x_1) > f(x_2)$ .

#### Constant function

A function  $f$  is constant on an open interval  $I$  if, for all choices of  $x$  in  $I$ , the values of  $f(x)$  are equal.

#### Local maximum

A function  $f$  has a local maximum at  $c$  if there is an interval  $I$  containing  $c$  so that, for all  $x \neq c$  in  $I$ ,  $f(x) < f(c)$ .

#### Local minimum

A function  $f$  has a local minimum at  $c$  if there is an interval  $I$  containing  $c$  so that, for all  $x \neq c$  in  $I$ ,  $f(x) > f(c)$ .

#### Even function $f$

$f(-x) = f(x)$  for every  $x$  in the domain ( $-x$  must also be in the domain).

#### Odd function $f$

$f(-x) = -f(x)$  for every  $x$  in the domain ( $-x$  must also be in the domain).

### HOW TO

Find the average rate of change of a function

Determine algebraically whether a function is even or odd without graphing it

Use a graphing utility to find the local maxima and local minima of a function

Use a graphing utility to determine where a function is increasing or decreasing

Graph certain functions by shifting, compressing, stretching, and/or reflecting (see Table 9)

Find the composite of two functions. Perform operation on functions

Construct functions in applications, including piecewise-defined functions

### FILL-IN-THE-BLANK ITEMS

- The average rate of change of a function equals the \_\_\_\_\_ of the secant line containing two points on its graph.
- A(n) \_\_\_\_\_ function  $f$  is one for which  $f(-x) = f(x)$  for every  $x$  in the domain of  $f$ ; a(n) \_\_\_\_\_ function  $f$  is one for which  $f(-x) = -f(x)$  for every  $x$  in the domain of  $f$ .
- Suppose that the graph of a function  $f$  is known. Then the graph of  $y = f(x - 2)$  may be obtained by a(n) \_\_\_\_\_ shift of the graph of  $f$  to the \_\_\_\_\_ a distance of 2 units.
- If  $f(x) = x + 1$  and  $g(x) = x^2$ , then \_\_\_\_\_ =  $(x + 1)^2$ .
- For the two functions  $f$  and  $g$ ,  $(g \circ f)(x) =$  \_\_\_\_\_.

### TRUE/FALSE ITEMS

- T F 1. A function  $f$  is decreasing on an open interval  $I$  if, for any choice of  $x_1$  and  $x_2$  in  $I$ , with  $x_1 < x_2$ , we have  $f(x_1) < f(x_2)$ .
- T F 2. Even functions have graphs that are symmetric with respect to the origin.
- T F 3. The graph of  $y = f(-x)$  is the reflection about the  $y$ -axis of the graph of  $y = f(x)$ .
- T F 4.  $f(g(x)) = f(x) \cdot g(x)$ .
- T F 5. The domain of the composite function  $(f \circ g)(x)$  is the same as that of  $g(x)$ .

### REVIEW EXERCISES

**Blue problem numbers indicate the authors' suggestions for use in a Practice Test.**

In Problems 1 and 2, use the graph of the function  $f$  shown on page 210 to find

- The domain and range of  $f$
- The intervals on which  $f$  is increasing, decreasing, or constant
- Whether it is even, odd, or neither
- The intercepts, if any

## CHAPTER REVIEW

The **Chapter Review** helps check your understanding of the chapter materials in several ways. "**Things to Know**" gives a general overview of review topics. The "**How To**" section provides a concept-by-concept listing of the operations you are expected to perform. The "**Review Exercises**" then serve as a chance to practice the concepts presented within the chapter. The review materials are designed to make you, the student, confident in knowing the chapter material.

# MULTIMEDIA

## Sullivan M@thP@k An Integrated Learning Environment

Today's textbooks offer a wide variety of ancillary materials to students, from solutions manuals to tutorial software to text-specific Websites. Making the most of all of these resources can be difficult. Sullivan M@thP@k helps students get it together. M@thP@k seamlessly integrates the following key products into an **integrated learning environment**.

### MathPro Explorer 4.0

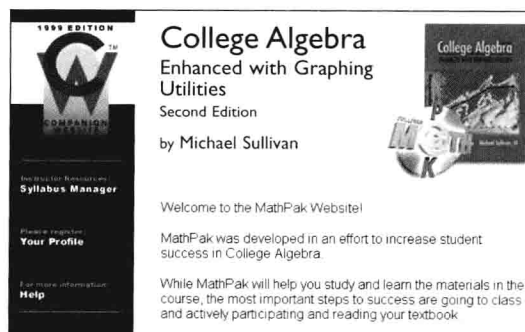
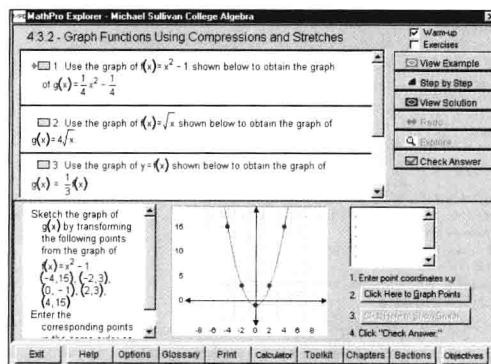
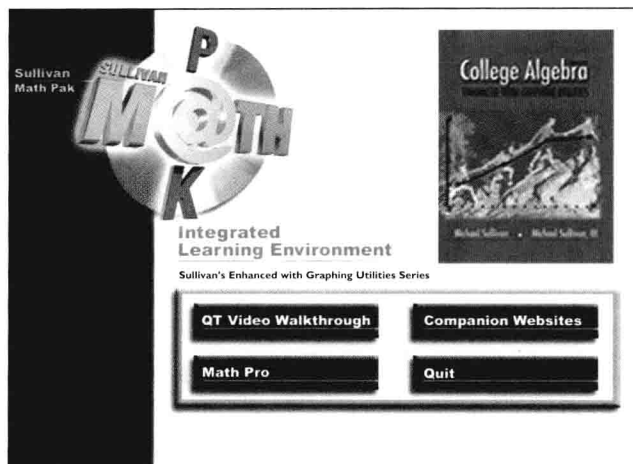
This hands-on tutorial software reinforces the material learned in class. More than 100 video clips, unlimited practice problems, and interactive step-by-step examples correspond directly to the text's sections and learning objectives. The exploratory component allows students to experiment with mathematical principles on their own in addition to the prescribed exercises.

### The Sullivan M@thP@k Website

This robust pass-code protected site features quizzes, homework starters, live animated examples, graphing calculator manuals, and much more. It offers the student many, many ways to test and reinforce their understanding of the course material.

### Student Solutions Manual

Written by Michael Sullivan III, co-author of both Sullivan series, and by Katy Murphy, the *Student Solutions Manual* offers thorough solutions that are consistent with the precise mathematics found in the text.



**Sullivan M@thP@k.**  
**Helping students Get it Together.**



## ADDITIONAL MEDIA

### Sullivan Companion Website

[www.prenhall.com/sullivan](http://www.prenhall.com/sullivan)

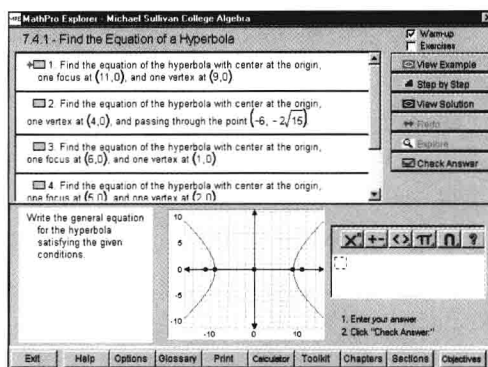
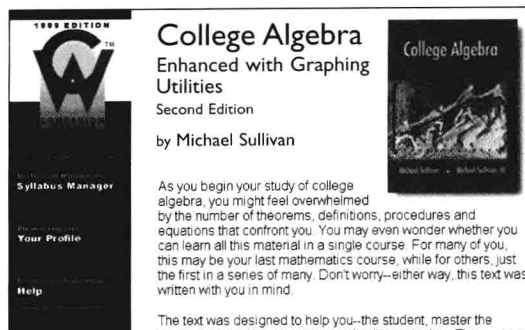
This text-specific website beautifully complements the text. Here students can find chapter tests, section-specific links, and PowerPoint downloads in addition to other helpful features.

### MathPro 4.0 (network version)

This networkable version of MathPro is free to adopters.

### TestPro 4.0

This algorithmically generated testing software features an equation editor and a graphing utility, and offers online testing capabilities. Windows/Macintosh CD-ROM.



## SUPPLEMENTS

### Student Supplements

#### Student Solutions Manual

Worked solutions to all odd-numbered exercises from the text and complete solutions for chapter review problems and chapter tests. ISBN: 0-13-085306-2

#### Lecture Videos

The instructional tapes, in a lecture format, feature worked-out examples and exercises, taken from each section of the text. ISBN: 0-13-040311-3

#### New York Times Themes of the Times

A free newspaper from Prentice Hall and *The New York Times*. Interesting and current articles on mathematics which invite discussion and writing about mathematics.

#### Mathematics on the Internet

Free guide providing a brief history of the Internet, discussing the use of the World Wide Web, and describing how to find your way within the Internet and how to find others on it.

### Instructor Supplements

#### Instructor's Edition

Includes answers to all exercises at the end of the book. ISBN: 0-13-085303-8

#### Instructor's Resource Manual

Contains complete step-by-step worked-out solutions to all even-numbered exercises in the textbook. Also included are strategies for using the Collaborative Learning projects found in each chapter. ISBN: 0-13-085307-0

#### Test Item File

Hard copy of the algorithmic computerized testing materials. ISBN: 0-13-085305-4

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