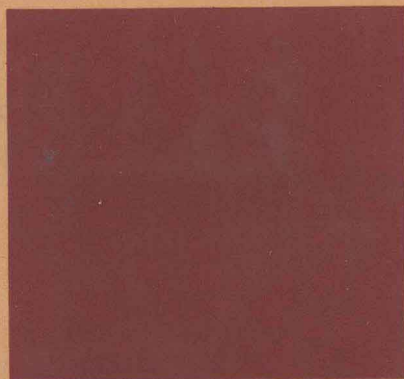


Newtonian Mechanics

A.P. FRENCH

THE M.I.T.
INTRODUCTORY
PHYSICS SERIES



Newtonian mechanics

THE M.I.T. INTRODUCTORY PHYSICS SERIES

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A. P. French

PROFESSOR OF PHYSICS, THE MASSACHUSETTS INSTITUTE OF TECHNOLOGY



Preface

THE WORK of the Education Research Center at M.I.T. (formerly the Science Teaching Center) is concerned with curriculum improvement, with the process of instruction and aids thereto, and with the learning process itself, primarily with respect to students at the college or university undergraduate level. The Center was established by M.I.T. in 1960, with the late Professor Francis L. Friedman as its Director. Since 1961 the Center has been supported mainly by the National Science Foundation; generous support has also been received from the Kettering Foundation, the Shell Companies Foundation, the Victoria Foundation, the W. T. Grant Foundation, and the Bing Foundation.

The M.I.T. Introductory Physics Series, a direct outgrowth of the Center's work, is designed to be a set of short books that, taken collectively, span the main areas of basic physics. The series seeks to emphasize the interaction of experiment and intuition in generating physical theories. The books in the series are intended to provide a variety of possible bases for introductory courses, ranging from those which chiefly emphasize classical physics to those which embody a considerable amount of atomic and quantum physics. The various volumes are intended to be compatible in level and style of treatment but are not conceived as a tightly knit package; on the contrary, each book is designed to be reasonably self-contained and usable as an individual component in many different course structures.

The text material in the present volume is designed to be a more or less self-contained introduction to Newtonian mechanics, such that a student with little or no grounding in the subject can, by beginning at the beginning, be brought gradually to a level of considerable proficiency. A rough guide to the possible use of the book is suggested by its division into three parts. Part I, *The Approach to Newtonian Dynamics*, is intended to serve two purposes. First, it does discuss the basic concepts of kinematics and dynamics, more or less from scratch. Second, it seeks to place the study of mechanics squarely in the context of the world of physical phenomena and of necessarily imperfect physical theories. This is a conscious reaction, on the author's part, against the presentation of mechanics as "applied mathematics," with the divorce-ment from reality and the misleading impression of rigor that this has engendered in generations of students (especially, alas, those brought up in the British educational system). The student who already has some expertise in using Newton's laws will find little of an analytical or quantitative sort to learn from Part I, but he may still derive some value and interest from reading through it for its broader implications.

Part II, *Classical Mechanics at Work*, is undoubtedly the heart of the book. Some instructors will wish to begin here, and relegate Part I to the status of background reading. The initial emphasis is on Newton's second law applied to individual objects. Later, the emphasis shifts to systems of two or more particles, and to the conservation laws for momentum and energy. A fairly lengthy chapter is devoted to the subject that deserves pride of place in the whole Newtonian scheme—the theory of universal gravitation and its successes, which can still be appreciated as a pinnacle in man's attempts to discover order in the vast universe in which he finds himself.

Part III, *Some Special Topics*, concerns itself with the problems of noninertial frames, central-force motions, and rotational dynamics. Most of this material, except perhaps the fundamental features of rotational motion and angular momentum, could be regarded as optional if this book is used as the basis of a genuinely introductory presentation of mechanics. Undoubtedly the book as a whole contains more material than could in its entirety be covered in a one-term course; one could, however, consider using Parts I and II as a manageable package for beginners, and Parts II and III as a text for students having some prior preparation.

One of the great satisfactions of classical mechanics lies in the vast range and variety of physical systems to which its principles can be applied. The attempt has been made in this book to make explicit reference to such applications and, as in other books in this series, to “document” the presentation with appropriate citations from original sources. Enriched in this way by its own history, classical mechanics has an excitement that is not, in this author’s view, surpassed by any of the more recent fields of physical theory.

This book, like the others in the series, owes much to the thoughts, criticisms, and suggestions of many people, both students and instructors. A special acknowledgment in connection with the present volume is due to Prof. A. M. Hudson, of Occidental College, Los Angeles, who worked with the present author in the preparation of the preliminary text from which, five years later, this final version evolved. Grateful thanks are also due to Eva M. Hakala and William H. Ingham for their invaluable help in preparing the manuscript for publication.

A. P. FRENCH

Cambridge, Massachusetts
July 1970

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2

In the Beginning was Mechanics.

MAX VON LAUE, *History of Physics* (1950)

I offer this work as the mathematical principles of philosophy, for the whole burden of philosophy seems to consist in this— from the phenomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phenomena.

NEWTON, Preface to the *Principia* (1686)

Prologue

ONE OF THE MOST prominent features of the universe is motion. Galaxies have motions with respect to other galaxies, all stars have motions, the planets have distinctive motions against the background of the stars, the events that capture our attention most quickly in everyday life are those involving motion, and even the apparently inert book that you are now reading is made up of atoms in rapid motion about their equilibrium positions. “Give me matter and motion,” said the seventeenth-century French philosopher René Descartes, “and I will construct the universe.” There can be no doubt that motion is a phenomenon we must learn to deal with at all levels if we are to understand the world around us.

Isaac Newton developed a precise and powerful theory regarding motion, according to which the *changes* of motion of any object are the result of *forces* acting on it. In so doing he created the subject with which this book is concerned and which is called classical or Newtonian mechanics. It was a landmark in the history of science, because it replaced a merely descriptive account of phenomena with a rational and marvelously successful scheme of cause and effect. Indeed, the strict causal nature of Newtonian mechanics had an impressive influence in the development of Western thought and civilization generally, provoking fundamental questions about the interrelationships of science, philosophy, and religion, with repercussions in social ideas and other areas of human endeavor.

Classical mechanics is a subject with a fascinating dual character. For it starts out from the kinds of everyday experiences