# *Inertial Navigation Systems Analysis*

**KENNETH R. BRITTING**

# *INERTIAL NAVIGATION SYSTEMS ANALYSIS*

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*Inertial Navigation Systems Analysis*

To the memory of

*KATHERINE ANNE*

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# Note From the Series Editors

We are pleased to welcome this text, *Inertial Navigation Systems Analysis*, into the Artech House GNSS Technology and Applications Series. This book, although originally published in 1971, is still in use today and frequently referred to by practicing navigation engineers. The underlying principles of operation of inertial navigation systems are thoroughly examined. This text includes discussions of inertial sensors (accelerometers and gyros), mechanizations (gimbaled vs. strap-down), reference frames, as well as the Earth's gravity field. Linearized error equations are carefully derived that provide great insight into inertial navigation system behavior. This text was the first to provide a unified treatment of these topics. A number of excellent texts have later been written covering modern inertial sensors and implementations. However, it is Dr. Britting's concise development of the basic mathematics, underlying the operation of any inertial navigator, that makes this a timeless text.

We express our sincere appreciation to Dr. Britting's family for permitting the reprinting of this classic text.

*Elliott Kaplan*
*Christopher Hegarty*
*December 2009*

# Foreword

Although the technique of inertial guidance can be said to have originated more than sixty years ago with the appearance of the gyrocompass, it did not attain full navigational status until the impetus of technology after World War II made it practical as well as feasible. During this period the basic inertial components—gyroscopes and accelerometers—remained essentially the same but continually improved in performance. Extensive studies led to a fairly complete understanding of the theory of inertial systems. Improvement both in inertial components and in the associated signal-processing equipment led from systems with a volume of over a cubic yard to those with less than a cubic foot. Accuracy and reliability produced systems that not only met the military needs in air and underwater but also allowed the successful accomplishment of the Apollo space missions and the installation of inertial systems in commercial aircraft.

During this time different general configurations produced systems with very different types of performance, although with the same basic components. Accordingly, a common basis for meaningful comparison of the performance of these systems was lacking, and discussions by proponents and opponents of a given configuration generated more heat than light. This book, based largely on the author's several years of study leading to his doctorate, is the first definitive attempt that successfully provides a basis for a realistic comparison of performance of various inertial system configurations—geometric, semi-analytic, or analytic. The solution is not a simple, rule-of-thumb technique, but it is of sufficient simplicity and directness for a skilled person to formulate his own comparisons reasonably quickly, effectively, and accurately.

In producing this book Dr. Britting presents a "Rosetta Stone" to the inertial guidance profession. As one of his faculty advisors during his doctoral studies, I am very pleased and proud to have the privilege of introducing a former student's noteworthy accomplishment.

WALTER WRIGLEY, Sc. D.
Professor of Instrumentation and Astronautics
Educational Director, Charles Stark Draper Laboratory
Massachusetts Institute of Technology

# Preface

Part of this book evolved from a set of lecture notes prepared in the Department of Aeronautics and Astronautics at the Massachusetts Institute of Technology for a one-semester graduate course in inertial navigation systems. In addition, a portion of the book was adapted from my doctoral thesis. Since the lecture notes were prepared for students who had already completed a comprehensive introductory course in classical mechanics, kinematics, inertial instrument theory, and inertial platform mechanization, a fairly advanced level of preparation is assumed. Nevertheless, it has been found through experience that the book is reasonably self-contained, allowing the student to follow the development with a modicum of referral to the technical literature.

The material is intended principally for the avionics system engineer who wishes to compare the performance of the various types of system mechanizations. While it is applicable to spacecraft and undersea navigation, the thrust of the book is aimed at terrestrial applications on, or slightly above, the surface of the earth. Because of the current interest in navigation systems for aircraft, the relevant navigation equations are developed for this application.

Perturbation techniques are extensively used to develop linearized system equations whose solutions closely approximate those obtained by a solution of the nonlinear differential equations. Since linear systems theory is applicable to linearized system equations, these equations are quite amenable to physical interpretation, providing an insight into system behavior not readily obtainable from computer solution of the nonlinear equations. The developed linear system models are, of course, essential to the application of optimal filtering techniques which are currently being applied to aided inertial systems.

Chapter 1 emphasizes concepts common to all inertial navigation system configurations. The mathematical notation and techniques used in this book are discussed in Chapter 2. Chapter 3 defines a number of coordinate frames that are essential to the description of the operation of inertial navigation systems. The relationships between the various coordinate frames are developed, and the nonorthogonal instrument-platform relationships are defined. Chapter 4 models certain of the geometric aspects of the earth as they apply to the study of inertial navigation systems. Analytic expressions for the earth's gravity and gravitational fields and the specific force vector are developed in this chapter. Chapter 5 contains material on the performance and mathematical modeling of the single-degree-of-freedom gyroscope. Material on gyro redundancy and reliability is included.

Chapters 6, 7, and 8 are devoted to the error analysis of terrestrial inertial navigation systems. In Chapters 6 and 7, the error equations for space-stabilized and local-level mechanizations are developed, and the effects of externally supplied altitude information are studied. A unified error analysis that applies to virtually all terrestrial inertial navigation systems is developed in Chapter 8. It is shown that if the system state vector is chosen to consist of the system's attitude and position errors, the error behavior can be described by one relatively simple vector differential equation. The unified theory is applied to obtain the error equations for space-stabilized, local-level, free azimuth, rotating azimuth, and strapdown configurations. In Chapter 9, self-alignment techniques are discussed and developed.

I express my sincerest appreciation to Professors Winston Markey and Walter Wrigley of the Massachusetts Institute of Technology and to Robert Wedan of the United States Department of Transportation for their criticism, advice, and encouragement throughout the preparation of this book. In addition each of these men deserves special recognition: Professor Markey, as director of the Measurement Systems Laboratory, provided the stimulating environment for the research summarized herein; Professor Wrigley, as Educational Director of the Draper Laboratory, was instrumental in guiding my academic research; and Robert Wedan was responsible for the support for much of my research over the years. I would also like to thank John Hatfield of the Man Vehicle Control Laboratory at MIT for providing many very helpful suggestions.

The National Aeronautics and Space Administration, Electronics Research Center, and the Department of Transportation, Transportation Systems Center, receive my thanks for the financial support for much of my research.

The many typists who have struggled over the years with the notation peculiar to inertial navigation and with my handwriting are to be thanked profusely. In particular, Mrs. Ann Preston deserves special recognition for her skill in preparing parts of this book. The many students of inertial

navigation at MIT who have contributed to this document, both directly and indirectly, are recognized and thanked.

Lastly, I would like to thank my family for their encouragement in this endeavor.

KENNETH R. BRITTING

*Cambridge, Massachusetts*
*April 1971*

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# Contents

CHAPTER 1

# Introduction

"If a man will begin with certainties, he shall end in doubts; but if he will be content to begin with doubts he shall end in certainties."

FRANCIS BACON

## 1.1 THE CONCEPT OF INERTIAL NAVIGATION

Navigation is the determination of a physical body's position and velocity relative to some reference coordinate frame or coordinate grid. A simple, one-dimensional example of navigation would consist of determining the position and speed of a train along a track connecting two points on the earth. The general case of terrestrial navigation involves the determination of a vehicle's position and velocity relative to the earth. The grid coordinates usually used for this application consist of the spherical coordinates, latitude, longitude, and altitude.

An inertial navigation system utilizes the inertial properties of sensors mounted aboard the vehicle to execute the navigation function. The system accomplishes this task through appropriate processing of the data obtained from force and inertial angular velocity measurements. Thus an appropriately initialized inertial navigation system is capable of continuous determination of vehicle position and velocity without the use of external radiation or optical information. While inertial navigation systems have obvious advantages for military applications, the trend toward automatic flight control systems in civil aircraft applications will require an extensive reliance on inertial systems.

All inertial navigation systems must perform the following functions:

- Instrument a reference frame
- Measure specific force
- Have knowledge of the gravitational field
- Time integrate the specific force data to obtain velocity and position information

The first function is accomplished by the use of gyroscopic instruments. Gyroscopes are bodies that display strong angular momentum characteristics. Since the inertially referred time rate of change of angular momentum is proportional to the applied torque, gyroscopic devices can maintain a known spatial direction through appropriate torque control. Thus three such devices are capable of instrumenting a three-dimensional cartesian coordinate frame. Each is typically used as the sensing element in a closed-loop servo system which operates to maintain the gyro's spatial direction. Thus if three untorqued gyros are mounted on a gimbaled platform whose gimbals are driven to maintain the gyros' orientation, an inertially nonrotating cartesian coordinate frame will be instrumented by the platform.* The inertial platform can, of course, be commanded to instrument rotating frames of reference, platforms instrumenting the local geographic coordinates, north, east, and down, being in common use. An alternate method of utilizing the gyro information structurally mounts the gyroscopic instruments on the vehicle. In this configuration each gyro is used as the sensing element in a closed-loop servo system which results in a torque being applied to the gyro which is proportional to the gyro's inertially referenced angular rotation. While the gyro no longer remains nonrotating relative to inertial space, the applied torquing signal which is proportional to the inertially referred angular velocity can be used to analytically calculate the relative angular orientation between the gyro's initial and present spatial direction. If three structurally mounted gyros are used, the relative orientation between the initial and present vehicle coordinate frame can be determined. Systems which analytically instrument a reference frame are popularly referred to as strapdown systems.

The second function, the specific force measurement, is accomplished with devices commonly called accelerometers. Although there are many ways of making force measurements, most of the devices in common use are sophisticated variations of the simple pendulum. The motion of the pendulous element is related to the motion of the platform or structural element upon which the accelerometer is mounted via Newton's second law of motion. According to Einstein's principle of equivalence, however, it is impossible to distinguish between the effects of inertial acceleration and gravitational fields, the two phenomena being manifestations of the same physical process. Thus in order for the navigation system to correlate the motion of the pendulous element with the inertial acceleration, detailed knowledge of the local gravitational field is necessary, the third function of the list.

* It is noted in passing, that two-degree-of-freedom instruments can be used; therefore two gyros are required to instrument a coordinate frame. The development in this book is confined to inertial systems that use single-degree-of-freedom instruments, although the material is easily adapted to apply to the two-degree-of-freedom case.

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