

WILEY FINANCE

Financial engineering and arbitrage in the financial markets

ROBERT DUBIL

Financial Engineering and Arbitrage
in the Financial Markets

Robert Dubil



 **WILEY**

A John Wiley & Sons, Ltd., Publication

This edition first published 2011
© 2011 John Wiley & Sons, Ltd

Registered Office

John Wiley & Sons Ltd, The Atrium, Southern Gate, Chichester, West Sussex, PO19 8SQ, United Kingdom

For details of our global editorial offices, for customer services and for information about how to apply for permission to reuse the copyright material in this book please see our website at www.wiley.com.

The right of the author to be identified as the author of this work has been asserted in accordance with the Copyright, Designs and Patents Act 1988.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, except as permitted by the UK Copyright, Designs and Patents Act 1988, without the prior permission of the publisher.

Wiley also publishes its books in a variety of electronic formats and by print-on-demand. Some content that appears in standard print versions of this book may not be available in other formats. For more information about Wiley products, visit us at www.wiley.com.

Designations used by companies to distinguish their products are often claimed as trademarks. All brand names and product names used in this book are trade names, service marks, trademarks or registered trademarks of their respective owners. The publisher is not associated with any product or vendor mentioned in this book. This publication is designed to provide accurate and authoritative information in regard to the subject matter covered. It is sold on the understanding that the publisher is not engaged in rendering professional services. If professional advice or other expert assistance is required, the services of a competent professional should be sought.

Library of Congress Cataloging-in-Publication Data

Dubil, Robert.

Financial engineering and arbitrage in the financial markets / Robert Dubil.

p. cm. — (The Wiley finance series)

Includes bibliographical references and index.

ISBN 978-0-470-74601-1 (cloth : alk. paper)

1. Financial engineering. 2. Arbitrage. 3. Capital market. 4. Investments—Mathematics. I. Title.

HG4523.D83 2011

332'.041—dc23

2011017543

A catalogue record for this book is available from the British Library.

ISBN 978-0-470-74601-1 (hardback) ISBN 978-1-119-95062-2 (ebk)

ISBN 978-1-119-95063-9 (ebk) ISBN 978-1-119-95064-6 (ebk)

Set in 10/12pt Times by Aptara Inc., New Delhi, India

Printed and bound by CPI Group (UK) Ltd, Croydon, CRO 4YY

Financial Engineering and Arbitrage
in the Financial Markets

To Britt, Elsa, Ethan, and illy

Introduction

This book is an update and an improvement on my original 2004 book.

The update is the discussion of the many new things that happened in the last 10 years. Hedge funds took over the relative value strategies instead of relying on sell-side dealers pitching structured products. Credit derivatives (default swaps and collateralized debt obligations) became part of the standard tool-kit. Statistical arbitrage and tactical asset allocation edged out a lot of buy-and-hold funds. Indexing has undergone a near revolution to include new asset classes and strategies.

The improvement is in the presentation. The method of the original book was to present the difficult subjects of arbitrage, derivative pricing, and financial engineering in terms of numbers – cash flow discounting, binomial trees, tables, and diagrams – rather than differential equations. This book follows that appealing formula. All examples are worked out numerically, but they are now enhanced with flow diagrams, and are connected across the markets and chapters. Another big improvement, hopefully, is the division of the book into three parts, each offering a different perspective. If you watched three boys play with Lego, you would want to know the types of blocks (colors, the number of pegs, etc.) they are playing with, what they are building with them (a robot or a fire station), and why each boy is playing. The three parts of this book are just like that: Part I is the basic spot, futures, swap, and option transactions as building blocks; Part II illustrates examples of engineering those building blocks into CDOs or mortgage-backed securities (MBSs); and Part III relates to the players: individuals, banks, hedge funds, and private equity.

The main premise is that all financial markets are organized in the same way. All have spot (cash) transactions, forward/futures transactions and options, as well as complex swap arrangements combining all three. If you master the spot-futures cash-and-carry trade in one market (stock index arbitrage), then you can easily grasp it in another (currency-covered interest parity). If you master how delta or vega risk is hedged in equity markets, then you are likely to understand the same process in commodities or interest rates. Building a fire station with red Lego blocks is similar to building a space ship with green and blue blocks. Instead of focusing on the purpose of individual financial markets, the book focuses on the common structure. What we are building is an arbitrage or a relative value trade to profit from the real or perceived mispricing of risk. The blocks we use have the same shape, only a different color.

The building clusters in all structured products and strategies are: spot and futures trading mechanics, spot-futures linkages, option pricing, option linkages to futures, and spot, swaps, and their decomposition into bonds and forwards. The improvements in Part I consist of new

chapters and easier-to-follow number and flow diagram presentations. The options discussion is split into two: options on price variables (equities, currencies, etc.) and options on non-price variables (interest rates). In the latter, we don't model prices directly – instead we model rates, derive the prices from the rates, then we price derivatives. The part includes a new chapter on credit derivatives. Part II introduces financial cash flow engineering. In addition to a survey of the perennially popular structured products, the mortgage section is substantially clearer and more complete; and the CDO section is entirely new. The analogy between prepayment and credit risk *tranching* should be very hard to miss. Part III is completely new and is mostly concerned with “why”. Chapter 11, taking an individual investor perspective, is a repository of all-you-need-to-know about modern portfolio theory and its morph into statistical arbitrage methods, as well as fundamental equity valuation methods. Chapter 12 scratches the surface of hedge fund strategies and the new area of strategy indexes as a beta way of getting the alpha. Chapter 13 looks at the traditional asset-liability management for banks, which in many cases is still more useful than the newer voluminously analyzed VaR methods. Chapter 14 focuses mainly on private equity, but it also looks at liability/politics, constrained pension funds, endowments, and sovereign funds. While Part III does not answer all the *whys*, it hopefully illustrates the main motivations and quantitative techniques pursued by the key players in the financial markets.

I apologize for any mathematical errors, long sentences, or awkward grammar. I also apologize for using the royal “we” throughout the book.

Now, please find a comfortable chair, grab a pencil and a calculator, and enjoy!

Contents

Introduction	xi
1 Purpose and Structure of Financial Markets	1
1.1 Overview of Financial Markets	1
1.2 Risk Sharing	2
1.3 Transactional Structure of Financial Markets	6
1.4 Arbitrage: Pure Versus Relative Value	8
1.5 Financial Institutions: Transforming Intermediaries vs Broker-Dealers	12
1.6 Primary (Issuance) and Secondary (Resale) Markets	13
1.7 Market Players: Hedgers vs Speculators	15
1.8 Preview of the Book	18
PART I RELATIVE VALUE BUILDING BLOCKS	
2 Spot Markets	23
2.1 Bonds and Annual Bond Math	23
2.1.1 Zero-Coupon Bond	23
2.1.2 Coupon Bond	25
2.1.3 Amortizing Bond	27
2.1.4 Floating Rate Bond	28
2.2 Intra-Year Compounding and Day-Count	30
2.2.1 Intra-Year Compounding	30
2.2.2 Day-Count	31
2.2.3 Accrued Interest	33
2.3 Term Structure of Interest Rates and the Discount Factor Bootstrap	34
2.3.1 Term Structure	34
2.3.2 Discount Factor Bootstrap	36
2.3.3 Valuation of an Arbitrary Bond	36
2.4 Interest Rate Risk: Duration and Convexity	39
2.4.1 Duration	41
2.4.2 Portfolio Duration	44
2.4.3 Convexity	45
2.4.4 Other Risk Measures	46

2.5	Equity, Commodity, and Currency Math	47
2.5.1	Equities	48
2.5.2	Currencies	49
2.6	Short Selling	51
2.6.1	Buying on Margin	52
2.6.2	Short Selling in a Margin Account	53
2.6.3	Short Selling of Bonds	54
3	Futures Markets	57
3.1	Fundamentals of Futures and Forwards	57
3.2	Futures Mechanics	59
3.2.1	Physical Commodity Futures	59
3.2.2	Interest Rate Futures	62
3.2.3	Stock Index Futures	69
3.2.4	Currency Futures and Forwards	70
3.3	Cash-and-Carry Arbitrage	73
3.3.1	Commodities	74
3.3.2	Stock Indexes	76
3.3.3	Currencies	79
3.4	Futures Not Subject to Cash-and-Carry	81
3.5	Yield Curve Construction with Interest Rate Futures	84
3.5.1	Certainty Equivalence of Eurodollar Futures	85
3.5.2	Forward Rate Agreements	86
3.5.3	Building Spot Zeros	88
3.5.4	Recovering the Forwards	91
3.5.5	Including Repo Rates in the Calculation of the Forwards	93
4	Swap Markets	95
4.1	Fundamentals of Swaps	95
4.1.1	The Dual Nature of Swaps	96
4.1.2	Implication for Pricing and Hedging	96
4.2	Interest Rate Swaps	97
4.2.1	Definition of an Interest Rate Swap	97
4.2.2	Valuation of Interest Rate Swaps	99
4.2.3	Hedging of Interest Rate Swaps	101
4.3	Cross-Currency Swaps	105
4.3.1	Definition of a Fixed-for-Fixed Cross-Currency Swap	105
4.3.2	Valuation and Settlement of Cross-Currency Swaps	107
4.3.3	Cross-Currency Swaps as Packages of Off-Market FX Forwards	109
4.3.4	Multicurrency and Combination Cross-Currency Swaps	110
4.4	Equity, Commodity, and Exotic Swaps	112
4.4.1	Equity Swaps	112
4.4.2	Commodity Swaps	114
4.4.3	Volatility Swaps	115
4.4.4	Index Principal Swaps	116

5	Options on Prices and Hedge-Based Valuation	119
5.1	Call and Put Payoffs at Expiry	120
5.2	Composite Payoffs at Expiry	122
5.2.1	Straddles and Strangles	122
5.2.2	Spreads and Combinations	123
5.3	Option Values Prior to Expiry	126
5.4	Options and Forwards, Risk Sharing and Put–Call Parity	127
5.5	Currency Options	128
5.6	Binomial Option Pricing	129
5.6.1	One-Step Examples	129
5.7	Black–Scholes Model and Extensions	141
5.7.1	Black–Scholes with No Dividends	141
5.7.2	Dividends	142
5.7.3	Options on Currency Rates	143
5.7.4	Black–Scholes Delta, Gamma, and Vega	144
5.8	Residual Risk of Options: Gamma, Vega, and Volatility	145
5.8.1	Implied Volatility	147
5.8.2	Volatility Smiles and Skews	148
5.9	A Real-Life Option Pricing Exercise	150
5.9.1	Consistency Checks: Put–Call Parity, Black–Scholes, and Binomial	150
6	Options on Non-Price Variables	155
6.1	Black Models For Bond Price Options, Caps/Floors, and European Swaptions	156
6.1.1	Options on Bond Prices	156
6.1.2	Cap and Floor Definitions	158
6.1.3	Relationship of Caps and Floors to FRAs and Swaps	159
6.1.4	A Cap Application	160
6.1.5	Pricing of Caps and Floors	163
6.1.6	European Swaption Definitions	164
6.1.7	Options to Cancel Swaps	165
6.1.8	Relationship of Swaptions to Forward Swaps	165
6.1.9	Pricing of European Swaptions	167
6.1.10	Limitations of the Black Model	168
6.2	Convexity-Adjusted Models For Libor Forwards, Quantos, and Constant Maturity Swaps	168
6.2.1	Convexity Adjustment for Eurodollar Futures	169
6.2.2	Convexity Adjustment for CMS Options	170
6.2.3	Quanto Adjustments	171
6.3	Arbitrage-Free Interest Rate Models	172
6.3.1	Short Rate Models	173
6.3.2	Trinomial Trees and Calibration	174
6.3.3	The Heath–Jarrow–Morton Model and the LIBOR Market Model	176
6.3.4	Bermudan Swaptions and Multifactor Models	180
6.4	Exotic Interest Rate Options	181

6.4.1	Periodic Caps	181
6.4.2	Digitals and Ranges	181
7	Default Risk and Credit Derivatives	183
7.1	Credit Default Swaps	184
7.1.1	Credit Default Swap	184
7.1.2	No Arbitrage: CDS vs Corporate Bond Spread	185
7.1.3	Bundled Single-Name Credit Derivatives	186
7.2	A Constant Default Probability Model	190
7.3	A Deterministic Credit Migration Model	193
7.4	A Poisson Model of Single Issuer Default	195
7.4.1	Poisson Distribution	195
7.4.2	A Single Issuer Default Model	196
7.4.3	Pricing a Credit Default Swap in a Single Issuer Default Model	198
7.5	The Default Correlation of the Reference Issuer and the Protection Seller	199
PART II CASH FLOW ENGINEERING		
8	Structured Finance	203
8.1	A Simple Classification of Structured Notes	204
8.2	Interest Rate and Yield Curve-Based Structured Products	206
8.2.1	An Inverse Floater	206
8.2.2	A Leveraged Inverse Floater	209
8.2.3	A Capped Floater	211
8.2.4	A Callable	211
8.2.5	A Range Floater	212
8.2.6	An Index Principal Swap	212
8.3	Asset Class-Linked Notes	213
8.3.1	Principal-Protected Equity-Linked Notes	213
8.3.2	A (Rainbow) Multi-Asset-Linked Note	216
8.3.3	Principal-At-Risk Notes and Commodity-Tracking ETNs	216
8.4	Insurance Risk Structured Products	219
9	Mortgage-Backed Securities	223
9.1	Mortgage Financing Basics	224
9.2	Prepayment Risk	226
9.3	Mortgage Pass-Through Securities	227
9.4	Collateralized Mortgage Obligations	232
9.4.1	Sequential-Pay CMO	232
9.4.2	Planned Amortization Class CMO	233
9.4.3	Interest-only (IO) and Principal-only (PO) Classes	237
9.5	Multiclass and Non-Vanilla CMOs	241
9.5.1	A Multiclass PAC Structure with a PAC I/O and a Floater/Inverse Coupon Split	241
9.5.2	Non-Accelerating Senior and Accrual Tranches in Sequential CMOs	242

10	Collateralized Debt Obligations and Basket Credit Derivatives	243
10.1	Collateralized Debt Obligations	243
10.1.1	Cash CDO	244
10.1.2	Synthetic CDO	246
10.2	Basket Credit Derivatives	249
10.2.1	First-to-Default Basket	249
10.2.2	<i>N</i> th-to-Default Basket, Arbitrage Conditions, and Hedging	251
10.2.3	Hedging of Basket Derivatives	252
10.3	Copulas and the Modeling of Default Correlation	252
10.3.1	A Gaussian Copula	254
10.3.2	General Copula Models	255
10.4	Synthetic CDO Tranche Pricing and Loss Analysis	256
10.4.1	Synthetic CDO Revisited	256
10.4.2	Synthetic CDO Pricing and Expected Loss	257
10.4.3	Synthetic CDO – Loss Rates, Ratings and the Crisis of 2008	259
10.5	Credit Derivative Indexes	260
 PART III THE PLAYERS		
11	Individual Investors: A Survey of Modern Investment Theory	265
11.1	A Brief History of Investment Thought	266
11.2	Free Cash Flow Valuation of Companies	269
11.2.1	Free Cash Flow Definitions	270
11.2.2	Growth and the Discounting of the Cash Flows	273
11.2.3	Terminal Multiple Models of Cash Flow Discounting	274
11.3	The Modern Portfolio Theory and the CAPM	276
11.3.1	Diversification and the Efficient Frontier	276
11.3.2	Two-Fund Separation	278
11.3.3	Systematic Risk and the CAPM	279
11.3.4	Using the CAPM as a Stock Screen to Discover Alpha	280
11.4	Multifactor Index Models	282
11.4.1	The Fama–French Three-Factor Model	283
11.4.2	The Carhart Fourth Factor: the Momentum	283
11.4.3	International Index Factors	284
11.5	Fundamental Indexing	284
11.5.1	A Brief History of Fundamental Indexing	285
11.5.2	Fundamental Indexing and Rebalancing	285
11.5.3	Tactical Asset Allocation	286
11.5.4	Fundamentally Indexed US Funds	286
12	Hedge Funds: Alpha, Beta, and Strategy Indexes	287
12.1	Hedge Fund Strategies	289
12.1.1	Relative Asset Value Funds	289
12.1.2	Relative Corporate/Credit Structure	292
12.1.3	Theoretical Relative Value	294
12.1.4	Statistical Relative Value Arbitrage	296

12.2	Portable Alpha and Market-Neutral Plays	298
12.3	Hedge Fund Replication and Strategy Indexes	299
13	Banks: Asset-Liability Management	303
13.1	Bank Balance Sheets and Income Statements	305
13.2	Interest-Sensitive Gap Management	313
13.3	Duration Gap Management	320
13.4	Value at Risk	322
14	Private Equity, Pension, and Sovereign Funds	329
14.1	Private Equity	329
14.1.1	Investment in Private Equity – Limited Partnership Funds	330
14.1.2	Leverage Buyouts	331
14.1.3	Private Equity Lending – Mezzanine Capital and Distressed Loans	332
14.1.4	Other Forms of Private Equity – PIPEs	333
14.1.5	Venture Capital	333
14.1.6	Exit Strategies – IPOs and Secondary Sales	334
14.2	Risk Allocation for Pension Funds and Sovereign Funds	335
14.2.1	Defined Benefit Pension Funds and Endowments	335
14.2.2	The Risk Budget Allocation Process	336
	Acknowledgment	338
	References	339
	Index	343

Purpose and Structure of Financial Markets

1.1 OVERVIEW OF FINANCIAL MARKETS

Financial markets play a major role in allocating excess savings to businesses in the economy. This desirable process takes various forms. Commercial banks take depositors' money and lend it to manufacturers, service firms, or home buyers who finance new construction or improvements. Investment banks bring to market equity and debt offerings of newly formed or expanding corporations. Governments issue short- and long-term bonds to finance the construction of new roads, schools, and transportation networks. Investors (bank depositors and securities buyers) supply their funds in order to shift their consumption into the future by earning interest, dividends, and capital gains.

The process of transferring savings into investment involves various participants: individuals, pension and mutual funds, banks, governments, insurance companies, industrial corporations, stock exchanges, over-the-counter (OTC) dealer networks, and others. All these agents can, at different times, serve as demanders and suppliers of funds, or as intermediaries. Economic theorists ponder the optimal design of securities and institutions, where "optimal" implies the best outcomes – lowest cost, least disputes, fastest – for security issuers and investors, as well as for the society as a whole. Are stocks, bonds, or mortgage-backed securities, the outcomes of optimal design or happenstance? Do we need "greedy" investment bankers, securities dealers, or brokers? What role do financial exchanges play in today's economy? Why do developing nations strive to establish stock exchanges even though they often have no stocks to trade on them? Once we answer these basic questions, it will not be difficult to see why all the financial markets are organically the same. In product markets, the four-cycle radiator-cooled engine-powered car and the RAM memory-bus-hard disk personal computer have withstood the test of time. And so has the spot-futures-options, primary-secondary, risk transfer-driven design of the financial market. In the wake of the 2008 crisis we have seen very limited tweaks to the design, because it is so robust.

All markets have two separate segments: *original issue* and *resale*. These are characterized by different buyers, sellers, and different intermediaries, and they perform different timing functions. The first transfers capital from the suppliers of funds (investors) to the demanders of capital (businesses); the second transfers capital from the suppliers of capital (investors) to other suppliers of capital (investors). The two segments are:

- *Primary markets* (issuer-to-investor transactions with investment banks as intermediaries in the securities markets, and banks, insurance companies and others in the loan markets);
- *Secondary markets* (investor-to-investor transactions with broker-dealers and exchanges as intermediaries in the securities markets, and mostly banks in the loan markets).

All markets have the originators, or issuers, of the claims traded in them (the original demanders of funds) and two distinctive groups of agents operating as investors, or suppliers

of funds. The two groups of funds suppliers have completely divergent motives. The first group aims to eliminate the undesirable risks of the traded assets and earn money on repackaging, the other actively seeks to take on those risks in exchange for uncertain compensation. The two groups are:

- *Hedgers* (dealers who aim to offset primary risks, be left with short-term or secondary risks, and earn spread from dealing);
- *Speculators* (investors who hold positions for longer periods without simultaneously holding positions which offset primary risks).

The claims traded in all financial markets can be delivered in three ways. The first is an immediate exchange of an asset for cash. The second is an agreement on the price to be paid with the exchange taking place at a predetermined time in the future. The last is a delivery in the future, contingent upon an outcome of a financial event, e.g. level of stock price or interest rate, with a fee paid up front for the right of delivery. The three market segments based on the delivery type are:

- *Spot or cash markets* (immediate delivery)
- *Forward markets* (mandatory future delivery or settlement)
- *Options markets* (contingent future delivery or settlement)

We focus on these structural distinctions to bring out the fact that all markets not only transfer funds from suppliers to users, but they also transfer risk from users to suppliers. They allow *risk transfer* or *risk sharing* between investors. The majority of the trading activity in today's market is motivated by risk transfer with the acquirer of risk receiving some form of certain or contingent compensation. The relative price of risk in the market is governed by a web of relatively simple arbitrage relationships that link all the markets. These allow market participants to assess instantaneously the relative attractiveness of various investments within each market segment or across all of them. Understanding these relationships is mandatory for anyone trying to make sense of the vast and complex web of today's markets.

1.2 RISK SHARING

All financial contracts, whether in the form of securities or not, can be viewed as bundles, or packages of unit payoff claims (mini-contracts), each for a specific date in the future and a specific set of outcomes. In financial economics, these are called *state-contingent claims*.

Let us start with the simplest illustration: an insurance contract. A 1-year life insurance policy promising to pay \$1,000,000 in the event of the insured's death can be viewed as a package of 12 monthly claims (lottery tickets), each paying \$1,000,000 if the holder dies during that month. The value of the policy up front (the premium) is equal to the sum of the values of all the individual tickets. As the holder of the policy goes through the year, he can discard tickets that did not pay off, and the value of the policy to him diminishes until it reaches zero at the end of the coverage period.

Let us apply the concept of state-contingent claims to known securities. Suppose you buy one share of XYZ SA stock currently trading at €45 per share and pays no dividends. You intend to hold the share for 2 years. To simplify things, we assume that the stock trades once a month and in increments of €1. The minimum price is €0 (a limited liability company cannot have a negative value) and the maximum price is €199. The share of XYZ SA can be viewed as a package of claims. Each claim represents a contingent cash flow from selling the share

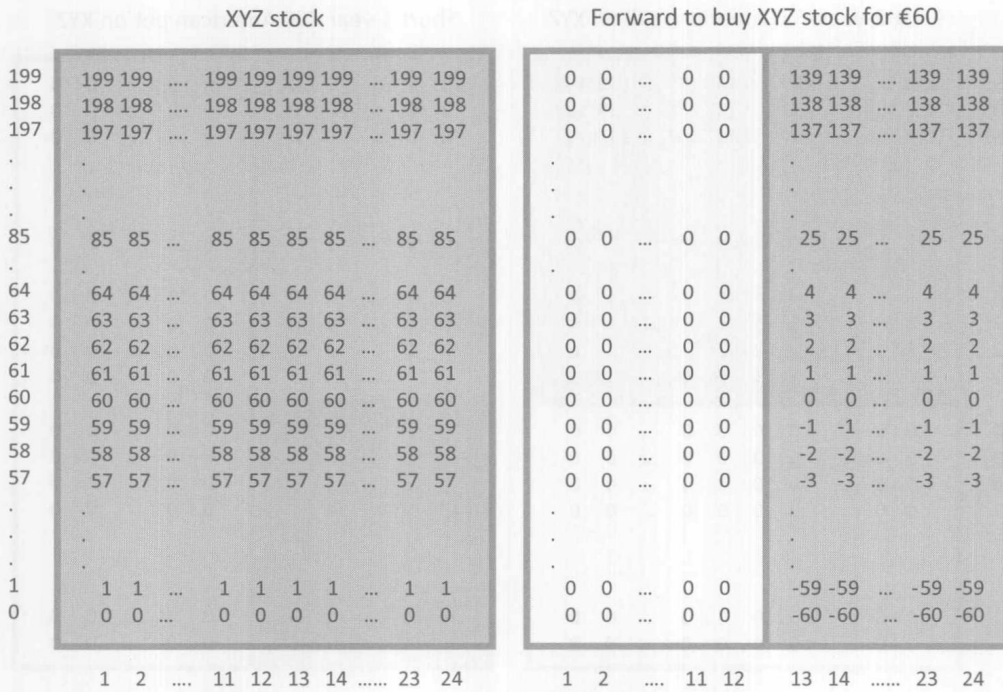


Figure 1.1 Stock and forward as packages of state-contingent claims

for a particular price in a particular month in the future. Only one of those claims will ever pay, say when we sell the stock for €78 in month 16. We can arrange the potential price levels from €0 to €199 in increments of €1 to have overall 200 possible price levels. We arrange the dates from today to 24 months from today (our holding horizon). The stock is equivalent to 200 times 24, or 480 claims. The easiest way to imagine this set of claims is as a rectangle with time on the horizontal axis and potential stock prices (states of nature) on the vertical axis. The price of the stock today is equal to the sum of the values of all the claims, i.e. all the state- and time-indexed squares of the rectangle.

Figure 1.1 shows the stock as a rectangle of 480 state-contingent claims. It also shows a forward contract on XYZ SA's stock viewed as a subset of this rectangle. Suppose we enter into a contract today to purchase the stock 13 months from today for €60. The forward can be viewed as a 200-by-24 rectangle with the first 12 months' worth of claims taken out (equal to zero, as no action can be taken). If, in month 13, the stock trades above €60, we have a gain; if the stock trades below €60, we have a loss equal to the difference between the actual stock price and the precontracted forward price.

Figure 1.2 shows a long American call option contract to buy XYZ SA's shares for €60 with an expiry 2 years from today as a 139 × 24 subset of our original rectangle, the rest zeroed out. The squares corresponding to the stock prices of €60 or below are eliminated, because they have no value. The payoff of each claim is equal to the intrinsic (exercise) value of the call. Figure 1.2 also shows a short American put struck at €60 with an expiry in 12 months.

The fundamental tenet of the option valuation methodology which applies to all securities is that if we can value each claim (one square of the rectangle) or small sets of claims (sections of the rectangle) in the package, then we can value the package as a whole sum of its parts.

Long 2-year €60 American call on XYZ										Short 1-year €60 American put on XYZ										
199	139	139	...	139	139	139	199	...	139	139	0	0	...	0	0	0	0	...	0	0
198	138	138	...	138	138	138	138	...	138	138	0	0	...	0	0	0	0	...	0	0
197	137	197	...	137	137	137	137	...	137	137	0	0	...	0	0	0	0	...	0	0
.
85	25	25	...	25	25	25	25	...	25	25	0	0	...	0	0	0	0	...	0	0
64	4	4	...	4	4	4	4	...	4	4	0	0	...	0	0	0	0	...	0	0
63	3	3	...	3	3	3	3	...	3	3	0	0	...	0	0	0	0	...	0	0
62	2	2	...	2	2	2	2	...	2	2	0	0	...	0	0	0	0	...	0	0
61	1	1	...	1	1	1	1	...	1	1	0	0	...	0	0	0	0	...	0	0
60	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0	0	0	...	0	0
59	0	0	...	0	0	0	0	...	0	0	-1	-1	...	-1	-1	0	0	...	0	0
58	0	0	...	0	0	0	0	...	0	0	-2	-2	...	-2	-2	0	0	...	0	0
57	0	0	...	0	0	0	0	...	0	0	-3	-3	...	-3	-3	0	0	...	0	0
.
1	0	0	...	0	0	0	0	...	0	0	-59	-59	...	-59	-59	0	0	...	0	0
0	0	0	...	0	0	0	0	...	0	0	-60	-60	...	-60	-60	0	0	...	0	0
	1	2	...	11	12	13	14	23	24	1	2	...	11	12	13	14	23	24

Figure 1.2 Long American call and short American put as packages of state-contingent claims. **aims**

Conversely, if we can value the package, then we are able to value subsets of claims through a subtraction of the whole minus a complement subset. Also, we may be able to combine disparate (dependent on different state variables) sets of claims (stocks on equity prices and bonds on interest rates) to form complex securities (a convertible bond). By subtracting one part (option) from the value of the combination (convertible bond), we can infer the value of a subset (straight bullet bond).

In general, the value of a contingent claim does not stay constant over time. If the holder of the life insurance becomes sick during the year and the likelihood of his death increases, then the value of all claims increases. In our stock example, the prices of the claims change as information about the company's earnings reaches the market. Not all the claims in the package have to change in value by the same amount, however. An improvement in the earnings may be only short term. The policyholder's likelihood of death may increase for the days immediately following his illness, but be less for more distant dates. As the prices of the individual claims fluctuate over time, so does the value of the entire bundle. However, at any given moment of time, the sum of the values of the claims must be equal to the value of the package, the insurance policy, or the stock. The valuation effort is restricted to here and now, and we have to repeat the exercise an instant later.

A good valuation model strives to make the claims in a package independent of each other. In our example, the payoff of the life insurance policy depends on the person *dying* during the month, not on whether the person is dead or alive. In that set-up, at most one claim of the whole set will pay. If we modeled the payoff to depend on being dead and not dying, all the claims after the morbid event would have positive prices and be contingent on each