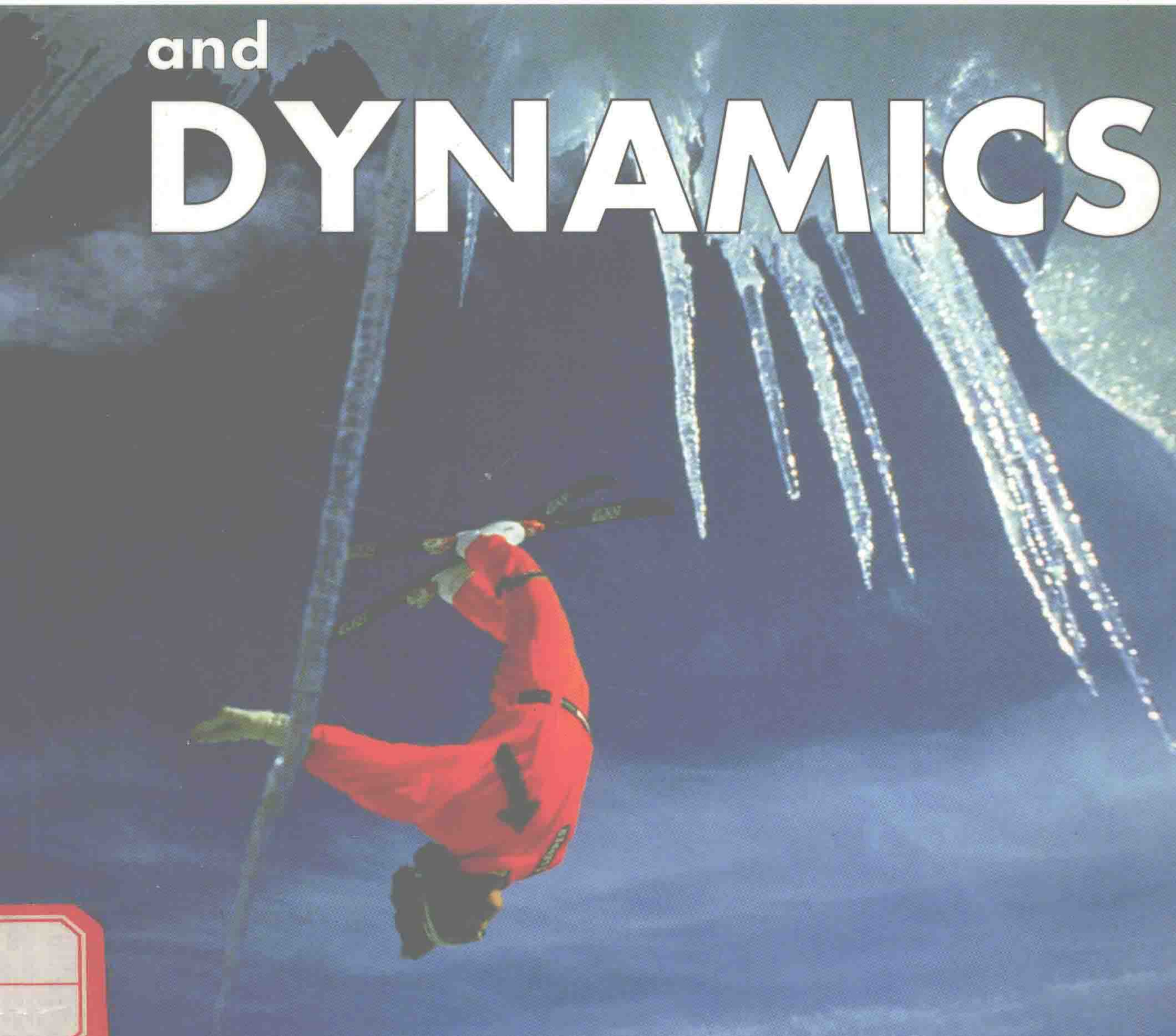
A FOUNDATION COURSE IN STATICS and DYNAMICS



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A Foundation Course in Statics and Dynamics

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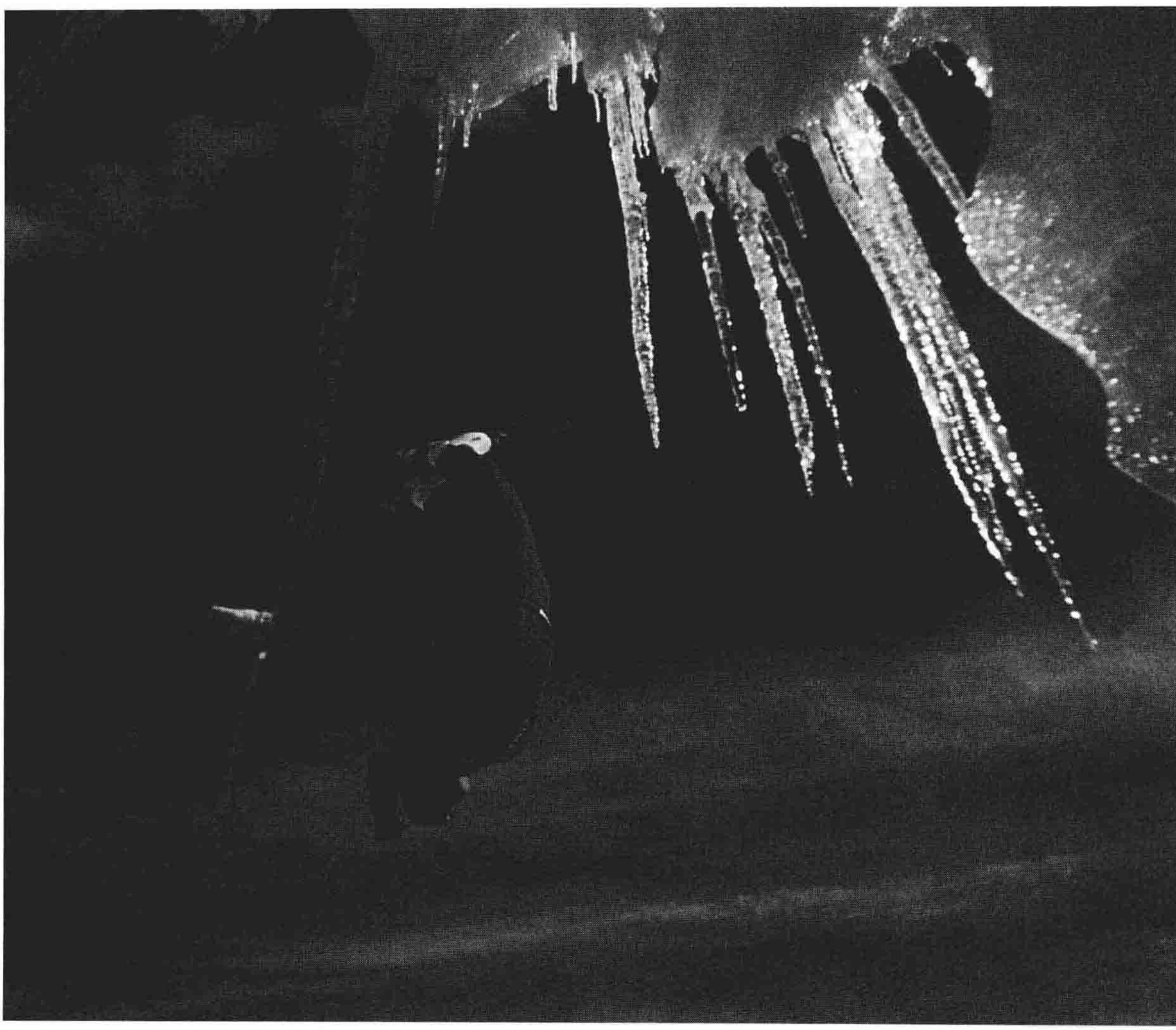

Preface

The aim of this book is to bridge the gap, which has steadily widened over the last decade, between the syllabuses for A-level mathematics and the requirements of first-year university courses in engineering and technology. This gap has occurred as a result of three movements:

- A-level syllabuses have reduced the force vector, statics and motion content of the most commonly taken mathematics courses. Much of what might have been described as applied mathematics is now to be found in specialist (second) mathematics courses. In the mid-eighties the JMB reported a drop from 7000 to 4000 candidates sitting this topic.
- In the late eighties a government-sponsored initiative encouraged a national movement to bring candidates into technology courses from a broader A-level background. This resulted in many university entrants to engineering and technology having an inadequate mathematics base. It gave rise to additional introductory courses at university in the form of foundation years, conversion courses, and so on.
- The more recent debate in the mid-nineties has highlighted the paradox between the increasing success of candidates at A-level mathematics and the increasing need for remedial classes at university. It has been suggested that this is most readily explained by the fact that syllabuses have widened and become less deep. Some topics can be dealt with in conceptual terms only, without the need for analytical ability.

Analytical ability and clear concepts are what is required for the innovative demands of university courses in engineering and technology. The aim of this book, therefore, is to enable university entrants who are weak in the applied mathematics topics to be better prepared for their first-year courses. It will be valuable as a text for introductory courses and as a reinforcement to the basic texts in engineering courses which presume prior knowledge of much foundational material.

The chapters are arranged to take the student from basic concepts of force, equilibrium, motion and energy through to their application in framed structures and force systems, rectilinear and curvilinear motion, friction, work and momentum. The basic relationships are described in each chapter and clarified by diagram or equation as appropriate. Topics are illustrated by worked examples and test-yourself problems. Recap boxes reinforce and summarize the principal teaching objectives at key points in each chapter. The text, examples and problems are all set, as far as possible, in the context of real engineering applications, which will help the student to visualize and remember the principles being explained.

The authors trust they have eliminated all the errors from the calculations. Identification of any errors will be gratefully received.

Acknowledgements

The authors would like to acknowledge the debt owed to many teachers and lecturers who over many years taught them the basics of statics and dynamics. The subject matter has a long history, but is as fundamental today as it was in its misty beginnings. Without a sound understanding of statics and dynamics, much of what we now call technology is flawed. What was taught to us, we now pass on, but in the process we have adapted these historical subjects to the needs of today.

Contents

1 Introduction

1.1 Statics and dynamics

Statics and dynamics are subjects which form the basis for understanding the action of forces, the nature of movements, and the way in which these phenomena interrelate with each other. Considerable parts of engineering depend on an accurate appreciation of the nature of forces and movements, so statics and dynamics are fundamental to many disciplines of engineering and technology. They have particular importance to aeronautical engineering, civil engineering, marine technology, mechanical engineering, and structural engineering. These branches of engineering and technology interrelate largely through their common foundations in statics and dynamics.

Statics is primarily concerned with the behaviour of forces acting on objects at rest. It is therefore of prime importance when considering the stability of buildings and similar large structures. But it is of equal significance when considering very small components such as switches and control mechanisms. And statics is just as relevant for components in buildings, aircraft, cars, ships and domestic appliances. Although statics usually assumes a state of rest for the object or component, it has equal meaning for states of uniform motion. And it may be applied to states of varying motion by use of equivalent static forces.

Dynamics is the study of the motions of objects and the forces that cause them. If the forces acting on an object or a system are not in equilibrium (balance each other out), it will be set in motion. The basic rules for predicting the behaviour of unbalanced systems were largely developed by Sir Isaac Newton towards the end of the seventeenth century. The application of these rules, called Newtonian mechanics, is covered in Parts II and III. The subject naturally divides into kinematics and kinetics. Kinematics studies the motions of objects without reference to the forces causing them. Kinetics studies the relationship between the motions of objects and the forces to which they are subjected. Nearly all engineering systems respond dynamically at some level. Sometimes the dynamics of a system are of supreme importance, as in the design of machines; sometimes they can be almost ignored. The twin towers of the World Trade Center in New York are so tall they can be felt swaying in the wind (responding dynamically to wind forces) but they were designed largely upon the principles of statics. Because of its relevance to virtually all aspects of engineering design and application, a thorough understanding of the subject is essential for scientists and engineers.

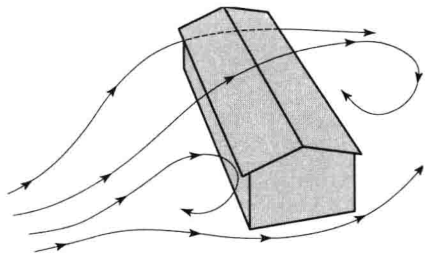

Figure 1.1 The relationship between wind speed, direction and pressure may follow a statistical distribution.

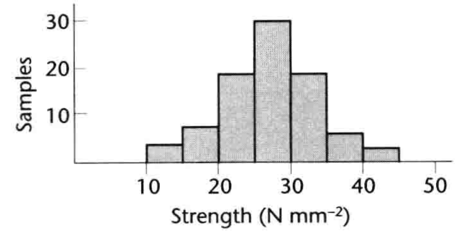

Figure 1.2 Strength of concrete on a construction site.

1.2 Laws of physics

Many natural phenomena take place in accordance with clearly defined laws. They have repeatability; if they were observed on more than one occasion, each time they would demonstrate the same relationship. In all such events there are some factors which are not readily controlled, so perfect repeatability is impossible to achieve. Our observations of natural phenomena therefore always contain an element of variability; here are some examples:

- Observation of the acceleration of a falling body
- The relationship of extension to applied force for a particular material
- Variation of volume, pressure and temperature for a gas

Minor variations in the object being observed, the surrounding influences and imperfections in our methods of observation, all will lead to some degree of uncertainty in the result. Any laws which may be proposed to express the observed events therefore contain this same degree of uncertainty. Physical laws may therefore be described as **best-fit statements** of observed events. Statements of this kind are essential to the development of engineering and technology, and for convenient use are presented in the form of equations or graphs.

Although many observed events appear to accord with clear laws, allowing for these variations, some other events may not show such clear relationships. Observation may show a wide scatter of results, which might be described as a **distribution** or **population**; here are some examples:

- The relationship of wind speeds, directions and pressure (Figure 1.1)
- Variation of the strength of concrete on a construction site (Figure 1.2)
- Observation of an irregular body falling in a viscous fluid (Figure 1.3)

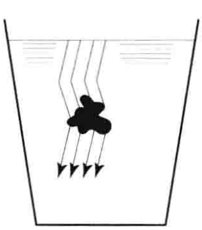

Figure 1.3 An irregular body falling in a viscous fluid.

For these and similar events the distributions of data could be assembled using statistical methods. Laws derived in this way would have much wider degrees of uncertainty, which themselves would need to be stated and quantified. Any physical law contains *some degree* of uncertainty, whether it be to a minor extent or a major extent.

Equations or graphs may be used to express laws governing physical events and may be simple or more complex in form:

- Force = mass × acceleration ($F = ma$)
- Final velocity = initial velocity + acceleration× time ($v = u + at$)
- Elastic modulus = stress/strain ($E = \sigma/\varepsilon$)

Scientists and engineers are interested in describing, recording and communicating physical processes for use in engineering design. If such information has been collected by observation it is usually processed in the form of discrete data. That is to say, measurements of quantities of interest are repeatedly taken and related to some independent variable such as time, displacement or temperature. In this case the data is only available at those instants when the measurements were made. Some experiments, to all intents and purposes, have output in the form of continuous data, where the data is recorded by equipment such as a pen recorder, which presents the information in the form of a continuous curve or signal. In these cases the data is available at any point during the period over which the experiment was conducted. If the data is well behaved it can often be represented by a curve. The curve's equation can be deduced from the data and sometimes a physical law may be formulated on the strength of it.

One set of physical laws fundamental to the understanding of statics and dynamics are Newton's laws of motion. These may be stated as follows:

(1) A particle remains at rest, or continues at uniform velocity in a straight line, unless acted on by a force.
(2) The acceleration of a particle is proportional to the force acting on it and is in the direction of the force.
(3) The forces of action and reaction between particles are equal in magnitude, opposite in direction, and are collinear.

These three laws will be explored and developed in later chapters, particularly Chapters 3 and 7, but the concepts expressed in them form the foundations of this text, and for statics and dynamics in general.

1.3 Concept and calculation

Physical phenomena and the laws describing them are discussed in Section 1.2. They are usually the result of observing natural events, and attempting to formulate an idea which governs their behaviour as seen. This idea, encapsulated in a law, is then passed on to others in the form of words, diagrams, models or even demonstrations. This is the **concept**; it must be capable of transmission in a form that is understandable to its audience. Because concepts embodied in physical laws describe actual natural events, some degree of visualization is possible, and even essential, to understanding the topic. A concept may be expressed in words or equations, but to make certain it is understood, any concept should be converted into visual form.

Calculation by contrast does not require understanding of the concepts contained in a physical law, but merely its application to obtain some desired information. All that is required in the calculation process is an acceptance of some equations, purporting to describe natural events, coupled with an ability in algebra and arithmetic. At the beginning of a course in engineering it is possible to make progress and even pass examinations by using calculation allied to a good memory. But to make secure progress in engineering it is essential to have grasped the concepts fully. In this sense the ability to learn rules of calculation, and to carry out the process successfully, may be a barrier to a proper understanding of the concepts.

However, once the proper concepts are visualized and grasped, calculation of examples will reinforce an understanding. Calculation will also give substance to the concepts, ensure that relative magnitudes are recognized, and give an appreciation of the correct units.

Calculation of specific information from given equations is usually carried out simply by use of a hand calculator. The only requirement for accuracy is to know the functions of the calculator and the proper sequences for inputting data. Most calculators will also include functions to process data in statistical form.

The use of computers in calculation is of greatest value when the equations and arithmetic become complex, or when one calculation process is to be repeated many times. Neither requirement is appropriate to examples in this text, but it is always worthwhile to obtain practice in computing, especially inputting data and presenting results.

1.4 Accuracy

The variation in most engineering data (Section 1.2) means it is usual for it to carry a limited accuracy. Hence information given for examples will usually be presented to two-figure or at most three-figure accuracy. No amount of calculation can increase that accuracy, and it is therefore inappropriate to give answers to a greater accuracy. Examples of the three-figure accuracy are numbers such as 1.27, 0.0164, 81 600 or 20.2.

Presentation of data in the form of graphs can lead to calculations which are themselves approximate. Where graphs are given in the form of defined mathematical relationships then slopes and areas associated with the graphs can be found precisely, as in Figures 1.4 and 1.5. Where the graph represents a more random set of data, as in Figure 1.6, calculation of slopes and areas may introduce an element of approximation. The accuracy of representing the physical process described by the

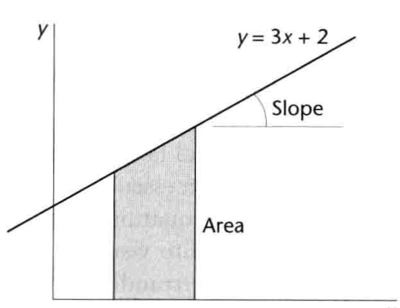

Figure 1.4 Straight line: slopes and areas can be found precisely.

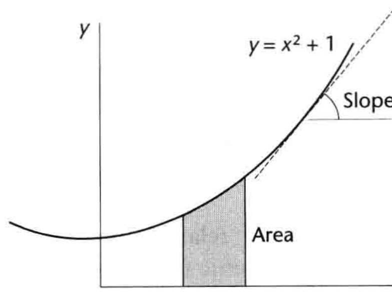

Figure 1.5 Quadratic curve: slopes and areas can be found precisely.

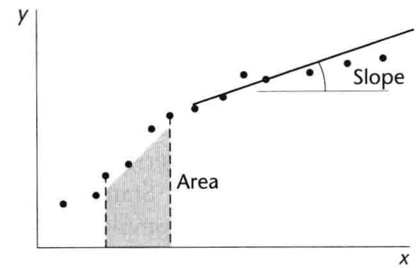

Figure 1.6 Random data: slopes and areas can only be estimated.

variable y depends on how rapidly it changes with x. Chapter 7 shows that information about an object's velocity can be derived from the slope of its displacement–time curve or the area under its acceleration–time curve. If these processes were represented by Figure 1.6, y would be the displacement or acceleration and x would be time. The velocity could only be estimated with any accuracy if the data for y were collected for frequent and regular small intervals of x in those regions where y changed rapidly with x. Similarly, if the data were continuous, as produced by a pen recorder, the accuracy of the estimate of the velocity at points in time during the experiment would increase; this is because the estimate would be computed over smaller and smaller intervals of time. A fuller explanation is given in Chapter 7.

1.5 Engineering units

Statics and dynamics involve the study of real events, and like engineering in general, the data will be measured in real quantities. As such the data will require **units** of measurement such as metres for distance and kilograms for mass. Normally the units will conform to the SI system, using the following quantities:

Distance	metre	m
Mass	kilogram	kg
Force	newton	N
Velocity	metre per second	$\mathrm{m\,s^{-1}}$
Pressure	newton per square metre	$\mathrm{N\,m^{-2}}$

In many cases it is convenient to use larger or smaller units, then multiples of 1000 are used such as kilo ($\mathrm{k} = 10^3$), mega ($\mathrm{M} = 10^6$), milli ($\mathrm{m} = 10^{-3}$). Intermediate powers of 10 are avoided.

It is often necessary in using equations to multiply or divide quantities involving units. This is acceptable and a new unit is usually produced. Hence metres (m) may be divided by seconds (s) to give $\mathrm{m\,s^{-1}}$. It is not possible, however, to add or subtract quantities having different units. Hence metres plus seconds has no meaning. This is a useful check in all calculations; it ensures that units are compatible with the calculation being carried out.

1.6 Algebra

In order to carry out calculations in engineering it is necessary to master the processes of algebra. Statements of physical laws, the equations for these laws and graphs representing them, all are usually cast in algebraic form. It is essential to understand how to manipulate algebraic expressions, to transpose and solve equations, and to derive information from graphs.

Statements of physical laws and other relationships can be quite lengthy and complex. Restatement of these laws in algebraic form is a form of shorthand, and gives great clarity to the relationship being described. Also, in the calculating process, algebra is a generalized form of arithmetic, and shows that the calculation applies to *all* numbers and not just the *particular* set of numbers being used at that time.

The processes of algebra are summarized below. For a full treatment of each item, reference should be made to one of the books listed in the further reading.

Symbols and notation

$$A + A = 2A$$
$$A \times A = A^2$$
$$A \times B = AB$$

Addition and subtraction

$$A + B = B + A$$
$$(A + B) + C = A + (B + C)$$
$$A - B = -B + A$$

Multiplication

$$A \times B = B \times A$$
$$A \times (B + C) = AB + AC$$

Division

$$(A + B)/C = A/C + B/C$$

Expansion

$$\begin{aligned}(A + B) \times (C + D) &= A(C + D) + B(C + D)\\ &= AC + AD + BC + BD\\ (A + B) \times (A + B) &= A(A + B) + B(A + B)\\ &= A^2 + 2AB + B^2\\ (A + B) \times (A - B) &= A(A - B) + B(A - B)\\ &= A^2 - B^2\end{aligned}$$