INTERMEDIATE FINANCIAL THEORY

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PREFACE

The market for financial textbooks is crowded at both the introductory and doctoral levels, but much less so at the intermediate level. Teaching opportunities at this level, however, are multiplying rapidly with the advent of masters of science programs in finance (master in computational finance, in mathematical finance, and the like) and the strengthening demand for higher-level courses in MBA programs.

The Masters in Banking and Finance Program at the University of Lausanne admitted its first class in the fall of 1993. One of the first such programs of its kind in Europe, its objective was to provide advanced training to finance specialists in the context of a one-year theory-based degree program. In designing the curriculum, it was felt that students should be exposed to an integrated course that would introduce a wide breadth of topics in financial economics, similar to what is found at the doctoral level. Such exposure could, however, ignore the particulars and detailed proofs and arguments and concentrate on the larger set of issues and concepts to which any advanced practitioner should be exposed.

Our ambition in this text is, accordingly, first to review rigorously and concisely the main themes of financial economics (those that students should have encountered in prior courses) and, second, to introduce a number of *frontier* ideas of importance for the evolution of the discipline and of relevance from a practitioner's perspective. We want our readers to be at ease with the main concepts of standard finance (MPT, CAPM, etc.) while also being aware of the principal new ideas that have marked the recent evolution of our discipline. Contrary to introductory texts, we aim at depth and rigor; contrary to higher-level texts, we do not emphasize generality. Whenever an idea can be conveyed through an example, this is the approach we chose. We have, similarly, ignored proofs and detailed technical matters unless a reasonable understanding of the related concept mandated their inclusion. Throughout the book the emphasis is on the notion of competitive financial equilibrium—what it means and how it is characterized in a variety of contexts ranging from the Arrow-Debreu model to the consumption capital asset pricing model. These concepts are presented as a platform for an in-depth understanding of the newer arbitrage pricing approaches.

Intermediate Financial Theory is intended primarily for masters level students with a professional orientation, a good quantitative background, and a preliminary education in business and finance. As such, the book is targeted for masters students in finance, but it is also appropriate for an advanced MBA class in financial economics, one with the objective of introducing students to the precise modeling of many of the concepts discussed in their capital markets and corporate finance classes. In addition, we believe the book will be a useful reference for entering doctoral candidates in finance

whose lack of prior background might prevent them from drawing the full benefits of the abstract material typically covered at that level. Finally, it is a useful refresher for well-trained practitioners.

As far as prerequisites go, we take the view that our readers will have completed at least one introductory course in finance (or read the corresponding text) and will not be intimidated by mathematical formalism. Although the mathematical requirements of the book are not large, some confidence in the use of calculus as well as matrix algebra is helpful.

Over the years, we have benefited from numerous discussions with colleagues over issues related to the material included in this book. We are especially grateful to Paolo Siconolfi and Jeremy Staum, both of Columbia University. We are also indebted to several generations of teaching assistants—François Christen, Philippe Gilliard, Tomas Hricko, Aydin Akgun, Paul Ehling—and of MBF students at the University of Lausanne who have participated in the shaping of this material. Their questions, corrections, and comments have lead to a continuous questioning of the approach we have adopted and have dramatically increased the usefulness of this text. In addition to these, we would like to acknowledge our reviewers, John Primus of California State University–Hayward and Victor Abraham of Pasadena City College. Finally, we would like to thank the Fondation du 450ème of the University of Lausanne for providing "seed financing" for this project.

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CONTENTS

CHAPTER 1

ON THE ROLE OF FINANCIAL MARKETS AND INSTITUTIONS

1.1 FINANCE: THE TIME DIMENSION

Why do we need financial markets and institutions? We have chosen to address this question as our introduction to this text on financial theory. In doing so we touch on some of the most difficult issues in finance and introduce concepts that will eventually require extensive developments. Our purpose here is to phrase this question as an appropriate background for the study of the more technical issues that will occupy us at length. We also want to introduce some important elements of the necessary terminology. We ask the reader's patience as most of the sometimes-difficult material introduced here will be taken up in more detail in the following chapters.

A financial system is a set of institutions and markets permitting the exchange of contracts and the provision of services for the purpose of allowing the income and consumption streams of economic agents to be desynchronized—that is, made less similar. It can, in fact, be argued that indeed the *primary* function of the financial system is to permit such desynchronization. There are two dimensions to this function: the time dimension and the risk dimension. Let us start with time. Why is it useful to dissociate consumption and income across time? Two reasons come immediately to mind. First, and somewhat trivially, income is typically received at discrete dates, say monthly, while it is customary to wish to consume continuously (i.e., every day).

Second, and more importantly, consumption spending defines a *standard of living* and most individuals find it difficult to alter their standard of living from month to month or even from year to year. There is a general, if not universal, desire for a *smooth* consumption stream. Because it deeply affects everyone, the most important manifestation of this desire is the need to save (consumption smaller than income) for retirement so as to permit a consumption stream in excess of income (dissaving) after retirement begins. The *lifecycle* patterns of income generation and consumption spending are not identical, and the latter must be created from the former. The same considerations apply to shorter horizons. Seasonal patterns of consumption and income, for

example, need not be identical. Certain individuals (car salespersons, department store salespersons) may experience variations in income arising from seasonal events (e.g., most new cars are purchased in the spring and summer), which they do not like to see transmitted to their ability to consume. There is also the problem created by temporary layoffs due to business cycle fluctuations. While temporarily laid off and without substantial income, workers do not want their family's consumption to be severely reduced.

Furthermore, and quite crucial for the growth process, some people—entrepreneurs, in particular—are willing to accept a relatively small income (but not consumption!) for a period of time in exchange for the prospect of high returns (and presumably high in-

BOX 1-1

Representing Preference for Smoothness

The preference for a smooth consumption stream has a natural counterpart in the form of the utility function, $U(\ )$, typically used to represent the relative benefit a consumer receives from a specific consumption bundle. Suppose the representative individual consumes a single consumption good (or a basket of goods) in each of two periods, now and tomorrow. Let c_1 denote today's consumption level and c_2 tomorrow's, and let $U(c_1) + U(c_2)$ represent the level of utility (benefit) obtained from a given consumption stream (c_1, c_2).

Therefore, preference for consumption smoothness must mean, for instance, that the consumption stream $(c_1, c_2) = (4, 4)$ is preferred to the alternative $(c_1, c_2) = (3, 5)$, or

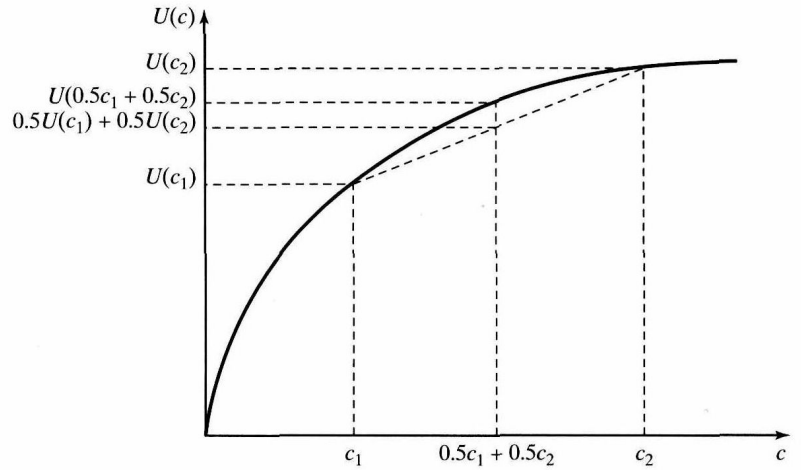
$$U(4) + U(4) > U(3) + U(5),$$

Dividing both sides of the inequality by 2, this implies

$$U(4) > \tfrac{1}{2}U(3) + \tfrac{1}{2}U(5).$$

As shown in Figure 1-1, when generalized to all possible alternative consumption pairs, this property implies that the function $U(.)$ has the rounded shape that we associate with "strict concavity."

FIGURE 1-1 A Strictly Concave Utility Representation



come) in the future. They are operating a sort of "arbitrage" over time. This does not disprove their desire for smooth consumption; rather they see opportunities that lead them to accept what is formally a low income level initially, against the prospect of a relatively high income level later (followed by a zero income level when they retire). They are investors who, typically, do not have enough liquid assets to finance their projects, hence the need to raise capital by borrowing or selling shares.

Therefore, the first key element in finance is **time.** In a timeless world, there would be no assets, no financial transactions (although money would be used, it would have only a transaction function), and no financial markets or institutions. The very notion of a (financial) contract implies a time dimension.

Asset holding permits the desynchronization of consumption and income streams. The peasant putting aside seeds, the miser burying his gold, or the grandmother putting a few hundred dollar bills under her mattress are all desynchronizing their consumption and income, and in doing so, presumably provide a higher level of well-being for themselves. A fully developed financial system should also have the property of fulfilling this same function *efficiently.* By that we mean that the financial system should provide versatile and diverse instruments to accommodate the widely differing needs of savers and borrowers in so far as size (many small lenders, a few big borrowers), timing and maturity of loans (how to finance long-term projects with short-term money), and the liquidity characteristics of instruments (precautionary saving cannot be tied up permanently). In other words, the elements composing the financial system should aim at *matching* as perfectly as possible the diverse financing needs of different economic agents.

1.2 DESYNCHRONIZATION: THE RISK DIMENSION

We argued above that time is of the essence in finance. When we talk of the importance of time in economic decisions, we think in particular of the relevance of choices involving the present versus the future. But the future is, by essence, uncertain: Financial decisions with implications (payouts) in the future are necessarily risky. Time and risk are inseparable. This is why **risk** is the second key word in finance.

For the moment let us compress the time dimension into the setting of a "Now and Then" (present vs. future) economy. The typical individual is motivated by the desire to smooth consumption between "Now" and "Then." This implies a desire to identify consumption opportunities that are as smooth as possible among the different possibilities that may arise "Then." In other words, *ceteris paribus*—most individuals would like to guarantee their family the same standard of living whatever events transpire tomorrow: whether they are sick or healthy; unemployed or working; confronted with bright or poor investment opportunities; fortunate or hit by unfavorable accidental events. This characteristic of preferences is generally described as "aversion to risk."

A productive way to start thinking about this issue is to introduce the notion of *states of nature.* A state of nature is a complete description of a possible scenario for the future across all the dimensions relevant for the problem at hand. In a "Now and Then" economy, all possible future events can be represented by an exhaustive list of states of nature or *states of the world.* We can thus extend our former argument for smoothing consumption across time by noting that the typical individual would also like to experience similar consumption levels across all future states of nature, whether good or bad.

An efficient financial system offers ways for savers to reduce or eliminate, at a fair price, the risks they are not willing to bear (risk shifting). Fire insurance contracts eliminate the financial risk of fire, and put contracts can prevent the loss in wealth associated with a stock's price declining below a predetermined level, to mention two examples. The financial system also makes it possible to obtain relatively safe aggregate returns from a large number of small, relatively risky investments. This is the process of diversification. By permitting economic agents to *diversify*, to *insure*, and to *hedge*, an efficient financial system fulfills the function of redistributing purchasing power not only over time, but also across states of nature.

1.3 THE SCREENING AND MONITORING FUNCTIONS OF THE FINANCIAL SYSTEM

The business of desynchronizing consumption from income streams across time and states of nature is often more complex than our initial description may suggest. If time implies uncertainty, uncertainty may imply not only risk, but often *asymmetric information* as well. By this term, we mean situations where the individuals involved have different information, with some being potentially better informed than others. How can a saver find a borrower with a good ability to repay or an investor with a good project, yielding the most attractive return for him and hopefully for society as well? What do "good" and "most attractive" mean? Do these terms refer to the highest potential return? What about risk? What if the return is itself affected by the actions of investors (a phenomenon labeled "moral hazard")? How does one share the risks of a project in such a way that both investors and savers are willing to proceed, taking actions acceptable to both? An efficient financial system not only assists in these information and monitoring tasks, but also provides a range of instruments (contractual arrangements) suitable for the largest number of savers and borrowers, thereby contributing to the channeling of savings toward the most efficient projects.

In the terms of the preeminent economist, Joseph Schumpeter (1961), "Bankers are the gatekeepers of capitalist economic development. Their strategic function is to screen potential innovators and advance the necessary purchasing power to the most

BOX 1-2

Representing Risk Aversion

Let us reinterpret the two-date consumption stream (c_1, c_2) of Box 1-1 as the consumption levels attained "Then" or "Tomorrow" in two alternative, equally likely, states of the world. The desire for a smooth consumption stream across the two states, which we associate with risk aversion, is obviously represented by the same inequality

$$U(4) > \tfrac{1}{2}U(3) + \tfrac{1}{2}U(5)$$

This implies the same general shape for the utility function. In other words, assuming plausibly that decision makers are **risk averse,** an assumption in conformity with most of financial theory, implies that the utility functions used to represent agents' preferences are **strictly concave.**

promising." For highly risky projects, such as the creation of a new firm exploiting a new technology, venture capitalists provide a similar function today.

1.4 THE FINANCIAL SYSTEM AND ECONOMIC GROWTH

The performance of the financial system matters at several levels. We shall argue that it matters for growth, that it impacts the characteristics of the business cycle, and most importantly, that it is a significant determinant of economic welfare. We tackle growth first. Channeling funds from savers to investors efficiently is obviously important. Whenever more efficient ways are found to perform this task, society can achieve a greater increase in tomorrow's consumption for a given sacrifice in current consumption.

Intuitively, more savings should lead to greater investment and thus greater future wealth. Figure 1-2 indeed suggests that, for 90 developing countries over the period 1971 to 1992, there was a strong positive association between saving rates and growth rates. When looked at more carefully, however, the evidence is usually not as strong.[1] One important reason may be that the hypothesized link is, of course, dependent on a ceteris

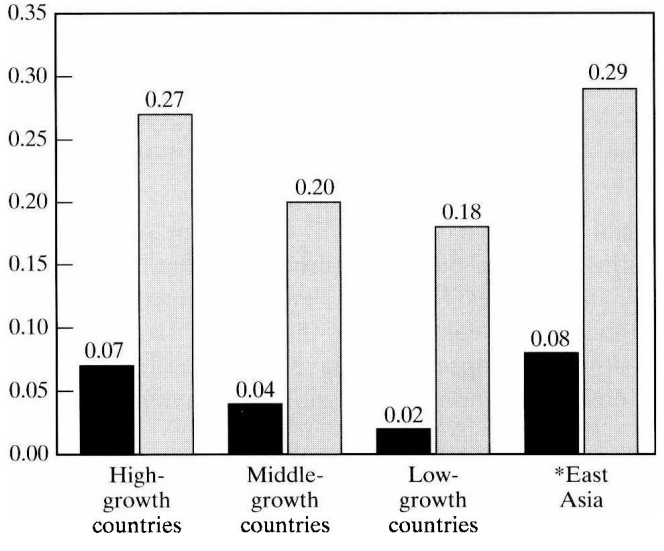


FIGURE 1-2 Savings and Growth in 90 Developing Countries

Source: IMF World Economic Outlook, May 1993 (Annual data, 1971–1992).

[1] In a straightforward regression in which the dependent variable is the growth rate in real per capita GNP, the coefficient on the average fraction of real GNP represented by investment (I/Y) over the prior five years is positive but insignificant. Together with other results, this is interpreted as suggesting a reverse causation from real per capita GNP growth to investment spending. See Barro and Sala-i-Martin (1995), Chapter 12, for a full discussion. There is also a theoretically important distinction between the effects of increasing investment (savings) (as a proportion of national income) on an economy's level of wealth and its growth rate. Countries that save more will *ceteris paribus* be wealthier. They need not grow more rapidly. The classic growth model of Solow (1956) illustrates this distinction.

paribus or "everything else maintained equal" clause: It applies only to the extent savings are invested in appropriate ways. The economic performance of the former Union of Soviet Socialist Republics reminds us that it is not enough only to save; it is also important to invest judiciously. Historically, the investment/GDP (Gross Domestic Product) ratio in the Soviet Union was very high in international comparisons, suggesting the potential for very high growth rates. After 1989, however, experts realized that the value of the existing stock of capital was not consistent with the former levels of investment. A great deal of the investment must have been effectively wasted, in other words, allocated to poor or even worthless projects. Equal savings rates can thus lead to investments of widely differing degrees of usefulness from the viewpoint of future growth. However, in line with the earlier quote from Schumpeter, there are reasons to believe that the financial system has some role to play here as well.

The following excerpt from *Economic Focus* (UBS Economic Research, 1993) is part of a discussion motivated by the observation that, even for high-saving countries of Southeast Asia, the correlation between savings and growth has not been uniform.

> The paradox of raising saving without commensurate growth performance may be closely linked to the inadequate development of the financial system in a number of Asian economies. Holding back financial development ('financial repression') was a deliberate policy of many governments in Asia and elsewhere who wished to maintain control over the flow of savings. (. . .) Typical measures of financial repression still include interest rate regulation, selective credit allocation, capital controls, and restricted entry into and competition within the banking sector.

These comments take on special significance in light of the recent Asian crisis, which provides another, dramatic, illustration of the growth-finance nexus. Economists do not fully agree on what causes financial crises. There is, however, a consensus that in the case of several East-Asian countries, the weaknesses of the financial and banking sectors, such as those described as "financial repression," have to take at least part of the blame for the collapse and the ensuing economic regression that have marked the end of the 1990s in Southern Asia.

Let us try to go further than these general statements in the analysis of the savings and growth nexus and of the role of the financial system. Following Barro and Sala-i-Martin (1995), one can view the process of transferring funds from savers to investors in the following way.[2] The least efficient system would be one in which all investments are made by the savers themselves. This is certainly inefficient because it requires a sort of "double coincidence" of intentions: Good investment ideas occurring in the mind of someone lacking past savings will not be realized. Funds that a non-entrepreneur would like to save would not be put to productive use. Yet, this unfortunate situation is a clear possibility if the necessary confidence in the financial system is lacking with the consequence that savers do not entrust the system with their savings. One can thus think of circumstances where savings never enter the financial system, or where only a small fraction does. When it does, it will typically enter via some sort of depository institution. In an international setting, a similar problem arises if national savings are primarily in-

[2]For a broader perspective and a more systematic connection with the relevant literature on this topic, see Levine (1997).

vested abroad, a situation that may reach alarming proportions in the case of less developed countries.[3] Let FS/S represent, then, the fraction of aggregate savings (S) being entrusted to the financial system (FS).

At a second level, the functioning of the financial system may be more or less costly. While funds transferred from a saver to a borrower via a direct loan are immediately and fully made available to the end user, the different functions of the financial system previously discussed are often best fulfilled, or sometimes can only be fulfilled, through some form of intermediation, which typically involves some cost. Let us think of these costs as administrative costs, on the one hand, and costs linked to the reserve requirements of banks, on the other. Different systems will have different operating costs in this large sense, and as a consequence, the amount of resources transferred to investors will also vary. Let us think of BOR/FS as the ratio of funds transferred from the financial system to borrowers and entrepreneurs.

Borrowers themselves may make diverse use of the funds borrowed. Some, for example, may have pure liquidity needs (analogous to the reserve needs of depository institutions), and if the borrower is the government, it may well be borrowing for consumption! For the savings and growth nexus, the issue is how much of the borrowed funds actually result in productive investments. Let I/BOR represent the fraction of borrowed funds actually invested. Note that BOR stands for borrowed funds whether private or public. In the latter case a key issue is what fraction of the borrowed funds are used to finance public investment as opposed to public consumption.

Finally let EFF denote the efficiency of the investment projects undertaken in society at a given time, with EFF normalized at unity; in other words, the average investment project has $EFF = 1$, the below-average project has $EFF < 1$, and conversely for the above average project—a project consisting of building a bridge leading nowhere—would have an $EFF = 0$; K is the aggregate capital stock, Y aggregate income, and Ω the depreciation rate. Then we may write

$$\dot{K} = EFF \cdot I - \Omega K \tag{1.1}$$

or, multiplying and dividing I with each of the newly defined variables

$$\dot{K} = EFF \cdot (I/BOR) \cdot (BOR/FS) \cdot (FS/S) \cdot (S/Y) \cdot Y - \Omega K \tag{1.2}$$

where our notation is meant to emphasize that the growth of the capital stock at a given savings rate might be influenced by the levels of the various ratios introduced above.[4] Let us now review how this might be the case.

One can see that a financial system performing its matching function efficiently will positively affect the savings rate (S/Y) and the fraction of savings entrusted to financial institutions (FS/S). This reflects the fact that savers can find the right savings instruments for their needs. In terms of overall services net of inconvenience, this acts like an

[3]The problem is slightly different here. Although capital flight is a problem from the viewpoint of building up a country's home capital stock, the acquisition of foreign assets may be a perfectly efficient way of building a national capital stock. The effect on growth may be negative when measured in terms of GDP (Gross Domestic Product), not necessarily so in terms of national income or GNP (Gross National Product). Switzerland is the example of a rich country investing heavily abroad and deriving a substantial income flow from it. It can be argued that the growth rate of the Swiss Gross National Product (but probably not GDP) has been enhanced rather than decreased by this fact.

[4]$\dot{K} = dK/dt$, that is, the change in K as a function of time.

increase in the return to the fraction of savings finding its way into the financial system. The matching function is also relevant for the *I/BOR* ratio. With the appropriate instruments (like flexible overnight loan facilities) firm's cash needs are reduced and a larger fraction of borrowed money can actually be used for investment.

By offering a large and diverse set of possibilities for spreading risks (insurance and hedging), an efficient financial system will also positively influence the savings ratio (*S/Y*) and the *FS/S* ratio. Essentially this works through improved return/risk opportunities, corresponding to an improved trade-off between future and present consumption (for savings intermediated through the financial system). Furthermore, in permitting entrepreneurs with risky projects to eliminate unnecessary risks by using appropriate instruments, an efficient financial system provides, somewhat paradoxically, a better platform for undertaking riskier projects. If, on average, riskier projects are also the ones with the highest returns, as most of financial theory reviewed later in this book leads us to believe, one would expect that the more efficiently this function is performed, the higher (ceteris paribus), the value of *EFF*; in other words, the higher, on average, the efficiency of the investment undertaken with the funds made available by savers.

Finally, a more efficient system may be expected to more effectively screen alternative investment projects and to better and more cost efficiently monitor the conduct of the investments (efforts of investors). The direct impact is to increase *EFF*. Indirectly this also means that, on average, the return/risk characteristics of the various instruments offered savers will be improved and one may expect, as a result, an increase in both *S/Y* and *FS/S* ratios.

The previous discussion thus tends to support the idea that the financial system plays an important role in permitting and promoting the growth of economies. Yet growth is not an objective in itself. There is such a thing as excessive capital accumulation. Jappelli and Pagano (1994) suggest that borrowing constraints,[5] in general a source of inefficiency and the mark of a less than perfect financial system, may have led to more savings (in part unwanted) and higher growth. While their work is tentative, it underscores the necessity of adopting a broader and more satisfactory viewpoint and of more generally studying the impact of the financial system on social welfare. This is best done in the context of the theory of general equilibrium, a subject to which we shall turn in Section 1.6.

1.5 FINANCIAL INTERMEDIATION AND THE BUSINESS CYCLE

Business cycles are the mark of all developed economies. According to much of current research, they are in part the result of external shocks with which these economies are repeatedly confronted. The depth and amplitude of these fluctuations, however, may well be affected by some characteristics of the financial system. This is at least the im-

[5]By "borrowing constraints" we mean the limitations that the average individual may experience in his or her ability to borrow, at current market rates, from financial institutions.