APPLICATIONS OF KALMAN FILTER TO HYDROLOGY, HYDRAULICS, AND WATER RESOURCES

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PREFACE

Seven years ago, in 1971, also at the University of Pittsburgh, I organized the First International Symposium on Stochastic Hydraulics, marking the establishment of "stochastic hydraulics." Its impacts are evidenced by various activities that have followed: The second one was held in Europe (Sweden) in 1976; and the third one will be held in Asia (Japan) in 1980. Committees and Task Committees on stochastic hydraulics have also been established under the International Association for Hydraulic Research, and the American Society of Civil Engineers. I hope that the methodology and applications of advanced estimation theory using Kalman filter and other estimators, described and illustrated in this book, will also help in stimulating expanded activities and in achieving a significant advancement of water science and engineering.

Many natural processes including those in hydraulics, hydrology, water resources, environmental quality studies, and other fields in geophysics are highly uncertain owing to their inherent randomness and/or man's ignorance. Modeling, simulation and estimation of these processes involve the use of: (a) a mathematical system model, (b) an observation (experimental, sampling, or monitoring, etc.) model, and (c) an estimation method that combines (a) and (b). Because of proven advantages of Kalman filtering over other estimation methods in many problems in other fields, water scientists and engineers around the world have begun to try it out in their fields. A considerable number of reports and papers have been published. While a group of investigators are enthusiastic about applications of the Kalman filtering theory and technique, a large number of water scientists are still unaware of the filter. The knowledge of Kalman filter should provide water scientists with a desirable, alternative tool in studying various problems, such as system modeling, parameter estimation, and design of experimental or monitoring systems, etc.

This book consists of lectures and papers presented at the American Geophysical Union Chapman Conference on Applications of Kalman Filtering Theory and Technique to Hydrology, Hydraulics and Water Resources, held at the University of Pittsburgh, Pittsburgh, Pennsylvania, May 22-24, 1978. The objective of the Conference was to give the Kalman filter a significant amount of exposure to water scientists by: (1) giving an introduction to the basic Kalman filtering theory and technique, along with other estimation techniques, (2) to identify and illustrate cases using Kalman filtering in hydrology, hydraulics, water resources, and (3) to determine directions of future study and to explore areas where applications of Kalman filter are most effective. The Conference provided an opportunity for participants to exchange views on applications of Kalman filter to specific problems or subjects.

As shown in the table of contents, this book covers Kalman filter and other estimators, along with applications of Kalman filter to a broad spectrum of subjects in hydrology, hydraulics, and water resources, such as design of experimental or monitoring systems, rainfall-runoff systems studies, streamflow modeling and forecasting, hydraulics of flow and other transport processes in streams and rivers, water quality studies, groundwater problems, and other areas of water resources and geophysics.

My proposal to hold the Conference received encouragement from many people in the field, especially Nicholas C. Matalas of the U.S. Geological Survey, President of AGU Hydrology Section. Because of space limitation I cannot list all other persons here, but I wish to mention that other members of the Organizing Committee, Joel I. Abrams (University of Pittsburgh), Rafael L. Bras (Massachusetts Institute of Technology), and C. C. Li (University of Pittsburgh) were among them.

The Organizing Committee handled the technical program, while the American Geophysical Union (A. F. Spilhaus, Executive Director; Susan Poling, Meetings Administrator) handled all other aspects (e.g., financial, advertising, mailing, and registration, etc.) of the Conference. The University of Pittsburgh hosted the Conference, provided Conference facilities and gave other necessary support. The National Science Foundation and the U.S. Geological Survey approved proposals submitted by AGU to provide partial financial support to the Conference.

Success of the Conference can be measured in part by the quality of participants and papers presented. Great interest and enthusiasm were evident throughout the Conference. I would like to thank all the participants for their interest and enthusiasm that assured success of the Conference. I am also indebted to other people. The efficient typing and other assistance of Mary Hall was instrumental in getting the Conference organizing work and the book editing to progress smoothly.

Chao-Lin Chiu
Pittsburgh, Pennsylvania
May, 1978

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EVOLUTION OF ANALYTICAL METHODS IN HYDRAULICS AND HYDROLOGY

EVOLUTION OF ANALYTICAL METHODS IN HYDAULICS

By

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SYNOPSIS

The development of analytical hydraulics is traced and the turning points occasioned by some representative milestone contributions are examined in some detail. The impact of the modern computer on the subject is discussed and the future courses of hydraulic analysis and experimentation are projected.

Over the years I have given many papers reporting detailed results of particular investigations at technical conferences. However, this is only the second time I have presented a "theme" or "keynote" lecture. The cynical French moralist Francois de la Rochefoucald claimed that when, with advancing years, our appetites desert us, we flatter ourselves that we have conquered our vices. Perhaps it is equally true that when one no longer can do creative scientific or engineering work, he flatters himself that he has become wise enough to be a generalist - a term which too often is synonymous with superficialist. I am not yet, however, quite to the age beyond which it is prohibitively hazardous to experiment indiscriminately with the Commandments, and so perhaps still have one or two research papers in me. They are not, however, for delivery at this particular Conference. The other general lecture, to which I just referred, was the opening day luncheon talk at the 1971 International Symposium on Stochastic Hydraulics, which also was organized by Professor Chiu and held at the University of Pittsburgh. I long have favored small conferences on specialized topics attended by active contributors to the subject. On the other hand, I also have been disturbed by overly zealous missionaries for any new development, be it fluid amplifers, nuclear power, stochastic hydraulics, or even Kalman filtering. It appears to me that the zealots tend to do their subject at a disservice by overselling it, and thereby promoting expectations that cannot be fulfilled. It was this concern, no doubt, which prompted me to state the following in my talk at the Symposium on Stochastic Hydraulics:

> "Stochastic tools and probabilistic methods in hydraulics (as in most other fields of science and engineering) are a crutch, a methodology born of our ignorance and necessitated by our incomplete knowledge about and inadequate information on our subject."

I had hoped, by means of this and similar statements, to promote some critical discussion of the limitations and proper role of stochastic methods in hydraulics, but among the champions of stochasticism present at that earlier Conference I succeeded primarily in provoking discussion of the limitations of my sanity. In any event, that lecture became the first of a series I prepared; others are entitled: "The Salutary Effects of Strong Drink", for presentation at temperance meetings; "Strong Drink: The Universal Menace to Mankind", for delivery at distillers' conventions; and "Deterministic Hydraulics - a Trivial Case of Stochastics (p=1)", for presentation at conferences on traditional approaches in hydraulics. In my remarks today, however, I shall limit myself to discussion of some long-time trends in analytical hydraulics rather than attempting to throw down the gauntlet concerning filtering techniques. Perhaps I am mellowing with age, or perhaps I have been so impressed by the usefulness and power of filtering methods that I am hesitant to be critical of them.

As I have undergone the evolution I referred to above, from specialist to superficialist, it has occurred to me that all aspects of our personal and professional lives are heavily influenced by - indeed, governed by - conservation laws. Everyone here is knowledgeable of the many conservation principles that are outgrowths of physical laws; these include conservation of mass, conservation of energy, conservation of linear and angular momentum, conservation of electrical charge, conservation of magnetic flux, and so on; the list is nearly endless. Equally as rigorous, or so it seems to me, are the nonphysical conservation laws. These include conservation of discipline, which states that the sum of self-discipline and imposed discipline is a constant for all individuals. If a person does not discipline himself, he will be disciplined by society or circumstances. Another example is conservation of joy, or if you prefer the negative view, conservation of anguish. The principle states that an individual's capacity for happiness minus his sensitivity to sorrow equals zero. The poet Gibran gave nonmathematical expression to this thought when he wrote: "The deeper sorrow carves into your being, the more joy you can contain". Yet another is the conservation of ratio of nobility of cause to magnitude of attendent abuses. History presents many elegant examples of this principle, including the Crusades, the Russian Revolution, and our own save-the-environment movement. A few other examples of nonphysical conservation principles are as follows: The ratio of the intensity of hunger to the joy of eating subsequently is a constant - provided the hunger does not become so intense that it results in starvation. (Cervantes Sancho Panza said it differently: "Hunger is the best sauce; that is why the poor always have a good appetite.") The ratio of the degree of mechanical sophistication of a kitchen to the tastiness of the food that comes out of it is a constant: (In other words, as kitchens have improved, our diets have deteriorated.) Over the years the quality of sound-reproduction equipment multiplied by the beauty of the popular music of the day has remained constant (and I note, with some dismay in this respect, that hi-fi sets are still being improved steadily.) Perhaps the ultimate conservation law is the constancy of death rate; despite the many life-prolonging advances of medical science and the ever changing threats from disease, weapons, accidents, and pollution, the death rate has remained constant: one per person.

I now would like to conjecture that there is a conservation law for analytical hydraulics, and examine its impact on the evolution of the subject and its implications for future developments in the field. The conservation principle may be stated as follows: The product of computer capability and the required exactness of formulation is invariant with time. A corollary is that as the power to compute becomes greater, the value of closed form solutions diminishes. (A cynical observer might be reminded here of Gresham's law for money.) This principle came to mind in the course of a conversation I had some fifteen years ago with a very well known MIT applied mathematician, whose specialty then was analysis of structural plates and shells. He stated that he felt that he was being made obsolete by the modern computer and problem-oriented computer languages. Even then, computer-based analysis of plates and shells had reached the point that an engineer could determine practically everything he needed to know about the stresses, moments, deflections, etc., for a shell by feeding into a computer the shell geometry, loading, and physical properties. He really had no need for approximate or even exact solutions, and even if they existed for his particular problem it likely would be faster or more convenient to obtain an approximate numerical solution by computational means via a computer. Indeed, the time and expense of even finding whether a traditional type of solution had been developed for the problem at hand well might exceed that of obtaining the computer solution. Needless to say, the impact of the computer on experimental determination of shell stresses and deflections (largely by means of small scale laboratory models) has been equally as devastating. The impact of computer analysis on hydraulics also has been revolutionary, but unlike the case of structural mechanics appears to have heightened the need for improved analytical techniques and experimentation; the reasons for this are discussed below.

Let us now trace, at least sketchily, the evolution of hydraulic analysis by means of certain examples from engineering hydraulics that were milestones in one sense or another. The foundation of hydrodynamics is generally acknowledged to have been established by Daniel Bernoulli, in his 1738 treatise, Hydrodynamica. There followed thereafter development of various approximate equations of fluid motion, and application of them to a variety of engineering problems, notably those concerned with turbines and windmills, water supply systems, and then as now, weapons. It was Euler, however, who finally established the science of hydrodynamics with the publication, in 1755, of his series of three now famous papers. The whole business of modern hydraulic analysis can be said really to have begun, at least in a defensibly rigorous and complete way, with the formulation of the full equations of motion - the Navier-Stokes equations - in 1822 (Navier) and 1845 (Stokes). As often has been the case, this new scientific development was not exploited by engineers for several years following its presentation. (Here it is worth noting that the elapsed time between scientific development and engineering utilization is steadily decreasing; the present Conference attests to this point.)

The full equations of fluid motion do not admit to general solution as yet, even by approximate numerical techniques, for reasons of nonlinearity and instability that are well known to this audience. Therefore it is not surprising that the early successful applications of the Navier-Stokes equations were to low Reynolds number flows which are sufficiently slow and weakly nonuniform that the instability and nonlinear features of the equations do not arise. These are the so-called creeping flow problems, for which the inertial terms can be deleted from the momentum equation, and uniform flows for which the convective inertial terms are zero. A beautiful example of the exploitation of the simplified equations of fluid motion is Reynolds' analysis of bearing lubrication, which was published in 1886 and practically laid the problem to rest from an engineering point of view, except for problems arising from cavitation and in bearings in which the Reynolds number becomes very high. Other early applications of the simplified equations of motion are Stokes' solution for parallel flow past a sphere, published in 1851, which gives an adequate prediction of the drag on a sphere in creeping motion but fails to predict the velocity field; and Oseen's improvement of Stokes' solution (1910), which takes an approximate accounting of the convective terms and thereby obtains an improved description of the velocity distribution away from the sphere surface. The Oseen approximation since has been used in a variety of viscous flow problems, and further improved upon, notably by Carrier. When one considers these and other triumphs of early analytical hydraulics it would appear that the analysts were succeeding only with those problems for which the equations of motion and continuity could be simplified to the extent they could be solved by means of classical mathematical techniques.

A very useful application of fully rigorous and generally applicable fluid-flow equations is ordering of the contributions of different constituent effects. Thus, by nondimensionalizing the Navier-Stokes, continuity, and energy equations, one can determine the relative contributions of inertial, viscous, gravity, pressure, and compressibility effects in a particular problem. Moreover, one can order the magnitudes of the components of the individual inertial, stress, and other terms. Thus even though the equations may not be solvable for the problem at hand, they can be used to appraise the importance of different effects. This is precisely what was done by Prandtl in the development of another milestone contribution to analytical hydraulics - boundary layer theory - which was presented in 1904. Here again the principal contribution was simplifying the equations to a form which was tractable with the available analytical and computational means. Although boundary layer theory now is universally acclaimed, it was not widely appreciated for many years after it was first presented. During the first decade of its existence (1904-14) there were no more than ten publications on the subject, and all of these came from Prandtl and his students. Not until the second decade (1914-24) were there contributions from outside Göttingen, and not until the third (1924-34) from beyond the borders of Germany (1). The simplest nontrivial boundary layer problem one can conceive is a developing boundary layer on a flat plate in a flow with zero pressure gradient. Even this relatively very simple problem gives rise to a third-order nonlinear ordinary differential equation which Blasius solved by means of a series expansion. Boundary layer theory gave birth to practically a whole industry which was concerned with development of clever means for solving the boundary layer equations. Names associated with now-classical contributions to the subject include von Kármán, Polhausen, von Mises, Howarth, Tolmien, and Schlichting, to mention but a few. Each of these developed one or another solution by a particular method, and their fundamental contributions often were as much to applied mathematics as to fluid mechanics. In other words, the factor limiting progress in the field was inadequate means for solving the equations. Prandtl recognized this, of course, and undertook to develop what later was called a "differential analyzer" for solving the boundary layer equations; his gadget is depicted in figure 1 (2). It is also noteworthy that the needs being presented by boundary layer theory (and other developments) were largely responsible for development of wide new subjects in mathematics; an example is similarity techniques for solving differential equations, which came out of boundary layer theory. Indeed, as Van Dyke (3) notes in his book on perturbation methods, fluid mechanicians pioneered the solution of nonlinear partial differential equations.

Here it is interesting to speculate how boundary layer theory might have evolved if development of computers had progressed faster than fluid mechanics. First, it is doubtful that boundary layer theory, as such, would have evolved at all. Instead, it is likely that the analysts would have undertaken to solve the full equations by launching computer programs, and might have discovered in the process that some terms are negligibly small, or that some had to be dropped in order to obtain convergence or stability. It is entirely possible that some of the classical contributions to the understanding of fluid motion (for example, Tollmien-Schlichting stability) which came about from detailed study of the boundary layer equations might have been bypassed, and this would have been unfortunate. Development of the byways of applied mathematics that are outgrowths of boundary layer theory would have been delayed, and perhaps overlooked. On the positive side of the ledger, practical problems related to fluid drag and flow around bodies likely would have been dealt with more successfully and earlier. Unquestionably, attention would have focused upon numerical rather than analytical means, and it is to these that hydrodynamicists would have made their mathematical contributions. When one recalls, however, that the numerical methodologies in this field make extensive use of the results of analytical treatment of the boundary layer equations, he is led to conclude that the progress of the computer-based approach would have been impeded by lack of these results, and overall I doubt that we would be significantly ahead of where we are now.

The evolution of the fluid-drag problem I have described was paralleled in several other fields of hydraulics, including potential flow theory and water waves (both of which predate boundary layer theory), flow in porous media, water-hammer analysis, flow in open channels, and many other free surface problems (e.g., supercavitation, bubble dynamics, etc.) In each of these a fairly rigorous set of descriptive equations was available, but these were too difficult to solve by classical means. Therefore, before the advent of the computer the approach was to simplify the equations until they became solvable with the available mathematical tools. In the process many new insights arose and several mathematical approaches evolved; many of these proved to be invaluable when the age of computer analysis finally arrived. In other fluid engineering problems, including sediment transporting river flows, rainfall-runoff, and dispersion, development of rigorous and generally applicable formulations were not forthcoming, and the analysts had to continue to rely on approximate analyses of particular aspects of the problem.

In summary, until the dawn of computer-based analytical hydraulics, success in analysis of fluids problems was heavily dependent on a rigorous yet sufficiently simple formulation for all aspects of the problem. In problems for which the equations of motion could not be rigorously formulated (e.g., multi-phase flows), or the boundary conditions were not well understood (e.g., flow past erodible boundaries), or for which the inputs could not be concisely described (e.g., rainfall-runoff), analytical success was modest, at best. In terms of the conservation principle, analytical success was proportional to the adequacy and the conciseness of the formulation and the fidelity with which it portrayed the physical processes.

The modern electronic computer came of age in the 1950's, and during the 1960's computers became generally available to scientists and engineers and much more readily usable as the software was improved and man-machine communication was simplified. Both the hardware and software have continued to improve at paces that can be described only as revolutionary; and the revolution shows no sign of ending, or even slowing down. The effects on practically every facet of hydraulics has been beyond what could have been foreseen by anyone except a true visionary. Exact solutions are now evaluated on small, personal, programmable calculators. Problems for which only approximate solutions of the simplified equations were available previously are now solved "exactly", by applying finite-difference or finite-element methods to the full equations. And problems which only a few years ago were treated analytically with little if any success now are proving tractable. It is this last category I now will discuss.

Computer analysis clearly has greatly relaxed the requirement for concise, rigorous formulation of fluids problems, and the dependence on traditional mathematical techniques. The second point already has been touched upon, and really is self-evident. Let us consider two classes of problems to examine how computers have relaxed in some ways the requirements of exactness and conciseness; first, calculation of turbulent flows; and second calculation of flows that respond in complex ways to random, or at least widely fluctuating, inputs.

One of the major stumbling blocks in the calculation of turbulent flows has been the so-called closure problem; that is, formulation of the relation between the turbulent momentum-exchange or mixing coefficient and other properties of the flow field, many of which are themselves dependent on the turbulent exchange processes. The difficulty is that the reasonably manageable closures (e.g., Boussinesq's eddy viscosity; Prandtl's mixing length; Taylor's vorticity transfer; Bradshaw's stress transport, etc.) proved to be reliable only in relatively simple situations. With the passage of time the so-called turbulence models evolved from algebraic equations to individual differential equations to sets of differential equations. Some of the more complex of these involve in excess of twenty differential equations. The two-differential-equation models contain expressions for the kinetic energy and length scale of turbulence, and the higher multi-order models describe still other features (production, dissipation, diffusion, etc.) of the turbulence and its manifestations. Needless to say, the resulting formulations can be solved only by means of computers. In a sense, a compact formulation for the turbulence is replaced by a somewhat loose description of various components of the turbulent processes, with heavy reliance placed on idealized experiments to evaluate the "tuning knob" constants the models contain. During the several years I have been observing